MULTICRITERIA POWER ENGINEERING PROBLEMS AND FUZZY SET BASED METHODS OF THEIR SOLUTION

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The results of research into the use of models and methods of multicriteria decision making in a fuzzy environment for solving power engineering problems are presented. Two general classes of models related to multiobjective (<X, M> models) and multiattribute (<X, R> models) problems as well as methods of their analysis based on the application of the Bellman-Zadeh approach to decision making in a fuzzy environment and techniques of fuzzy preference modeling, respectively, are briefly considered. A review of the authors' results associated with the application of these models and methods for solving diverse types of problems of power system and subsystems planning, operation, and control is presented. The recent results on the use of <X, M> and <X, R> models and methods of their analysis for the allocation of reactive power sources in distribution systems and for the prioritization in maintenance planning in distribution systems, respectively, are considered. References 35, table 1.
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Introduction. Diverse types of uncertainty [1] are commonly encountered in a wide range of optimization and decision making problems related to planning, operation, and control of power systems and subsystems. The incorporation of the uncertainty factor in the construction of mathematical models serves for increasing their adequacy and, as a result, the credibility and factual efficiency of decisions based on their analysis. Investigations of recent years show the benefits of applying fuzzy set theory [2, 3] to deal with diverse types of uncertainty. Its use offers advantages of both a fundamental nature (the possibility of obtaining more effective, less "cautious" solutions as well as considering simultaneously different types of uncertainty) and a computational character [4, 5].

The uncertainty of goals is an essential kind of uncertainty related to a multicriteria nature of many power engineering problems. Issues related to the classification of situations which need the use of a multicriteria approach are discussed in [6]. However, from the substantial point of view, it is possible to identify the following classes of situations [5, 7] which need the application of a multicriteria approach:

- problems whose solution consequences cannot be estimated on the basis of a single criterion: these problems are associated with the analysis of models which include economic as well as physical indices (when alternatives cannot be reduced to comparable form) and also by the need to consider indices whose cost estimations are hampered;
- problems those may be solved on the basis of a single criterion or several criteria. However, if the uncertainty of information does not permit one to obtain unique solutions, it is possible to solve these problems, applying additional criteria, including criteria of qualitative character (whose utilization is based on knowledge, experience, and intuition of involved experts).

Therefore, two classes of models (so-called $< X, M >$ models and $< X, R >$ models) may be constructed [5, 7]. The present paper briefly describes these models as well as methods of their analysis (based on the use of the Bellman-Zadeh approach to decision making in a fuzzy environment and the application of techniques involving fuzzy preference modeling, respectively).

The paper includes a review of the authors' results related to using $<X, M>$ and $<X, R>$ models and methods of their analysis to solve diverse problems of power systems and subsystems. The recent results associated with their utilization for the allocation of reactive power sources in distribution systems and the prioritization in maintenance planning in distribution systems are considered in more detail.

<X, M> Models, Their Analysis, and Applications. When analyzing $<X, M>$ models, a vector of objective functions $F(X) = \{F_1(X),...,F_q(X)\}$ is considered, and the problem consists of optimizing all of them, i.e.

$$F_p(X) \to \underset{X \in L}{\text{extr}}, \qquad p = 1,...,q , \qquad (1)$$

where L is a feasible region in R^n.

The first step in solving the problem (1) is associated with determining a set of Pareto solutions $\Omega \subseteq L$ [8]. This step is useful; however it does not permit one to obtain unique solutions. It is necessary to choose a particular Pareto solution applying information of a decision maker (DM). There are three approaches to its use [5]: a priori, a posteriori, and adaptive. The most preferable approach is the adaptive one. When using this approach, the procedure of successive improving the solution quality is realized as a transition from $X_\alpha^0 \in \Omega \subset L$ to $X_{\alpha+1}^0 \in \Omega \subset L$ with considering information I_α of DM (α is a number step of the decision making process). The solution search may be presented as

$$X_1^0, F(X_1^0) \xrightarrow{I_1} ... \xrightarrow{I_{\alpha-1}} X_\alpha^0, F(X_\alpha^0) \xrightarrow{I_\alpha} ... \xrightarrow{I_{\omega-1}} X_\omega^0, F(X_\omega^0). \qquad (2)$$

The process (2) serves for two types of adaptation: computer to preferences of DM and DM to the problem. The first type of adaptation is based on information of DM. The second type is realized as a result of carrying out steps $X_\alpha^0, F(X_\alpha^0) \xrightarrow{I_\alpha} X_{\alpha+1}^0, F(X_{\alpha+1}^0)$ which permit DM to understand the correlation between own needs and possibilities of their satisfaction by the model (1).

