DOI: https://doi.org/10.15407/techned2024.02.003

THE ELECTROMAGNETIC FIELD AT THE FLAT SURFACE OF CONDUCTING BODY CAUSED BY BASIC SOURCES OF A NON-UNIFORM EXTERNAL FIELD

Yu.M. Vasetsky*
Institute of Electrodynamics National Academy of Sciences of Ukraine,
Beresteiskyi Ave., 56, Kyiv, 03057, Ukraine, e-mail: yuriy.vasetsky@gmail.com.

The purpose of the work is to obtain the expressions for the intensities of the electric and magnetic components of electromagnetic field at the action of two types of the basic sources of external field located near conducting half-space, where the eddy currents flow. The basic sources are the rectilinear current of infinite length, parallel to the interface between the dielectric and conducting media, and the magnetic moment, oriented along normal to the interface. The solution for electromagnetic field at surface between the media is applied. It is valid for strong skin effect in the form of expansion into an asymptotic series. Each term of the series is proportional to the derivative of corresponding order of external field components. This allows taking into account the external field non-uniformity. It is shown that the mathematical models with ideal skin effect have limited application. Then it is necessary to use more correct mathematical models for non-uniform field and a bounded depth of skin layer. The obtained expressions for electromagnetic field at the action of the basic sources of non-uniform external field allow us to use the principle of superposition to determine the field distributions in electromagnetic systems with more complex three-dimensional configuration. References 25, figures 5.

Key words: three-dimensional quasi-stationary electromagnetic field, strong skin effect, external field of rectilinear current and magnetic moment, asymptotic method, analytical solution**.**

Introduction. Despite significant progress in numerical calculation methods, the development of effective methods for solving three-dimensional problems of electromagnetic field, including analytical ones, in a fairly general formulation remains a topical problem. This is due to the wide use of the harmonic and pulse fields in electrotechnical, electrophysical devices and in technological processes where the electromagnetic field interacts with conducting bodies. As examples, we can mention the devices for magnetic pulse forming of thin-walled metal products [1–3], the heat treatment technology for metal products using induction heating [4–8], processing metal, in particular welded joints at the action of strong electromagnetic field and high-intensity current to improve mechanical properties [9–14].

The general analytical solution of three-dimensional problem in the system of electromagnetic field with arbitrary external sources near conducting half-space is summarized in [15] by results of numerous articles. The solution has no restrictions on the electrophysical properties of media and field frequency. In the case of the high-frequency and short-time pulse electromagnetic fields the strong skin effect occurs, then the current and field are localized in the thin surface layer of conducting body. Under such conditions, the problems are significantly simplified and the perturbation methods can be used [16, 17]. M. Leontovych proposed to use the approximate impedance boundary condition in order to take into account the bounded penetration depth $\delta = \sqrt{2/(\omega\mu\mu_0\gamma)}$ of electromagnetic field with cyclic frequency ω and such properties of conducting medium as conductivity γ and relative magnetic permeability μ. At approximate approach, it is assumed that the electromagnetic field locally penetrates into metallic body by the same way as a uniform field penetrates into conducting half-space. The local value of the field is determined by the model of a body with ideal conductivity, when the penetration depth is equal to zero $\delta \to 0$ [18, 19]. Further, the approximate approach using the impedance boundary condition was extended to bodies with different geometric and physical properties of the boundary surfaces, and the acceptable methods for electromagnetic field calculation were also developed [20–22].

This work demonstrates the results of simplifying the determined exact analytical solution of the initial three-dimensional electromagnetic field problem in the case of strong skin effect in expanded

* ORCID: https://orcid.org/0000-0002-4738-9872

statement, when the field penetration depth is small not only as compared with the characteristic size of the body, but also with the distance r between the external field sources and observation points at the surface of the body. The approximate solution was obtained by expansion of the potentials and field vectors into an asymptotic series in term of small parameter $\varepsilon = \mu\delta / \left(\sqrt{2r}\right) < 1$ [23].

The main difference from works on perturbation method consists in the substantiation of the results, since they are obtained on the basis of the exact solution of the problem. The extended formulation of strong skin effect along with the obtained results of series expansion allowed us to make conclusions regarding the general features of three-dimensional electromagnetic field forming. In particular, it was established that at flat boundary the field intensities are determined not only by the magnitude of the components of known external field, as in the model for ideal skin effect, but also by its derivatives with respect to coordinate perpendicular to media interface. So, the effect of field non-uniformity at the surface is determined, and the field distribution at the surface does not require solving the additional boundary-value problems.

Despite the fact that the results are obtained for arbitrary configuration of external field sources, the application of calculation expressions to solve a number of problems still requires an unreasonably large volume of calculations. Such problems include, for example, solving the inverse problems of electromagnetic field theory, optimizing the external source location, determining the necessary surface area exposed to the field, other problems. Therefore, within the framework of the simplified method for calculating three-dimensional electromagnetic fields, it is relevant to obtain the specific expressions for typical elements of external field, which, using the principle of superposition with summation of the fields created by a system of such elements, will allow developing the calculation algorithms for a wide range of external field sources.

The purpose of the work is to obtain specific expressions for the intensities of the electric and magnetic components of electromagnetic field under the action of two types of basic sources of the external field located near conducting half-space, taking into account the eddy currents induced by a rectilinear current contour of infinite length, parallel to the interface between the dielectric and conducting media, and by magnetic moment, oriented along the normal to interface surface.

