DOI: https://doi.org/10.15407/techned2025.01.024

# GENERALIZED DEFINITION OF THE APPARENT POWER AND ENERGY-EFFICIENT STRATEGIES OF ACTIVE FILTRATION IN THE REDUCED COORDINATE BASIS OF A MULTIPHASE POWER SUPPLY SYSTEM

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*The paper substantiates the equivalence of determining the apparent power of a multiphase power supply system with different transmission line impedances using the Fryze-Buchholz-Deppenbrock method and on a reduced coordinate basis. Two energy-efficient control strategies for shunt active filtering in the reduced coordinate basis are proposed. The first strategy provides a unit value of the power factor, and the second strategy minimizes power losses in the transmission line while maintaining symmetry and the quasi-sinusoidal shape of the consumed currents. The advantages of using a reduced coordinate basis are a reduction in the number of sensors and key active filter regulators as well as the absence of the problem to organize the artificial grounding point for phase voltage measurements. A correction factor for the apparent power and power factor formulas was determined and verified in the presence of restrictions on the symmetrical and sinusoidal shape of the consumed currents.* References 22, figures 5, tables 2.
***Keywords:*** apparent power, power factor, minimization of power losses, control strategy for shunt active filter, reduced coordinate basis.

**Introduction.** The apparent power is one of the key concepts of power theory, which has been evolving for more than a century and a half, and an overview of its main achievements is given in [1]. In theoretical terms, the correct determination of the apparent power involves solving the optimization problem of maximizing the active power, which is carried out using the methods of Lagrange multipliers [2] and integral inequalities [3]. In decomposing the apparent power into quadratic components, complex calculus [4, 5], electric field theory [6], and geometric algebra [7] are used. In practical terms, the study of the components of apparent power and power losses in a transmission line [8] is important for selecting and compensating associated currents or voltages through shunt or series filtering. Despite the variety of power theories [9] with corresponding definitions of apparent power and its decompositions, which are partly reflected in the current standard [10], there is a clear criterion [11] for the correctness of the apparent power definition for the needs of energy-efficient shunt active filtering. It corresponds to the physical meaning of the power factor and allows experimental verification for minimization of the power loss in the transmission line. Proposed a long time ago, it does not consider the requirements of modern standards for symmetry and sinusoidal shape of the consumed currents.

**The aim of the paper** is to generalize the apparent power formula for a multiphase power transmission system with different transmission line wire resistances in a reduced coordinate basis, substantiate and develop energy-efficient strategies for shunt active filtering while ensuring the symmetry and sinusoidal shape of the consumed currents, evaluate and verify the energy-saving effect of the proposed strategies.

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**The apparent power definition as the geometric mean value of the voltage and current power losses.** The determination of the apparent power of a multiphase power supply system based on the Fryze-Depenbrock-Buchholz (FBD) method [12] for the case of different transmission line impedances is given in [2, 3]. At the same time, each of the wires of the m-phase system (Fig. 1) with resistance $r_k, k = 1,2,...,m$ is represented by a separate phase, the energy processes in which are determined by the current $i_k$ and the potential difference $v_{k0} = v_k - v_0$ relative to the point of artificial grounding with the potential $v_0 = \sum_{k=1}^{m} v_k g_k \Big/ \sum_{k=1}^{m} g_k ; g_k = 1/r_k$.

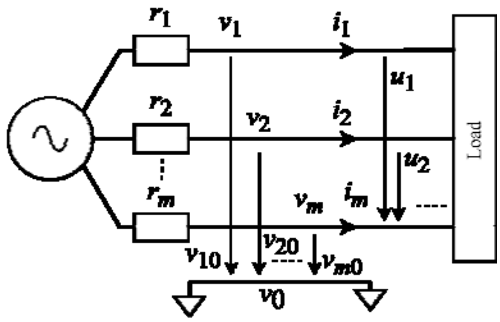



**Fig. 1.** Multi-phase power supply system with a resistive transmission line model and coordinates of different reference frames