When analyzing multiobjective problems, it is necessary to solve some questions related to normalizing objective functions, selecting principles of optimality, and considering priorities of objectives. Their solution and, therefore, development of multiobjective methods is carried out in several directions [9, 10]. Without discussing them, it is necessary to point out that an important question in multiobjective analysis is the solution quality. It is considered high if levels of satisfying objective functions are equal or close to each other (harmonious solutions) if we do not differentiate their importance [5, 12]. From this point of view, it should be recorded the validity and advisability of the direction related to the principle of guaranteed result. Other directions may lead to solutions with high levels of satisfying some objectives that is reached by low levels of satisfying other objectives (for instance [11, 12]).

The lack of clarity in the concept of "optimal solution" is the basic methodological complexity in solving multiobjective problems. When applying the Bellman-Zadeh approach to decision making in a fuzzy environment [13], this concept is defined with reasonable validity: the maximum degree of implementing goals serves as a criterion of optimality. This conforms to the principle of guaranteed result and provides constructive lines in obtaining harmonious solutions. The use of the Bellman-Zadeh approach permits one to realize an effective (from the computational standpoint) as well as rigorous (from the standpoint of obtaining solutions $X^0 \in \Omega \subseteq L$) method of analyzing multicriteria models [5, 7]. Its use also allows one to consider indices, criteria, and constraints of qualitative character.

When using the Bellman-Zadeh approach, each objective function $F_p(X)$ is replaced by a fuzzy set

$$A_p = \{X, \mu_{A_p}(X)\}, \qquad X \in L, \qquad p = 1,...,q , \qquad (3)$$

where $\mu_{A_p}(X)$ is a membership function of A_p [2,3]. A fuzzy solution D with setting up the fuzzy sets (3) is obtained as a result of the intersection $D = \bigcap_{p=1}^{q} A_p$ with a membership function

$$\mu_D(X) = \min_{p=1,..,q} \mu_{A_p}(X), \quad X \in L . \qquad (4)$$

Its use permits one to obtain the solution proving the maximum degree

$$\max \mu_D(X) = \max_{X \in L} \min_{p=1,..,q} \mu_{A_p}(X) \qquad (5)$$

of belonging to the fuzzy solution D and reduced the problem (1) to

$$X^0 = \arg\max_{X \in L} \min_{p=1,...,q} \mu_{A_p}(X). \tag{6}$$

The use of (6) requires to build membership functions $\mu_{A_p}(X), \quad p=1,...,q$ reflecting a degree of achieving "own" optima by $F_p(X), X \in L$, $p=1,...,q$. This condition is satisfied by the use of membership functions

$$\mu_{A_p}(X) = \left[\frac{F_p(X) - \min\limits_{X \in L} F_p(X)}{\max\limits_{X \in L} F_p(X) - \min\limits_{X \in L} F_p(X)} \right]^{\lambda_p} \tag{7}$$

for maximized objective functions or by the use of membership functions

$$\mu_{A_p}(X) = \left[\frac{\max F_p(X) - F_p(X)}{\max\limits_{X \in L} F_p(X) - \min\limits_{X \in L} F_p(X)} \right]^{\lambda_p} \tag{8}$$

for minimized objective functions. In (7) and (8), $\lambda_p, \quad p=1,...,q$ are importance factors for objective functions.