Mathematical model. Approximate solution of three-dimensional problem. The general analytical solution of the linear three-dimensional conjugation problem of quasi-stationary electromagnetic field at the flat boundary between the dielectric and conducting media is given in [15]. In the case of strong skin effect (at $\varepsilon < 1$), the problem solution in dielectric region is presented in the form of asymptotic series, each term of which is proportional to the derivative with corresponding order of the given components of external field sources [23]. These expressions are basic to determine the field at boundary surface for typical sources of external magnetic intensity $\dot{\boldsymbol{H}}_0$. The calculations with sufficient accuracy can be performed for the small parameter $\varepsilon \le 0.3$. This condition is fulfilled for many technological processes, where it is necessary to ensure a strong interaction between the inductor and high-conductivity body. The resulting expressions are presented separately for the tangential and normal components of electric $\dot{\boldsymbol{E}} = \dot{\boldsymbol{E}}_{||} + \dot{\boldsymbol{E}}_{\perp}$ and magnetic $\dot{\boldsymbol{H}} = \dot{\boldsymbol{H}}_{||} + \dot{\boldsymbol{H}}_{\perp}$ intensities.

The tangential component of electric intensity at the media interface is the same in the dielectric and conducting media:

$$\dot{\boldsymbol{E}}_{||}(z=0) = \varsigma \sum_{n=0}^{N} 2a_n(\mu)\left(\frac{\varepsilon\, r}{\sqrt{i}}\right)^n \left\{\frac{\partial^{(n)}}{\partial z^n} \boldsymbol{e}_z \times \dot{\boldsymbol{H}}_{0||}\right\}\Bigg|_{z=0} . \tag{1}$$

Here $\varsigma = p/\gamma$ is the surface impedance, $p = \sqrt{i\omega\mu\mu_0\gamma}$ is the propagation constant, and i is imaginary unit. The coordinate directed along the axis perpendicular to the surface is denoted as z. The factors $a_n(\mu)$ are the coefficients of the Taylor series for function $1/w = \sum_{n=0}^{\infty} a_n(\mu)\left(\chi/\sqrt{i}\right)^n$, where $w(\chi) = \frac{\chi}{\sqrt{i}} + \sqrt{1 + \left(\frac{\chi}{\mu\sqrt{i}}\right)^2}$. The number N of the terms of limited asymptotic series is determined, first of all, by the value of small parameter ε.

The normal component of the electric field intensity is different at the sides of the dielectric and conducting areas. The general property of field forming follows from the exact solution of the boundary-value problem, and consists in the fact that for any ε at any point of conducting half-space, the component of the electric field intensity directed perpendicularly to the surface is equal to zero [15]. And then at the surface of the body in dielectric medium, the normal component of electric intensity $\dot{E}_{\perp}^{+}$ is completely determined by the induced electric field of external sources, which is considered as known:

$$\dot{\boldsymbol{E}}_z(z<0)=0\,.\ \ \dot{E}_{\perp}^{+}=-2i\omega\dot{A}_{0z}(z=0). \tag{2}$$

Here $\dot{A}_{0z}(z=0)$ is the normal component of the vector potential of the magnetic field of external sources.

The same values for the tangential component of magnetic intensity at the different sides of the media interface are as follows

$$\dot{\boldsymbol{H}}_{\|}(z=0)=-\sum_{n=0}^{N+1}2a_{n-1}(\mu)\left(\frac{\varepsilon\, r}{\sqrt{i}}\right)^{n}\left\{\frac{\partial^{(n)}\dot{\boldsymbol{H}}_{0\|}}{\partial z^{n}}\right\}\Bigg|_{z=0}, \tag{3}$$

where it is accepted that $a_{-1}=-1$.

Taking into account the continuity of the normal component of magnetic flux density, the expressions for the normal components of intensities in the dielectric $\dot{\boldsymbol{H}}_{\perp}^{+}$ and conducting $\dot{\boldsymbol{H}}_{\perp}^{-}$ areas at interface surface are determined by

$$\dot{\boldsymbol{H}}_{\perp}^{+}=\mu\dot{\boldsymbol{H}}_{\perp}^{-}=-\sum_{n=0}^{N}2a_{n}(\mu)\left(\frac{\varepsilon\, r}{\sqrt{i}}\right)^{n+1}\left\{\frac{\partial^{(n+1)}\dot{\boldsymbol{H}}_{0\perp}}{\partial z^{n+1}}\right\}\Bigg|_{z=0}. \tag{4}$$

In the given expressions, the first term of the series expansion ($n=0$) corresponds to the simplified model of the field at the surface of the body: the tangential component of the magnetic intensity $\dot{\boldsymbol{H}}_{\|}(z=0)$ corresponds to the field for body with ideal conductivity; the tangential component of the electric intensity $\dot{\boldsymbol{E}}_{\|}(z=0)$ is related with the tangent magnetic field component by Leontovich's approximate impedance boundary condition; the normal component of magnetic field $\dot{\boldsymbol{H}}_{\perp}^{+}$ for body with ideal conductivity is equal to zero, so the first term of the series corresponds to the normal derivative of external field.

As seen from (1), (3) and (4), the powers of small parameter ε for each separate term of the series coincide with the order of the derivative of external field components, which as a whole characterize the non-uniformity of this field. The greater the value of ε, the greater the value of the corresponding asymptotic series term. From this point of view, the value of ε can be considered as a characteristic of external field non-uniformity near the media interface.

The electric and magnetic components of the field at the flat surface of conducting body are calculated according to expressions (1)–(4) for external field created by the rectilinear current parallel to the flat surface and the magnetic moment oriented perpendicularly to the media interface.