As a result of solving the problem of maximizing the active power at given asymmetric sinusoidal source voltages and transmission line loss power using the Lagrange multiplier method in [2], and in the general case of periodic currents and voltages using Schwarz's inequality [3], the authors obtained identical expressions for the apparent power as the product of effective voltage and current values. Using notation $T^{-1}\int_T \mathbf{x}^\wedge(t)\mathbf{y}(t)dt = \mathbf{x} \circ \mathbf{y}$ for the scalar product of arbitrary one-dimensional vectors, where $^\wedge$ is the transpose sign, these expressions are of the form

$$S = \sqrt{\mathbf{v}_+ \circ \mathbf{v}_+} \times \sqrt{\mathbf{i}_+ \circ \mathbf{i}_+} = VI, \tag{1}$$

where $\mathbf{i}_+(t) = \left\| i_1\sqrt{r_1/r} \quad i_2\sqrt{r_2/r} \quad ... \quad i_m\sqrt{r_m/r} \right\|$; $\mathbf{v}_+(t) = \left\| v_{10}\sqrt{r/r_1} \quad v_{20}\sqrt{r/r_2} \quad ... \quad v_{m0}\sqrt{r/r_m} \right\|$, $r$ is the normalizing resistance of the transmission line.

The disadvantages of this approach are the increased dimensionality of voltage and current vectors describing the multiphase power supply system, and the associated increased number of sensors active filter regulators; the opaque physical content of the quantity $v_0$ and the difficulty of measuring this value for practical implementation; the difficulty of calculating the ratio (1) when the resistance of one of the line wires is close to zero, since the resistance of this wire appears in the denominator of the corresponding coordinate vector $\mathbf{v}_+(t)$.

Let's take a closer look at the first drawback. Since the algebraic sum of the instantaneous values of the currents of a multiphase system (Fig. 1) is zero according to Kirchhoff's first law, only *m-1* of the current coordinates are independent. Let us choose as independent quantities *m-1* of the currents corresponding to the line wires, which can be represented by a vector $\mathbf{i}(t) = \left\| i_1 \quad i_2 \quad ... \quad i_{m-1} \right\|^\wedge$, then the current of the neutral wire is given by the equation $i_m(t) = -\mathbf{j}^\wedge \mathbf{i}(t)$, where $\mathbf{j}$ is a vector consisting of $m$-1 unit.

The set of $m$ voltages, appearing in the vector $\mathbf{v}_+(t)$, is also linearly dependent, since given the expression for the potential $v_0$ of the artificial ground point: $\sum_{k=1}^{m} v_{k0} g_k = \sum_{k=1}^{m} v_k g_k - v_0 \sum_{k=1}^{m} g_k = 0$. Therefore, as shown in a number of authors' works [13-19], the analysis of energy processes of multiphase power supply systems can be carried out in a reduced coordinate basis using a voltage vector of smaller dimensionality, $\mathbf{u}(t) = \left\| u_1 \quad u_2 \quad ... \quad u_{m-1} \right\|; u_k = v_k - v_m;\ k = 1,2,...,m-1,$ which are calculated with respect to the potential $v_m$ of neutral wire, which eliminates the problem of organizing an artificial grounding point. The determination of the apparent power using the specified reduced coordinate basis in the notation of this article corresponds to the expression

$$S = \sqrt{(\mathbf{i} \circ \mathbf{Ri}) \times (\mathbf{u} \circ \mathbf{R}^{-1}\mathbf{u})}, \tag{2}$$

where $\mathbf{R} = \begin{Vmatrix} r_1 & 0 & 0 \\ 0 & ... & 0 \\ 0 & 0 & r_{m-1} \end{Vmatrix} + r_m \mathbf{j}^\wedge \mathbf{j}$ is a matrix of transmission line resistances that satisfies the matrix-vector equation

$$\mathbf{e}(t) = \mathbf{Ri}(t) + \mathbf{u}(t), \tag{3}$$

where $\mathbf{e}(t)$ is an EMF vector of a multi-phase power source.