The construction of (7) or (8) demands the solution of the following problems:

$$F_p(X) \to \min_{X \in L}, \tag{9}$$

$$F_p(X) \to \max_{X \in L}. \tag{10}$$

Thus, the solution of the problem (1) demands analysis of ($2q+1$) monocriteria problems (9), (10), and (5), respectively.

Since X^0 is to belong to $\Omega \subseteq L$, it is necessary to build

$$\bar{\mu}_D(X) = \bigwedge_{p=1}^{q} \mu_{A_p}(X) \wedge \mu_{\pi}(X) = \min \left\{ \min_{p=1,..,q} \mu_{A_p}(X), \mu_{\pi}(X) \right\}, \tag{11}$$

where $\mu_{\pi}(X) = 1$ if $X \in \Omega$ and $\mu_{\pi}(X) = 0$ if $X \notin \Omega$.

Finally, the existence of additional conditions (indices, criteria or constraints) of qualitative character, defined by linguistic variables [2, 3], reduces (6) to

$$X^0 = \arg\max_{X \in L} \min_{p=1,...,q+s} \mu_{A_p}(X), \tag{12}$$

where $\mu_{A_p}(X), X \in L, p = q+1,...,s$ are membership functions of fuzzy values of linguistic variables which reflect these additional conditions.

Power and Energy Shortage Allocation as Applied to Load Management. The statement of the problem of multicriteria allocation of resources or their shortages (these problems are equivalent from the general, mathematical, and information points of view) among consumers supposes the possibility to use diverse types of objective functions (linear, fractional, quadratic, etc. [14]) in (1) defined in a feasible region

$$L = \{X \in R^n \mid 0 \le x_i \le A_i, \qquad \sum_{i=1}^{n} x_i = A\}, \tag{14}$$

where $X = (x_1,...,x_n)$ is a vector of limitations (for definiteness) for consumers, A_i is a permissible value of limitation for the i-th consumer, A is a total value of limitations for all involved consumers.

A general scheme for solving the problem formalized within the framework of the model (1), (14) is given in [5]. It has served for developing an adaptive interactive decision making system AIDMS1.

The problems of power and energy shortage allocation are of a fundamental importance in a family of load management problems. They are to be analyzed not only from the economical and technological points of view, but also from social and ecological points of view. When solving these problems, it is necessary to account for considerations of creating incentive influences for consumers. Taking this into account, methods of power and energy shortage allocation, based on fundamental principles of allocating resources have drawbacks [14, 15]. Their overcoming is possible by formulating and solving the problems within the framework of multiobjective models. This permits one to consider and to minimize diverse

consequences of shortage allocation and to create incentive influences for consumers (the first attempts to apply the models (1), (14) were associated with the necessity to overcome the situation in the USSR energy sector emerged after the Chernobyl accident).

A substantial analysis of problems of power and energy shortage allocation, systems of economics management, and real, readily available reported and planned information has permitted the construction of a general set of goals (the first version of this set was proposed in [16]. The general set of goals is directed at decreasing diverse negative consequences for consumers and creating incentive influences for them. This set is universal because can serve as the basis for models at different levels of load management hierarchy by aggregation of information and posterior decomposition of the problems. Examples of solving power and shortage allocation problems are given in [12, 17].

Multicriteria Power System Operation. The use of the results described above permits one to apply the multicriteria approach to power system operation to realize dispatch on several objectives. In [17], this is exemplified by a case study with the standard IEEE 30-bus system, where three objective functions are to be minimized: losses, sulfur oxide emissions, and nitrogen oxide emissions.

The consideration of the objective functions of emissions creates no difficulties at all. However, the presentation of the objective function of losses gives rise to difficulties. One possible workaround is the application of sequential multicriteria optimization procedures [5] with using sensitivity models reflecting the loss changes at each optimization step [17, 18].

Optimization of Network Configuration and Voltage Control in Distribution Systems. The most important functions of Distribution Management Systems are implemented on the basis of solving problems of optimal network configuration (network reconfiguration) and optimal voltage control in distribution systems.