Rectilinear current of infinite length parallel to media interface. Since at quasi-stationary approximation, contours with current are necessarily closed [24], the application of the model for straight-line current can be considered with certain restrictions. For external field sources in the form of contours with current, which are represented by a set of rectilinear sections (Fig. 1, *a*), the field can be considered at distances r which are small as compared to the characteristic dimensions of the contour D, i.e. $r/D<<1$. Since the field is considered at media interface, this condition is extended to the distance from the current to the surface $h/D<<1$, where h is the distance between the conductors of the contour and boundary surface for each rectilinear section. The areas near the corner points of the contour are excluded from consideration.

In the case of external field source, which contains the curvilinear current contour, coplanar to boundary surface (Fig. 1, *b*), the local replacement by rectilinear thread with current can be performed for each specific point of the contour. The direction of the current thread is to be coincided with the direction of the tangent vector to the contour $\boldsymbol{t}$. The local replacement of the curvilinear contour with rectilinear thread is often used to determine the magnitude of magnetic field. The replacement is possible in a limited area near the current at the distance that is much smaller than the radius of curvature of the contour $r/R<<1$. This condition is added to the restrictions on contours with rectilinear sections.

As a result, we have the simple calculation model for rectilinear current, parallel to the flat media interface at distance h from it and in this case directed along of the axis x of Cartesian coordinate system

(x, y, z) (Fig. 1, *c*).

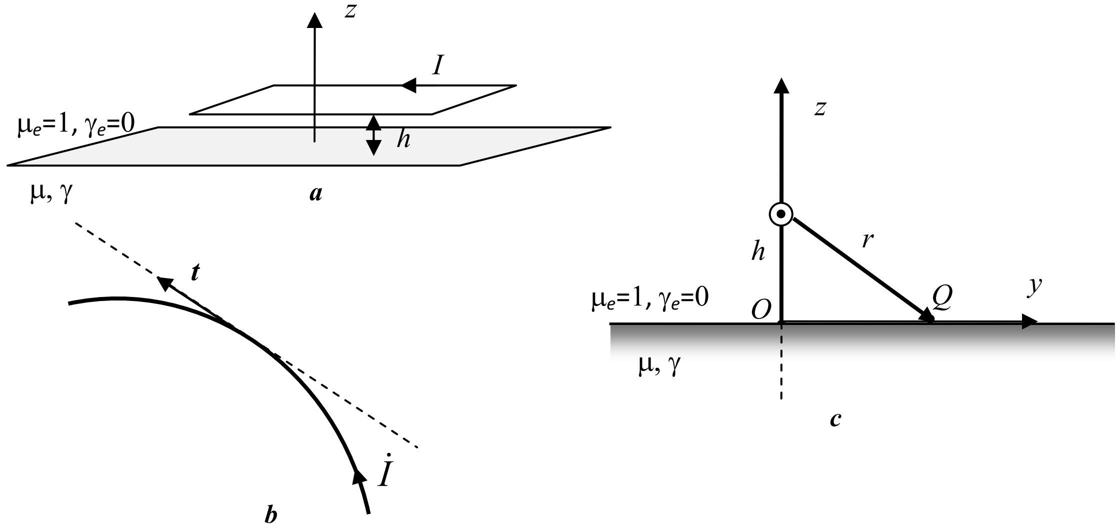


Fig. 1

The magnetic field created by direct current is well known. The field intensity at arbitrary point of the space is determine by the following expression

$$\dot{\boldsymbol{H}}_0 = \frac{\dot{I}}{2\pi} \frac{y\boldsymbol{e}_z - (z-h)\boldsymbol{e}_y}{y^2 + (z-h)^2} . \tag{5}$$

Here ($\boldsymbol{e}_x, \boldsymbol{e}_y, \boldsymbol{e}_z$) are the standard basis vectors of the coordinate system.

The field of rectilinear current is non-uniform. Therefore, the field distribution at the surface, which is determined by the field of the external current and eddy currents in conducting half-space, is related with the level of field non-uniformity, i.e. with the value of parameter ε and the corresponding derivatives of external field in expressions (1), (3) and (4). The expressions are approximate, and for the specific calculations presented below, the small parameter does not exceed permissible value $-\varepsilon \le 0.3$.

The parameter ε depends on the distance r between the source and points of observation. At the point O at the surface, the distance $r = h$ is minimal and then the parameter ε takes the maximum value $\varepsilon_m = \mu\delta \big/ \left(h\sqrt{2}\right) = \varepsilon r / h$. At this point, the error of approximate calculation method is the largest. Using h and ε_m , the surface impedance is equal to $\varsigma = \sqrt{i}\omega\mu_0 h \varepsilon_m$. Below we will use one common small parameter ε_m for analysis.

By substituting the value of external magnetic field (5) into the expressions for field intensities (1), (3) and (4), we obtain their distribution along the media interface. The field distribution along coordinate y depending on both parameter ε_m and the height of location h is carried out with dimensionless quantities. For the selected type of non-uniform external field, we introduce the following normalization of field intensities: $\dot{\boldsymbol{E}}_{\parallel}^* = \dot{\boldsymbol{E}}_{\parallel} \Big/ \left(\frac{\mu_0 \omega \dot{I}}{2\pi} \right)$ and $\dot{\boldsymbol{H}}^* = \dot{\boldsymbol{H}} \Big/ \left(\frac{\dot{I}}{2\pi h} \right)$.