The multipliers of the root in (2) have a clear physical meaning [14]: $\mathbf{i} \circ \mathbf{Ri} = T^{-1} \int_T [i_1^2(t) r_1 + i_2^2(t) r_2 + ... + i_m^2(t) r_m] dt = P_{LS}$ is the power loss in the transmission line due to current flow; $\mathbf{u} \circ \mathbf{R}^{-1} \mathbf{u} = P_{SC}$ is the power loss on the same transmission line impedances caused by short-circuit mode of the voltages measured at points of common coupling and $\mathbf{i}_{SC}(t) = \mathbf{R}^{-1} \mathbf{u}(t)$ is the current of this short-circuit mode.

Thus, according to (2), the apparent power of a multiphase system is the geometric mean value of the power losses from currents and voltages measured at the points of common coupling

$$S = \sqrt{P_{LS} P_{SC}}. \tag{4}$$

We will show that the definition of (1) has the same physical meaning. It is easy to see that the square of the second multiplier in (1)

$$I^2 = \mathbf{i}_+ \circ \mathbf{i}_+ = T^{-1} r^{-1} \int_T [i_1^2(t) r_1 + i_2^2(t) r_2 + ... + i_m^2(t) r_m] dt = P_{LS} / r \tag{5}$$

is the normalized current power loss on the transmission line impedances.

Let's give a different physical meaning for the square of the first multiplier from (1). To do this, we denote $\mathbf{D}_r = diag(r_1 \quad r_2 \quad ... \quad r_{m-1})$ and, provided $\prod_{k=1}^{m} r_k \neq 0$ we find an equation for the inverse matrix

$$\begin{gathered}\mathbf{R}^{-1} = (\mathbf{D}_r + r_m \mathbf{jj}^\wedge)^{-1} = [\mathbf{D}_r (\mathbf{I} + r_m \mathbf{D}_r^{-1} \mathbf{jj}^\wedge)]^{-1} = (\mathbf{I} + r_m \mathbf{D}_r^{-1} \mathbf{jj}^\wedge)^{-1} \mathbf{D}_r^{-1} = \\ (\mathbf{I} - \frac{1}{g_m + \mathbf{j}^\wedge \mathbf{D}_r^{-1} \mathbf{j}} \mathbf{D}_r^{-1} \mathbf{jj}^\wedge) \mathbf{D}_r^{-1} = \mathbf{D}_r^{-1} - \frac{1}{g_m + \mathbf{j}^\wedge \mathbf{D}_r^{-1} \mathbf{j}} \mathbf{D}_r^{-1} \mathbf{jj}^\wedge \mathbf{D}_r^{-1} = \mathbf{D}_g - \frac{1}{g_+} \mathbf{gg}^\wedge = \mathbf{G},\end{gathered} \tag{6}$$

where $\mathbf{D}_g = \mathbf{D}_r^{-1} = diag(g_1 \quad g_2 \quad ... \quad g_{m-1}); \mathbf{g}^\wedge = \mathbf{j}^\wedge \mathbf{D}_g = \| g_1 \quad g_2 \quad ... \quad g_{m-1} \|; g_+ = \sum_{k=1}^{m} g_k$.

Given (6) and the relation between the coordinates of the vector $\mathbf{u}(t)$ and the voltages $v_{k0}$,

$$\mathbf{u} = \begin{Vmatrix} u_1 \\ u_2 \\ ... \\ u_{m-1} \end{Vmatrix} = \begin{Vmatrix} v_1 - v_m \\ v_2 - v_m \\ ... \\ v_{m-1} - v_m \end{Vmatrix} = \begin{Vmatrix} v_{10} \\ v_{20} \\ ... \\ v_{m-10} \end{Vmatrix} - \begin{Vmatrix} v_{m0} \\ v_{m0} \\ ... \\ v_{m0} \end{Vmatrix} = \mathbf{v}_0 - v_{m0} \mathbf{j},$$

the equation for the vector of short-circuit currents will be as follows

$$\mathbf{i}_{SC}(t) = \mathbf{R}^{-1} \mathbf{u}(t) = (\mathbf{D}_g - \frac{1}{g_+} \mathbf{gg}^\wedge)(\mathbf{v}_0 - v_{m0} \mathbf{j}) = \mathbf{D}_g \mathbf{v}_0 - \left( v_{m0} + \frac{\mathbf{g}^\wedge \mathbf{v}_0 - v_{m0} \mathbf{g}^\wedge \mathbf{j}}{g_+} \right) \mathbf{g}.$$