The problems of optimal network configuration are associated with altering network topology by changing locations of their disconnections. Works dedicated to the analysis of such problems seem to "compete" for achieving "more optimal" solutions. However, this competition is not convincing, considering that a combination of the information uncertainty and relative stability of optimal solutions produces uncertainty regions of indistinguishable solutions. Besides, these works do not permit one to consider a power system reaction (the results of [18], for instance, allow one to overcome this drawback on the basis of constructing so-called functionally oriented equivalents). Furthermore, the majority of results in this field is directed at the solution of problems which are defined under a monocriteria statement (usually, power or energy losses are minimized). However, reconfiguration problems are multicriteria in nature because they have an impact on reliability, service quality, and economical feasibility of power supply [19]. Considering this, a developed computing system DNOS permits one to consider and to minimize objective functions of power losses, energy losses, system average interruption frequency index (SAIFI), system average interruption duration index (SAIDI), undersupply energy, poor quality energy consumption (consumption of energy outside of permissible voltage limits), and integrated overload of network elements in diverse combinations. Examples of solving reconfiguration problems are given in [17]. They show that the use of the multicriteria approach leads to the harmonious solution with small deviations from locally optimal solutions.

The techniques for optimal voltage control, which are implemented within the framework of a computing system VCOS, are directed at minimizing poor quality energy consumption. However, in accordance with a situational hierarchy, it may be necessary to implement an energetically efficient control, which considers the static characteristics of loads. Thus, it becomes necessary to speak about the second statement directed at the minimization of poor quality energy consumption and the reduction of peak load and/or energy consumption. This consideration has served for implementing another function of VCOS. Examples of its applications are given in [19].

Power Compensation in Distribution Systems. Traditionally, problems of reactive power compensation in distribution systems are associated with the selection of the locations, sizes, and types of capacitors to minimize the objective function of economical nature, while the constraints on upper and lower voltage limits at different load levels are satisfied.

Considering the discrete nature of such problems, the generalized algorithms of discrete optimization [20, 21] can be applied for their solution. These algorithms combine formal procedures (related to the method of normalized functions) and informal procedures (based on greedy heuristics) and allow one to obtain quasi-optimal solutions after a small number of steps, overcoming the NP-completeness. They do not require an analytical description of objective functions and constraints, ensuring the flexibility and the possibility to correctly reflect diverse types of initial data using so called discrete sequences [20, 21]. However, our experience in solving problems of reactive power compensation shows that the necessity to

simultaneously observe constraints on upper and lower voltage limits at different buses creates essential obstacles. It is not uncommon to face situations when these constraints induce an empty feasible region. These obstacles can be avoided by minimizing the objective function of an economical nature as well as the objective function which reflects a volume of poor quality energy consumption. Besides, if the problem is associated with the determination of capacitor types (fixed or switched), the quantity of objectives should be increased.

Taking the above into account, the algorithms [20, 21] have been modified to solve discrete multicriteria problems considering the results presented above.

The computing platform EPODIAN was developed to solve reactive power compensation problems in monocriteria and multicriteria settings for large-scale distribution networks.

Solution results

Solution	Economic objective function (R$·10³)	Poor quality energy consumption (MWh/year)	Total installed nominal reactive power (kVAr)
I	708	826	600
A	543	472	630
B	661	384	1215
C	568	396	900

As an example, Table demonstrates the results of the application of EPODIAN for the allocation of reactive power sources in a distribution network 13.8/0.22 kV of one of substations of the Energy Company of Minas Gerais (CEMIG). This network includes 2 feeders with more than 5000 consumers connected to them. The total length of feeders is 729 km. They are modeled by 9660 electrical nodes. In Table, I is an initial state, A is a monocriteria solution minimizing the objective function of the economical nature, B is a monocriteria solution minimizing the objective function which reflects a volume of poor quality energy consumption, and C is a multicriteria solution providing a compromise between the solutions A and B. All solutions were obtained less than 30 s on a conventional dual-core desktop PC.

Although the presented results do not take into account the uncertainty of initial information (associated with power demand curves, energy tariffs, etc.), EPODIAN allows their consideration into a general scheme of multicriteria analysis under information uncertainty [5, 22] with evaluation of particular (monocriteria) and aggregated (multicriteria) risks in multiple scenarios.