The expressions for normalized intensities under the action of external direct current field are as follows

$$\dot{\boldsymbol{E}}_{\parallel}^* = \boldsymbol{e}_x 2\sqrt{i}\varepsilon_m \sum_{n=0}^{N} a_n \left(\frac{\varepsilon_m}{\sqrt{i}} \right)^n h^{n+1} \frac{\partial^{(n)}}{\partial z^n} \left(\frac{z-h}{r^2} \right) \Bigg|_{z=0} , \tag{6}$$

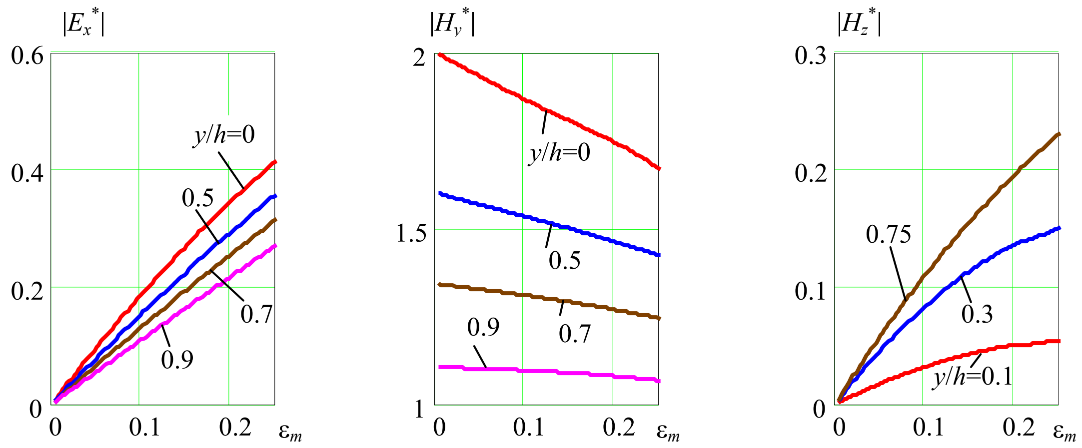
$$\dot{\boldsymbol{H}}_{\parallel}^* = \boldsymbol{e}_y 2 \sum_{n=0}^{N+1} a_{n-1} \left(\frac{\varepsilon_m}{\sqrt{i}} \right)^n h^{n+1} \frac{\partial^{(n)}}{\partial z^n} \left(\frac{z-h}{r^2} \right) \Bigg|_{z=0} , \tag{7}$$

$$\dot{\boldsymbol{H}}_\perp^* = \boldsymbol{e}_z 2\frac{y}{h}\sum_{n=0}^{N} a_n\left(\frac{\varepsilon_m}{\sqrt{i}}\right)^{n+1} h^{n+3}\frac{\partial^{(n+1)}}{\partial z^{n+1}}\left(\frac{1}{r^2}\right)\Bigg|_{z=0}, \qquad (8)$$

where $r=\sqrt{y^2+(z-h)^2}$.

Fig. 2 illustrates the effect of parameter ε_m for the electric and magnetic field intensities at different points of the surface. Since this parameter is proportional to the penetration depth of electromagnetic field, it simultaneously shows the influence of the depth on field distribution for fixed distance h. The electric field intensity, which is zero for ideal skin effect at $\varepsilon_m=0$, increases with growth of ε_m almost linearly (Fig. 2, *a*). The influence of external field non-uniformity is most strong for the points located at the smallest distance from field source. The same tendency occurs for magnetic field intensity (Fig. 2, *b*), which has the greatest value at ideal skin effect. As seen the decrease in magnetic field intensity with increasing parameter ε_m becomes less appreciable for more remote points of the surface from the source.

For ideal skin effect (ideal electrical conductivity of the medium), the normal component of magnetic field intensity is equal to zero. But this component occurs in the case of diffusion of non-uniform electromagnetic field into body at $\varepsilon_m>0$ (Fig. 2, *c*). As following from (8), the normal component of magnetic field is an odd function of coordinate y and therefore directly under external current at $y=0$ the field intensity is equal to zero. The non-zero value of the normal component of magnetic field intensity takes place at points on the surface at $|y|>0$ and $\varepsilon_m>0$.



a ***b*** ***c***

Fig. 2

The electromagnetic field distributions in the form of dependences on the relative value y/h at different parameter ε_m are shown in Fig. 3. The more external field non-uniformity, which, as noted above, is characterized by parameter ε_m, the greater the magnitude of electric field intensity (Fig. 3, *a*). It should be noted that the value of $\varepsilon=\varepsilon_m\Big/\sqrt{1+(y/h)^2}$ decreases as the distance from point O is closest to current. It is shown in the reduction of the effect of field non-uniformity at more remote points of the surface.

In the case of ideal skin effect, the tangent magnetic field component is equal to double value of tangent external field component (Fig. 3, *b*, curve for $\varepsilon_m=0$). With increasing of ε_m, i.e. an increasing in the depth of eddy current flow, the field intensity at the surface decreases.

Fig. 3, *c* shows that the normal component of magnetic field intensity has a non-monotonic character. It reaches its maximum value at the points of the surface $y/h\approx 0.5\div 0.75$ for the range of $\varepsilon_m\le 0.3$. The greater the value of parameter ε_m, the greater the maximum value of this magnetic field component. When $\varepsilon_m=0.3$ the maximum normal component constitutes 23% of tangential component, and the normal component cannot be neglected.