The last expression in parentheses is zero because

$$\mathbf{g}^\wedge \mathbf{v}_0 - v_{m0} \mathbf{g}^\wedge \mathbf{j} + g_+ v_{m0} = \sum_{k=1}^{m-1} v_{k0} g_k + v_{m0} (g_+ - \sum_{k=1}^{m-1} g_k) = \sum_{k=1}^{m} v_{k0} g_k = 0$$

due to the marked linear dependence of $m$ voltages $v_{k0}$.

Let's express the short-circuit power loss in the voltage coordinates of the FBD method:

$$\begin{gathered}P_{SC} = \mathbf{u} \circ \mathbf{i}_{SC} = (\mathbf{v}_0 - v_{m0} \mathbf{j}) \circ \mathbf{D}_g \mathbf{v}_0 = \mathbf{v}_0 \circ \mathbf{D}_g \mathbf{v}_0 - T^{-1} \int_T v_{m0}(t) [v_{10}(t) g_1 + v_{20}(t) g_2 + ... + v_{m-10}(t) g_{m-1}] dt = \\ = T^{-1} \int_T [v_{10}^2(t) r_1 + v_{20}^2(t) / r_2 + ... + v_{m0}^2(t) / r_m] dt = (\mathbf{v}_+ \circ \mathbf{v}_+) / r = V^2 / r.\end{gathered} \tag{7}$$

Therefore, it is the square of the first multiplier in (1) divided by the normalizing resistance *r*.

After substituting (5), (7) in (1) and reducing the normalizing resistance, we obtain an expression for the apparent power formula similar to (4). The values that determine the apparent power in the two coordinate systems discussed above are summarized in Table 1 for comparative analysis.

**Table 1**

| | By FBD method | Proposed |
|---|---|---|
| Coordinate basis | $\mathbf{i}_+(t) = \left\| i_1\sqrt{r_1/r} \quad i_2\sqrt{r_2/r} \quad \ldots \quad i_m\sqrt{r_m/r} \right\|$; $\mathbf{v}_+(t) = \left\| v_{10}\sqrt{r/r_1} \quad v_{20}\sqrt{r/r_2} \quad \ldots \quad v_{m0}\sqrt{r/r_m} \right\|$ | $\mathbf{i}(t) = \left\| i_1 \quad i_2 \quad \ldots \quad i_{m-1} \right\|$; $\mathbf{u}(t) = \left\| u_1 \quad u_2 \quad \ldots \quad u_{m-1} \right\|$; |
| Voltage power loss $P_{SC}$ | $(\mathbf{i}_+ \circ \mathbf{i}_+)r$ | $\mathbf{i} \circ \mathbf{Ri}$ |
| Current power loss $P_{LS}$ | $(\mathbf{v}_+ \circ \mathbf{v}_+)/r$ | $\mathbf{u} \circ \mathbf{R}^{-1}\mathbf{u}$ |
| Apparent power $S$ | $\sqrt{(\mathbf{v}_+ \circ \mathbf{v}_+) \times (\mathbf{i}_+ \circ \mathbf{i}_+)}$ | $\sqrt{(\mathbf{i} \circ \mathbf{Ri}) \times (\mathbf{u} \circ \mathbf{R}^{-1}\mathbf{u})}$ |

An important practical aspect of changing the paradigm of the apparent power definition as the geometric mean value of current and voltage power losses is the possibility of its experimental verification as follows [1, 11]

$$S_E = P\sqrt{P_{LS} / P_{LS}^{MIN}}, \tag{8}$$

where $P = \mathbf{v}_+ \circ \mathbf{i}_+ = \mathbf{u} \circ \mathbf{i}$ is the active power; $P_{LS}^{MIN}$ is the minimum possible power loss in the transmission line caused by the active current according to the Fryze concept [20]. Measurement of three active powers with transparent physical content, which are included in the right part of (8), makes it possible to verify the theoretical determination of the apparent power [21], as well as its modification under additional restrictions on the consumed currents and voltages to improve the quality of electrical energy at the points of common coupling.