<X, R> Models, Their Analysis, and Applications. Some criteria considered in the process of decision making can be reflected only in a qualitative (semantic, contextual) manner. For instance, concepts such as "flexibility of development", "investment utility", etc. are based on experience, knowledge, and intuition of experts. This generates the necessity of the evaluation, comparison, choice, prioritization and/or ordering of alternatives in strong correspondence with the preferences of DM.

The application of fuzzy set theory to preference modeling [23] provides a flexible framework for dealing with "fuzziness" of the assessments which incorporates human consistency into the applied models. Next we briefly discuss three basic techniques for processing fuzzy preferences.

Assume we are given a set X of alternatives which are to be examined according to q criteria of quantitative and/or qualitative nature. The decision making problem can be represented as a pair $<X, R>$ where $R = \{R_1,...,R_q\}$ is a vector of fuzzy preference relations [5, 24]. In this case, we have

$$R_p = [X \times X, \mu_{R_p}(X_k, X_l)], \qquad p = 1,...,q, \qquad X_k, X_l \in X, \qquad (15)$$

where $\mu_{R_p}(X_k, X_l)$ is a membership function of nonstrict fuzzy preference relation, which represents the degree to which X_k weakly dominates X_l, i.e., the degree to which X_k is at least as good as X_l, according to the *p*-th criterion.

The availability of fuzzy or linguistic estimates $\tilde{F}_p(X_k)$, $p = 1,...,q$, $X_k \in X$ with membership functions $\mu[f_p(X_k)]$, $p = 1,...,q, X_k \in X$ permits one [5, 24] to construct elements of R_p, $p = 1,...,q$ as follows:

$$\mu_{R_p}(X_k, X_l) = \sup_{\substack{X_k, X_l \in X \\ f_p(X_k) \leq f_p(X_l)}} \min\{\mu[f_p(X_k)], \mu[f_p(X_l)]\}, \qquad (16)$$

$$\mu_{R_p}(X_l, X_k) = \sup_{\substack{X_k, X_l \in X \\ f_p(X_l) \leq f_p(X_k)}} \min\{\mu[f_p(X_k)], \mu[f_p(X_l)]\} \qquad (17)$$

if the *p*-th criterion is associated with minimization. If it is associated with maximization, then the domain conditions of suprema in (23) and (24) should be changed to $f_p(X_k) \geq f_p(X_l)$ and $f_p(X_l) \geq f_p(X_k)$, respectively.

However, fuzzy preference relations are not a unique form of preference representation. For instance, the authors of [25] indicate eight formats which can be used to establish preferences among analyzed alternatives. It is natural that application of multiple preference formats demands a subsequent conversion to a unique format which can be processed and analyzed. Considering the advantages and rationality of the use of fuzzy preference relations for this objective, the results of [5], for instance, permit one to construct so-called transformation functions to convert different formats to fuzzy preference relations.

Let us consider the situation of setting up a single nonstrict preference relation R defined as follows:
- $(X_k, X_l) \in R$ means "X_k is not worse than X_l",
- $(X_l, X_k) \in R$ means "X_l is not worse than X_k",
- $(X_k, X_l) \notin R$ or $(X_l, X_k) \notin R$ means "X_k and X_l are not comparable".

The nonstrict fuzzy preference relation R can be represented [5, 26] by a strict fuzzy preference relation R^S and an indifference relation R^I: $R = R^S \bigcup R^I$. The strict preference relation R^S is constituted by all pairs of alternatives that satisfy the conditions $(X_k, X_l) \in R$ and $(X_l, X_k) \notin R$. If $(X_k, X_l) \in R^S$, it can be said that X_k is strictly better than X_l (or X_k dominates X_l). The indifference relation is constituted by all pairs of alternatives that simultaneously satisfy the conditions: $(X_k, X_l) \in R$ and $(X_l, X_k) \in R$. If $(X_k, X_l) \in R^I$, it can be said that X_k is indifferent to X_l.