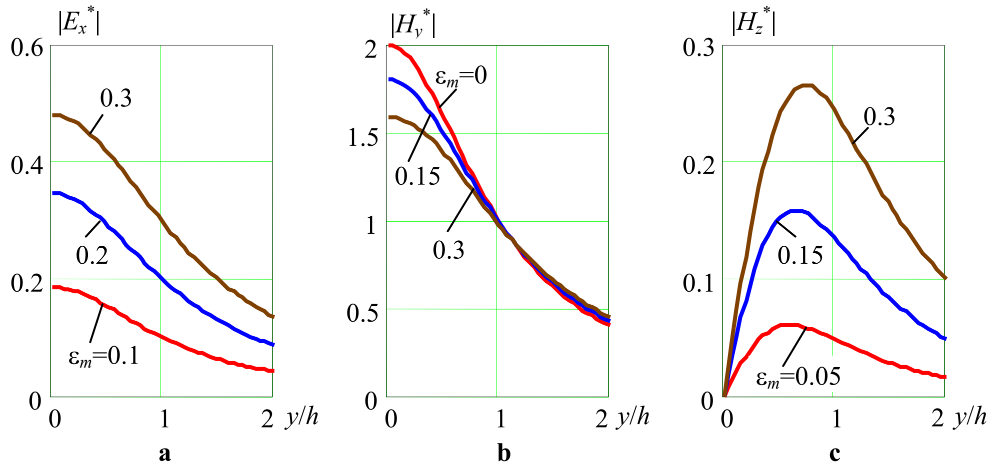


Fig. 3

Magnetic moment oriented perpendicularly to the interface of media. The sources of the external field are not only contours with alternating current. The external field sources can also be represented by a system of magnetic moments [25]. The field of the magnetic moments is also non-uniform, and therefore for its determination with eddy currents in conducting body having flat surface, it is necessary to use calculation method that takes into account the field non-uniformity. When strong skin effect takes place, the application of analytical method, using expressions (1)–(4) is advisable.

The use of the field of magnetic moments is also useful for calculating the electromagnetic field when the external field is represented by contours with alternating current. This is due to the fact that for quasi-stationary approximation the contours should be closed and each of them cannot be divided into parts [24]. At the same time, the principle of superposition is valid for magnetic field of the system of magnetic moments. Each contour of arbitrary configuration with current I can be replaced by the surface bounded by closed contour with field sources in the form of a double layer of magnetic charges (magnetic moments), $\boldsymbol{dm} = \mu_0 I dS\boldsymbol{n}$, where $\mu_0 I\boldsymbol{n}$ is the vector of surface density of the distributed magnetic moment which is directed along the normal $\boldsymbol{n}$ to the surface (Fig. 4).

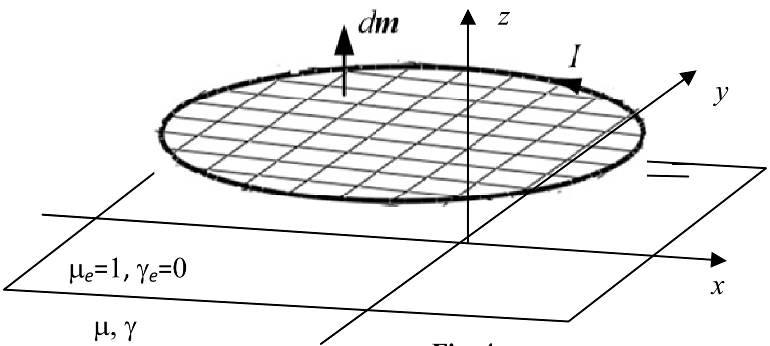


Fig. 4

Each magnetic moment $\boldsymbol{m}$ is a separate field source with magnetic field intensity. The expression in vector form for the intensity at arbitrary point of space is as follows

$$\boldsymbol{H}_0 = \frac{1}{4\pi}\left[\frac{3\boldsymbol{m}\cdot\boldsymbol{r}}{r^4}\frac{\boldsymbol{r}}{r} - \frac{\boldsymbol{m}}{r^3}\right], \tag{9}$$

where vector $\boldsymbol{r}$ is directed from source point (from moment $\boldsymbol{m}$) to point of observation.

As in the previous case, we will consider a basic external field source in the form of one magnetic moment that varies in time according to a sinusoidal law. In this paper, we will consider only the magnetic moments directed perpendicular to the flat surface of body $\boldsymbol{m} = m_z\boldsymbol{e}_z$, which is located at distance h above the surface of conducting body. The system of such moments with expressions (1)–(4) makes it possible to determine, in particular, the field under the action of planar current contours parallel to media interface. In this case there is the same limitation on configuration as in the previous case with the rectilinear current.

The external magnetic field of magnetic moment $\dot{\mathbf{m}}_\perp = \dot{m}_z\mathbf{e}_z$ is characterized by axial symmetry and it is convenient to write the expression in cylindrical coordinate system (ρ, θ, z) with unit vectors $\boldsymbol{e}_\rho, \boldsymbol{e}_\theta, \boldsymbol{e}_z$ directed along the corresponding coordinates. In this coordinate system, the external magnetic field (9) can be represented as two components along the directions $\boldsymbol{e}_\rho$ and $\boldsymbol{e}_z$

$$\dot{\boldsymbol{H}}_0 = \frac{1}{4\pi}\left[3\frac{(\dot{\boldsymbol{m}}\cdot\boldsymbol{r})\boldsymbol{r}}{r^5} - \frac{\dot{\boldsymbol{m}}}{r^3}\right] = \frac{\dot{m}_z}{4\pi}\left[3\frac{(z-h)^2}{r^5} - \frac{1}{r^3}\right]\boldsymbol{e}_z + \frac{\dot{m}_z}{4\pi}3\frac{(z-h)\rho}{r^5}\boldsymbol{e}_\rho . \tag{10}$$

We will analyze the electric and magnetic field intensities using normalized values, defined in this case as follows: $\dot{\boldsymbol{E}}_{||}^* = \dot{\boldsymbol{E}}_{||} \Big/ \left(\frac{\mu_0 \dot{m}_z \omega}{4\pi h^2}\right)$ i $\dot{\boldsymbol{H}}^* = \dot{\boldsymbol{H}} \Big/ \left(\frac{\dot{m}_z}{4\pi h^3}\right)$.