**The Influence of the Transmission Line Impedance Ratio on the Determination of Apparent Power and Energy-Efficient Active Filtering Strategies.** Of great importance for the theory of energy-efficient active filtration is the correct definition of the active current vector which, according to S. Fryze's conception [20], provides active power $P$ at the points of common coupling at a unit value of the power factor $\lambda = P/S$ and guarantees the minimum power loss $P_{LS}^{MIN}$ in the transmission line. In [14–19] it is shown that the active current is a fraction of the short-circuit current vector $\mathbf{i}_{SC}(t)$, that is equal to the ratio of active power to voltage short-circuit power loss:

$$\mathbf{i}_A(t) = \frac{P}{P_{SC}}\mathbf{i}_{SC}(t) = \frac{P}{\mathbf{u} \circ \mathbf{R}^{-1}\mathbf{u}}\mathbf{R}^{-1}\mathbf{u}(t). \tag{9}$$

The formation of such a current by means of shunt active filtration ensures the minimum power loss in the transmission line [17]

$$P_{LS}^{MIN} = \mathbf{i}_A \circ \mathbf{Ri}_A = \frac{P^2}{\mathbf{u} \circ \mathbf{R}^{-1}\mathbf{u}} \tag{10}$$

and the energy-saving effect, which is estimated by the ratio of power losses in the transmission line in the absence and presence of a filter as follows

$$W = \frac{P_{LS}}{P_{LS}^{MIN}} = \frac{\mathbf{i} \circ \mathbf{Ri}}{\mathbf{i}_A \circ \mathbf{Ri}_A} = \frac{\mathbf{i} \circ \mathbf{Ri}}{P^2 / (\mathbf{u} \circ \mathbf{R}^{-1}\mathbf{u})} = \frac{S^2}{P^2} = \lambda^{-2}. \tag{11}$$

When deriving (11), the determination of the apparent power by (2) was used, which indicates its compliance with the criterion of compatibility with the physical content of the power factor [1, 11] in the most general case of periodic currents and voltages.

Since one of the constituent part of the apparent power $P_{SC}$ appears in the denominator of the scalar coefficient (9), the determination of the apparent power affects the correct formation of the active current vector. Thus, for the case of identical resistances of line wires $r_1 = r_2 = r_{m-1} = 0$ and zero resistance of neutral, the matrix of transmission line resistances is proportional to the unit matrix $\mathbf{R} = r\mathbf{I}$, $(\mathbf{i} \circ \mathbf{Ri}) \times (\mathbf{u} \circ \mathbf{R}^{-1}\mathbf{u}) = r(\mathbf{i} \circ \mathbf{i}) \times r^{-1}(\mathbf{u} \circ \mathbf{u}) = (\mathbf{i} \circ \mathbf{i}) \times (\mathbf{u} \circ \mathbf{u})$, and the apparent power can be determined by the Buchholz formula as a product of the effective values of voltage and current, and the active current becomes proportional to the vector of Fryze's formula [20] squared the effective value of the voltage vector in the denominator. However, as shown in [18], when the resistances of the line wires are equal and the neutral resistance is non-zero, the formation of active current according to the Fryze's formula does not provide a unit value of the power factor.