With the use of the inverse relation R^{-1} the strict preference relation R^S can be defined as $R^S = R \setminus R^{-1}$ with

$$\mu_R^S(X_k, X_l) = \max \{\mu_R(X_k, X_l) - \mu_R(X_l, X_k),\, 0\}. \tag{18}$$

The expression (18) serves as the basis for the choice procedure which has been studied by many authors (its properties as well as its axiomatic characterization are discussed, for instance, in [27, 28]).

If $\mu_R^S(X_l, X_k)$ is the membership function of the fuzzy set of all X_k which are strictly dominated by X_l, then the complementary relation $1 - \mu_R^S(X_l, X_k)$ gives the fuzzy set of alternatives which are not dominated by other alternatives from X. Therefore, in order to meet the set of alternatives from X that are not dominated by other alternatives, it is sufficient to find the intersection of all $1 - \mu_R^S(X_l, X_k)$, $X_k \in X$ on all $X_l \in X$ [26]. This intersection is the fuzzy set of nondominated alternatives with a membership function

$$\mu_R^{ND}(X_k) = \inf_{X_l \in X}[1 - \mu_R^S(X_l, X_k)] = 1 - \sup_{X_l \in X} \mu_R^S(X_l, X_k). \tag{19}$$

Considering that a rational criterion for choice is to select a solution providing the highest level of nondominance, one can choose alternatives as follows:

$$X^{ND} = \{X_k^{ND} \mid X_k^{ND} \in X,\, \mu_R^{ND}(X_k^{ND}) = \sup_{X_k \in X} \mu_R^{ND}(X_k)\}. \tag{20}$$

Let us consider the first procedure [26] to deal with multiple criteria, when preferences are modeled as a vector R of fuzzy preference relations. The expressions (18)-(20) are applicable if we take $R = \bigcap_{p=1}^{q} R_p$

with
$$\mu_R(X_k, X_l) = \min_{1 \leq p \leq q} \mu_{R_p}(X_k, X_l), \qquad X_k, X_l \in X. \tag{21}$$

When using (28), X^{ND} fulfils the role of a Pareto set. Its contraction is possible on the basis of distinguishing the importance of each R_p, $p = 1,...,q$ by means of the convolution:

$$\mu_T(X_k, X_l) = \sum_{p=1}^{q} \lambda_p \mu_{R_p}(X_k, X_l), \qquad X_k, X_l \in X, \tag{22}$$

where $\lambda_p \geq 0$, $p = 1,...,q$ are importance factors normalized as

$$\sum_{p=1}^{q} \lambda_p = 1 . \tag{23}$$

The construction of $\mu_T(X_k, X_l)$, $X_k, X_l \in X$ allows one to obtain the membership function $\mu_T^{ND}(X_k)$ of the set of nondominated alternatives according to an expression similar to (25). The intersection

$$\mu^{ND}(X_k) = \min\{\mu_R^{ND}(X_k), \ \mu_T^{ND}(X_k)\}, \qquad X_k \in X \tag{24}$$

provides us with

$$X^{ND} = \{X_k^{ND} \mid X_k^{ND} \in X, \mu^{ND}(X_k^{ND}) = \sup_{X_k \in X} \mu^{ND}(X_k)\} . \tag{25}$$

The expressions (19) and (20) can serve as the basis for the second procedure [5, 24], which has a lexicographic character. It is based on the step-by-step application of criteria for comparing alternatives. The procedure permits one to build a sequence $X^1, X^2, ..., X^q$ so that $X \supseteq X^1 \supseteq X^2 \supseteq ... \supseteq X^q$ by applying the following expressions:

$$\mu_{R_p}^{ND}(X_k) = \inf_{X_l \in X^{p-1}}[1 - \mu_{R_p}^{S}(X_l, X_k)] = 1 - \sup_{X_l \in X^{p-1}} \mu_{R_p}^{S}(X_l, X_k), \qquad p = 1, ..., q, \tag{26}$$

$$X^p = \{X_k^{ND,p} \mid X_k^{ND,p} \in X^{p-1}, \mu_{R_p}^{ND}(X_k^{ND,p}) = \sup_{X_l \in X^{p-1}} \mu_{R_p}^{ND}(X_k)\} . \tag{27}$$