The external field of the magnetic moment is non-uniform. Therefore the expressions for field components at the surface will contain the terms of asymptotic series that are proportional to the derivatives with respect to the vertical coordinate and parameter ε_m raised to corresponding power. The tangential component of electric field intensity has only azimuthal component

$$\dot{\boldsymbol{E}}_{||}^* = \boldsymbol{e}_\theta \sqrt{i}\, 6\frac{\rho}{h}\varepsilon_m \sum_{n=0}^{N} a_n \left(\frac{\varepsilon_m}{\sqrt{i}}\right)^n h^{n+4} \frac{\partial^{(n)}}{\partial z^n}\left(\frac{z-h}{r^5}\right)\Bigg|_{z=0} , \tag{11}$$

where $r = \sqrt{\rho^2 + (z-h)^2}$.

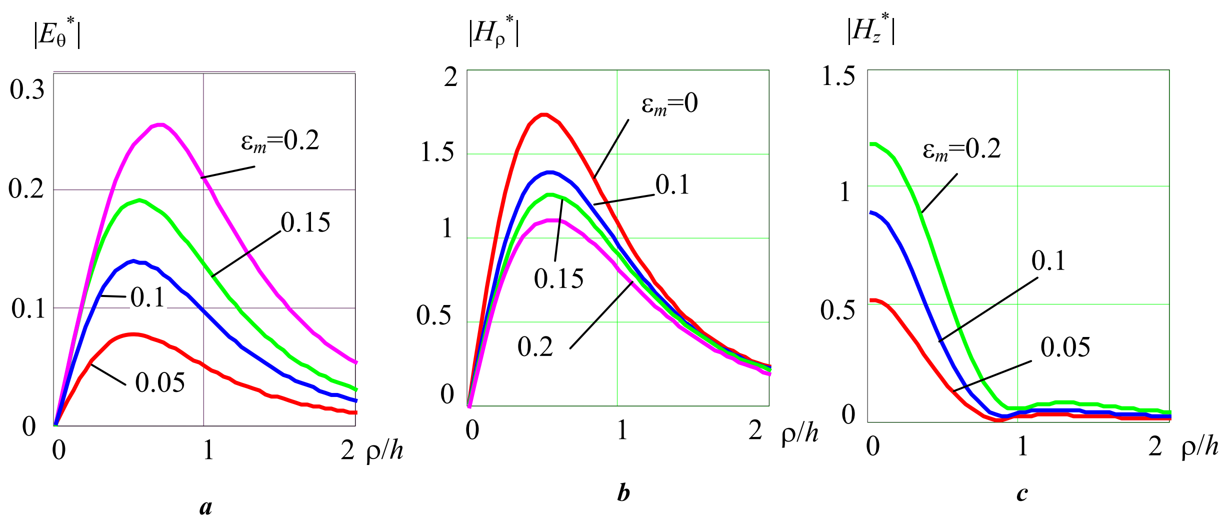
The normalized values of the tangent and normal components of magnetic field intensity are given as

$$\dot{\boldsymbol{H}}_{||}^* = -\boldsymbol{e}_\rho 6\frac{\rho}{h} \sum_{n=0}^{N+1} a_{n-1} \left(\frac{\varepsilon_m}{\sqrt{i}}\right)^n h^{n+4} \frac{\partial^{(n)}}{\partial z^n}\left(\frac{z-h}{r^5}\right)\Bigg|_{z=0} , \tag{12}$$

$$\dot{\boldsymbol{H}}_{\perp}^* = \boldsymbol{e}_z 2\sum_{n=0}^{N} a_n \left(\frac{\varepsilon_m}{\sqrt{i}}\right)^{n+1} h^{n+4} \frac{\partial^{(n+1)}}{\partial z^{n+1}}\left(\frac{2(z-h)^2 - \rho^2}{r^5}\right)\Bigg|_{z=0} . \tag{13}$$

Fig. 5 shows the dependences of the field variation along the surface at different small parameter ε_m for the modules of normalized complex amplitudes of electromagnetic field intensities.

For ideal skin effect at $\varepsilon_m \to 0$, the tangential electric field intensity and normal magnetic field component are equal to zero. That is why these values are not presented in Fig. 5, *a*, *c*. On the contrary, the tangential magnetic field $\dot{H}_\rho$ is equal to double value of the tangential component of external magnetic field [23]. It reaches its maximum at the points of the circle with radius $\rho = h/2$ and, as following from (12), it is equal to $\dot{H}_{||\max}^* = -\left|\dot{H}_\rho^*\right|_{\max} = -6\cdot 0{,}5 \Big/ \sqrt{0{,}5^2 + 1} = -1{,}717$.



a ***b*** ***c***

Fig. 5

When parameter ε_m increases, that is, with the increase in external electromagnetic field non-uniformity, the field distribution within the surface changes. The tangential component of electric field

(Fig. 5, *a*) is no longer equal to zero and increases with increase of ε_m. On the contrary, the tangential component of magnetic field intensity decreases with increase of ε_m. This is due to the increase of skin layer thickness and, accordingly, the decrease in field of the currents flowing through surface layer.

For non-uniform field and bounded skin layer thickness, that is, at $\varepsilon_m > 0$, the normal component of magnetic field intensity is no longer zero. Note, already for $\varepsilon_m \approx 0.2$ the normal component $\left|\dot{H}_z^*\right|$ becomes commensurate with the tangential component $\left|\dot{H}_\rho^*\right|$, and the neglect of this field component in simplified models can lead to significant calculation errors. Also note that the normal component of magnetic field becomes insignificant at distance $\rho/h \geq 0.8$. As the analysis shows, this is due to the fact that the phase shift of normal field component changes sharply within the range of $\rho/h \approx 0.8 \div 1.0$ by approximately 180^0. This means that over this range the change in the direction of normal field component as compared to the direction at $\rho/h < 0.8$ takes place.

Conclusion. The proposed specific expressions for electromagnetic field at the interface between the dielectric and conducting media in the case of strong skin effect are directly determined by the known field of external sources. This condition greatly simplifies the solution of field problems, as there is no need to solve separately the field equations at the surface. The principle of superposition with the help of obtained specific expressions for the basic elements of external field in the form of the rectilinear current and magnetic moment, which change in time according to sinusoidal law, can be used to solve a certain class of three-dimensional field problems with strong skin effect.

As following from the analysis of obtained results for the basic elements of external field, the mathematical models with ideal skin effect have a limited scope of application. In the case of non-uniform field of the external sources, when the penetration depth of the field is commensurate with the distance between the source and conducting body, it is necessary to use more correct mathematical models for electromagnetic field. The analytical approaches using the expansion of field intensity components into an asymptotic series with introduced small parameter are a convenient way to describe the electromagnetic field.

Further development of the study can be aimed to determine the field under the action of other sources of non-uniform external field and to find the impedance boundary condition for such fields. It allows applying the known approaches to solve the boundary-value problems of electrodynamics.

Роботу виконано за рахунок держбюджетної теми «Розвиток теорії та моделювання нестаціонарних електрофізичних процесів в електропровідних і діелектричних середовищах імпульсних електромагнітних систем (шифр : Бар'єр-3)», державний реєстраційний номер теми 0123U100671,КПКВК 6541030.

1. Psyk V., Risch D., Kinsey B.L., Tekkaya A.E., Kleiner M. Electromagnetic forming – A review. *Journal of Materials Processing Technology*. 2011. Vol. 211. Issue 5. Pp. 787-829. DOI: https://doi.org/10.1016/j.jmatprotec.2010.12.012.
2. Gayakwada D., Dargara M.K., Sharmaa P.K., Rajesh purohitb, Ranab R.S. A Review on Electromagnetic Forming Process. *Procedia Materials Science*. 2014. Vol. 6. Pp. 520-527. DOI: https://doi.org/10.1016/j.mspro.2014.07.066.
3. Batygin Y., Barbashova M., Sabokar O. Electromagnetic Metal Forming for Advanced Processing Technologies. Springer, Cham., 2018. DOI: https://doi.org/10.1007/978-3-319-74570-1.
4. Rudnev V., Loveless D., Cook R., Black M. Handbook of induction heating. London: Taylor & Francis Ltd, 2017. 772 p. DOI: https://doi.org/10.1201/9781315117485.
5. Lucía O., Maussion P., Dede E.J., Burdío J.M. Induction Heating Technology and Its Applications: Past Developments, Current Technology, and Future Challenges. *IEEE Transactions on Industrial Electronics.* 2014. Vol. 61. Issue 5. Pp. 2509-2520. DOI: https://doi.org/10.1109/TIE.2013.2281162.
6. Acero J., Alonso R., Burdio J.M., Barragan L.A., Puyal D. Analytical equivalent impedance for a planar induction heating system. *IEEE Transaction on Magnetics.* 2006. Vol. 42. No 1. Pp. 84-86. DOI: https://doi.org/10.1109/TMAG.2005.854443.
7. Kondratenko I., Vasetsky Yu., Zaporozhets A. Interactions Between Electromagnetic Field and Moving Conducting Strip. Lecture Notes in Electrical Engineering. 2023. Vol. 1111. Springer, Cham. VII. 124 p. DOI: https://doi.org/10.1007/978-3-031-48274-8.
8. Rashchepkin A., Kondratenko I., Karlov O., Kryshchuk R., Zhiltsov A., Vasyuk V. Electromagnetic stirring of metals in spatially orthogonal magnetic fields. *Tekhnichna elektrodynamika*. 2020. No 2. Pp. 86-92. DOI: https://doi.org/10.15407/techned2020.02.086.
9. Babutsky A., Chrysanthou A., Ioannou J. Influence of pulsed electric current treatment on corrosion of structural metals. *Strength of materials.* 2009. Vol. 41. No 4. Pp. 387-391. DOI: https://doi.org/10.1007/s11223-009-9142-3.