Finally, let us consider the third technique [5, 20]. The application of (19) in the form

$$\mu_{R_p}^{ND}(X_k) = 1 - \sup_{X_l \in X} \mu_{R_p}^{S}(X_l, X_k), \qquad p = 1, ..., q \tag{28}$$

permits one to construct the membership functions of the set of nondominated alternatives for each fuzzy preference relation. The membership functions $\mu_{R_p}^{ND}(X_k)$, $p = 1, ..., q$ play a similar role to the membership functions which replace objective functions $F_p(X)$, $p = 1, ..., q$ in $< X, M >$ model analysis. Therefore, it is possible to construct

$$\mu^{ND}(X_k) = \min_{1 \le p \le q} \mu_{R_p}^{ND}(X_k) \tag{29}$$

to obtain X^{ND}. If it is necessary to distinguish the importance of preference relations, it is possible to extend (36) to

$$\mu^{ND}(X_k) = \min_{1 \le p \le q}[\mu_{R_p}^{ND}(X_k)]^{\lambda_p} . \tag{30}$$

The use of (37) does not require the normalization of λ_p, $p = 1, ..., q$ in the way similar to (23).

The use of the *second procedure* may lead to solutions different from the results obtained on the basis of the *first procedure*. Besides, solutions based on the *first procedure* and the *third procedure* having a single generic basis, may also be different. At the same time, the *third procedure* is preferable from the substantial point of view. In particular, the *first procedure* may lead to the choice of alternatives with unit degree of nondominance, though these alternatives are not the best ones from the point of view of all preference relations. The *third procedure* provides this result only for those alternatives which are the best ones from the point of view of all preference relations. The possibility to obtain different solutions on the basis of different approaches is considered natural, and the choice of an approach is a prerogative of DM.

The presented procedures have been implemented within the framework of an interactive decision making system MDMS1 [5].

The described techniques, as it was indicated above, are considered as the basic ones. They have served for developing other techniques:
- analysis of alternatives with fuzzy ordering of criteria [5, 26];
- analysis of alternatives with the use of the concept of fuzzy majority (based on the application of the so-called OWA operator, originally proposed in [29]) [5];
- analysis of alternatives based on an outranking approach (by a means of a fuzzy version of the PROMETHEE [30]) [5].

The results described above have also served as a basis for developing and improving methods for multicriteria group decision making ([5, 31-33], for instance).

Substation Planning in Power Systems. Among a family of power system expansion problems, the problem of substation planning is of a fundamental importance. The paper [34] discusses how to consider the uncertainty factor in the solution of this problem. In [34], the analysis of a power utility group of substations is carried out to select a solution among three alternatives, considering their total costs with the uncertainty of interest rates modeled as trapezoidal fuzzy numbers [2, 3]. However, as it is shown in [5], this analysis does not give ground to proceed with a convincing decision. One possible way to overcome this difficulty is the application of the well-established approach in decision making practice: the multicriteria analysis of alternatives in a fuzzy environment [5], when the application of additional criteria, including criteria of qualitative character, serves as convincing means to contract the decision uncertainty regions. In [5], the use of this approach is exemplified by analysing the same power utility group of substations as [34], with the use of the three basic techniques for decision-making described above.

Selection of Technology in Renewable Energy Diffusion Plan. The problem of selecting the appropriated technology in a renewable energy diffusion plan for the Sardinia region is stated in [35]. In [5], this problem is resolved on the basis of analyzing alternatives with using the concept of fuzzy majority. In particular, the following options of energy sources have been considered as alternatives: 1) solar (domestic solar water heaters), 2) wind (wind turbines of type grid connected), 3) hydraulic (hydro plants in derivation schemes), and 4) biomass (combined heat and power plants fed by agricultural wastes or energy crops).

The following criteria have been considered: 1) targets of primary energy saving in a regional scale, 2) sustainability according to Greenhouse pollutant emissions, 3) consistence of installation and maintenance requirements with local technical conditions, 4) continuity and predictability of performances, 5) market maturity, and 6) compatibility with political, legislative, and administrative situation.