10. Gallo F., Satapathy S., Ravi-Chandar K. Melting and crack growth in electrical conductors subjected to short-duration current pulses. *International Journal of Fracture.* 2011. Vol. 16. Pp. 183-193. DOI: https://doi.org/10.1007/s10704-010-9543-0.
11. Vasetsky Yu. M., Kondratenko I.P. Electromagnetic field of the inductor for local electric pulse effects on metal products. *Tekhnichna Elektrodynamika*. 2020. No 4. Pp. 11-14. DOI: https://doi.org/10.15407/techned2020.04.011. (Ukr)
12. Olevsky E.A., Jiang R., Xu W., Maximenko A., Grippi T., Torresani E. Quasi-instantaneous materials processing technology via high-intensity electrical nano pulsing. *Scientific Reports*. 2024. Vol. 14. Pp. 434. DOI: https://doi.org/10.1038/s41598-023-50698-w.
13. Kondratenko I., Zhiltsov A., Vasyuk V. Mathematical modeling of electrophysical processes in systems of operational non-destructive determination of residual stresses. *Tekhnichna Elektrodynamika*. 2012. No 3. Pp. 21-22. (Ukr).
14. Kondratenko I., Zhiltsov A., Vasyuk V. Simulation of discharge capacity axle symmetric systems 'coil – Non-ferromagnetic massive disk' by the method of integral equations. Proceedings 6th International Conference on *Computational Problems of Electrical Engineering* (CPEE), Lviv Ukraine, 02-05 September 2015. Art. no. 7333340. Pp. 71-73. DOI: https://doi.org/10.1109/CPEE.2015.7333340 .
15. Vasetsky Yu., Zaporozhets A. Electromagnetic Field of Arbitrary Spatial Current Contour Located Near Conducting Body with Flat Surface. Electromagnetic Field Near Conducting Half-Space. Chapter 1. *Lecture Notes in Electrical Engineering*. 2023. Vol. 1070. Springer, Cham. Pp. 1-32. DOI: https://doi.org/10.1007/978-3-031-38423-3_1
16. Rytov S.M., Calculation of skin effect by perturbation method. *Journal of Experimental and Theoretical Physics.* 1940. Vol. 10. Issue 2. Pp. 180-190. (Rus).
17. Mitzner K.M. An integral equation approach to scattering from a body of finite conductivity. *Radio Science*. 1967. Vol. 2. Issue 12. Pp. 1459–1470. DOI: https://doi.org/10.1002/rds19672121459.
18. Landau L.D., Lifshitz E.M. Electrodynamics of Continuous Media. Elsevier Ltd, 1984. 475 p. DOI: https://doi.org/10.1016/B978-0-08-030275-1.50024-2.
19. Simonyi K. Foundation of electrical engineering. Elsevier Ltd, 1963. 865 p. DOI: https://doi.org/10.1016/C2013-0-02694-1.
20. Yuferev S., Ida N. Surface Impedance Boundary Conditions: A Comprehensive Approach. CRC Press, 2018. 412 p. DOI: https://doi.org/10.1201/9781315219929.
21. Liu X., Yang F., Li M., Xu S. Generalized Boundary Conditions in Surface Electromagnetics: Fundamental Theorems and Surface Characterizations. *Applied Sciences*. 2019. Vol. 9. Issue 9. Pp. 1891-1918. DOI: https://doi.org/10.3390/app9091891.
22. Berdnyk S., Gomozov A., Gretskih D., Kartich V., Nesterenko M. Approximate boundary conditions for electromagnetic fields in electromagnetism. *Radioelectronic and Computer Systems*. 2022. No 3. Pp. 141-160. DOI: https://doi.org/10.32620/reks.2022.3.11.
23. Vasetsky Y., Zaporozhets A. Approximate Mathematical Models for Analysis of Alternating Electromagnetic Field of Sources Near Conducting Body. Electromagnetic Field Near Conducting Half-Space. *Lecture Notes in Electrical Engineering.* 2023. Vol. 1070. Springer, Cham. Pp. 33-67. DOI: https://doi.org/10.1007/978-3-031-38423-3_2.
24. Tamm I.E. *Fundamentals of the Theory of Electricity*. Moskva: Mir, 1979. 684 p. (Rus)
25. Polivanov K. *Theory of Electromagnetic Field*. Imported Pubn, 1984.

ЕЛЕКТРОМАГНІТНЕ ПОЛЕ НА ПЛОСКІЙ ПОВЕРХНІ ЕЛЕКТРОПРОВІДНОГО ТІЛА, ОБУМОВЛЕНЕ СТАНДАРТНИМИ ДЖЕРЕЛАМИ НЕОДНОРІДНОГО ЗОВНІШНЬОГО ПОЛЯ

Ю.М. Васецький, докт. техн. наук
Інститут електродинаміки НАН України,
пр. Берестейський, 56, Київ, 03057, Україна, e-mail: yuriy.vasetsky@gmail.com.

Метою роботи є отримання конкретних виразів для напруженостей електричної та магнітної складових електромагнітного поля через дію двох типів елементарних джерел зовнішнього поля, розташованих поблизу електропровідного півпростору, з урахуванням вихрових струмів: прямолінійного струму, паралельного поверхні поділу діелектричного та електропровідного середовищ, і магнітного моменту, орієнтованого вздовж нормалі до поверхні електропровідного тіла у разі прояву сильного скін-ефекту. Застосовано розв'язок для електромагнітного поля на поверхні поділу середовищ, що справедливий за сильного скін-ефекту у вигляді розкладання в асимптотичний ряд, кожен член якого пропорційний похідної відповідного порядку від компонент зовнішнього поля, що дає змогу врахувати вплив неоднорідності зовнішнього поля. Показано, що математичні моделі з ідеальним скін-ефектом мають обмежену область застосування, що обумовлює для неоднорідного поля і кінцевої глибини скін шару застосовування більш коректних математичних моделей. Отримані вирази для електромагнітного поля за дії елементарних джерел неоднорідного зовнішнього поля дають змогу використовувати принцип суперпозиції задля визначення розподілу полів в електромагнітних системах більш складної тривимірної конфігурації. Бібл. 25, рис. 5.
Ключеві слова: тривимірне квазістаціонарне електромагнітне поле, сильний скін-ефект, зовнішнє поле прямого струму і магнітного моменту, асимптотичний метод, аналітичний розв'язок.

Надійшла 23.02.2024
Остаточний варіант 25.03.2024