The first two criteria are of a quantitative character. The third, fourth, and sixth criteria are of a qualitative character and have been evaluated through the set of three fuzzy values. The fifth criterion is also of a qualitative character and has been evaluated through the set of five fuzzy values.

Prioritization in Distribution System Maintenance Planning. The problem of prioritization in maintenance planning aims at assuring reliability and service quality of power supply through network and equipment preventive maintenance. CEMIG implements a strategy to realize the maintenance in distribution systems according to the plans suggested by manufacturers of network elements and equipment.

The prioritization techniques allow one to take into account not only parameters of network elements and equipment, but also factors related to the conditions of their operation. In addition to the failure risks, which can be assessed through the statistical analysis, factors associated with the impact of these failures are considered in the prioritization process. Among them it is possible to mention quantitative factors (number of consumers, energy demands, etc.) as well as qualitative factors (political impact, maintenance complexity, etc.).

The computing system for prioritization in maintenance planning GIMPRIS implemented for CEMIG provides group decision making environment in the web-based platform. The flexible tools for preference and aggregation modeling are designed to extend the capabilities of $< X, R >$ models for prioritization of more than 50000 equipment items. The dynamic group management scheme implemented within the system allows a supervisor to efficiently control the process of convergence to consensus, while moderating up to 10 experts simultaneously.

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МНОГОКРИТЕРИАЛЬНЫЕ ЗАДАЧИ В ЭНЕРГЕТИКЕ И МЕТОДЫ ИХ РЕШЕНИЯ НА ОСНОВЕ ТЕОРИИ НЕЧЕТКИХ МНОЖЕСТВ

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Представлены результаты исследований по использованию моделей и методов многокритериального принятия решений в нечеткой среде при решении задач электроэнергетического характера. При этом рассматривается два класса моделей, относящихся к многоцелевым (<X, M> модель) и многоатрибутным (<X, R> модель) задачам, а также методы их анализа, основанные как на использовании подхода Беллмана-Заде к принятию решений в нечеткой среде, так и технике, связанной с определением нечетких предпочтений. Представлен обзор результатов, полученных авторами при использовании указанных моделей и методов для решения различных задач, связанных с проектированием и управлением работой энергетических систем и подсистем. Последние результаты, связанные с применением <X, M> и <X, R> моделей и методов их анализа, относятся к задачам распределения источников реактивной мощности в распределительных сетях и определения приоритетов при планировании их обслуживания. Библ. 35, табл. 1.
Ключевые слова: энергетические системы, проектирование, управление, подход Беллмана-Заде, нечеткие множества.

БАГАТОКРИТЕРІАЛЬНІ ЗАДАЧІ В ЕНЕРГЕТИЦІ ТА МЕТОДИ ЇХНЬОГО РОЗВ'ЯЗАННЯ НА ОСНОВІ ТЕОРІЇ НЕЧІТКИХ МНОЖИН

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Наведено результати щодо досліджень використання моделей та методів багатокритеріального прийняття рішень в нечіткому середовищі при розв'язанні задач електроенергетичного спрямування. Розглянуто два класи моделей, які належать до багатоцільових (<X, M> модель) та багатоатрибутних (<X, R> модель) задач, а також методи їхнього аналізу, що засновані як на використанні підхода Беллмана-Заде до прийняття рішень у нечіткому середовищі, так і на техніці, пов'язаній з визначенням нечітких переваг.
Представлено огляд результатів, які отримано авторами при використанні вказаних моделей та методів для вирішення задач проектування та управління роботою енергетичних систем і підсистем. Останні результати, пов'язані з використанням <X, M> і <X, R> моделей та методів їхнього аналізу, належать до задач розподілу джерел реактивної потужності у розподільних мережах та визначення пріоритетів при плануванні їхнього обслуговування. Бібл. 35, табл. 1.
Ключові слова: енергетичні системи, проектування, управління, підхід Беллмана-Заде, нечіткі множини.