Let us substantiate the active current formula for the purposes of practical implementation of active current in active filtration control systems for the general case of different resistances of line wires and neutral. Since in (2) and (9) the matrices $\mathbf{R}, \mathbf{R}^{-1}$ can be determined with an accuracy of a constant multiplier, consider the matrices normalized with respect to the resistance of the neutral wire $r_m$

$$\bar{\mathbf{R}} = \frac{1}{r_m}\mathbf{R} = \begin{Vmatrix} \rho_1^{-1} & 0 & 0 \\ 0 & ... & 0 \\ 0 & 0 & \rho_{m-1}^{-1} \end{Vmatrix} + \mathbf{j}^{\wedge}\mathbf{j}; \rho_k = r_m / r_k; k = 1, 2, ..., m-1;$$

$$\bar{\mathbf{G}} = \bar{\mathbf{R}}^{-1} = \begin{Vmatrix} \rho_1 & 0 & 0 \\ 0 & ... & 0 \\ 0 & 0 & \rho_{m-1} \end{Vmatrix} - \frac{1}{\rho_+}\begin{Vmatrix} \rho_1 \\ ... \\ \rho_{m-1} \end{Vmatrix}\begin{Vmatrix} \rho_1 \\ ... \\ \rho_{m-1} \end{Vmatrix}^{\wedge}; \rho_+ = 1 + \sum_{k=1}^{m-1}\rho_k$$

and find the equation for the reference vector of active current voltages

$$\mathbf{u}_\rho(t) = \bar{\mathbf{G}}\mathbf{u}(t) = \begin{Vmatrix} \rho_1 u_1 \\ ... \\ \rho_{m-1}u_{m-1} \end{Vmatrix} - \frac{\sum_{k=1}^{m-1} u_k \rho_k}{\rho_+}\begin{Vmatrix} \rho_1 \\ ... \\ \rho_{m-1} \end{Vmatrix} = \begin{Vmatrix} (u_1 - u_0)\rho_1 \\ ... \\ (u_{m-1} - u_0)\rho_{m-1} \end{Vmatrix}; u_0 = \frac{\sum_{k=1}^{m-1} u_k \rho_k}{\rho_+}. \quad (12)$$

Taking into account the new notation, the equation for active current (9) will be as follows

$$\mathbf{i}_A(t) = \frac{P}{\mathbf{u} \circ \mathbf{u}_\rho}\mathbf{u}_\rho(t) = \frac{P}{T^{-1}\int_T \sum_{k=1}^{m-1} \rho_k u_k(t)[u_k(t) - u_0(t)]dt}\begin{Vmatrix} [u_1(t) - u_0(t)]\rho_1 \\ ... \\ [u_{m-1}(t) - u_0(t)]\rho_{m-1} \end{Vmatrix}. \quad (13)$$

Normalized short-circuit power is simplified according to

$$\begin{aligned} \mathbf{u} \circ \bar{\mathbf{G}}\mathbf{u} = r_m P_{SC} &= T^{-1}\int_T \sum_{k=1}^{m-1} \rho_k u_k(t)[u_k(t) - u_0(t)]dt = T^{-1}\int_T \sum_{k=1}^{m-1} \rho_k u_k^2(t)dt - T^{-1}\int_T u_0(t)\sum_{k=1}^{m-1} \rho_k u_k(t)dt = \\ &= T^{-1}\int_T \sum_{k=1}^{m-1} \rho_k u_k^2(t)dt - \rho_+ T^{-1}\int_T u_0^2(t)dt = \sum_{k=1}^{m-1} \rho_k U_k^2 - \rho_+ U_0^2, \end{aligned} \quad (14)$$

and the equation for apparent power will be as follows

$$S = \sqrt{(\mathbf{i} \circ \bar{\mathbf{R}}\mathbf{i}) \times (\mathbf{u} \circ \bar{\mathbf{G}}\mathbf{u})} = \sqrt{\left(\sum_{k=1}^{m-1} I_k^2(t)\rho_k^{-1} + I_m^2\right) \times \left(\sum_{k=1}^{m-1} \rho_k U_k^2 - \rho_+ U_0^2\right)}. \quad (15)$$

If the resistance of one of the line wires (for the *n*-th is certain) is zero, the equation for the reference vector of voltages can be found using the limit junction in the following

$$\lim_{\rho_n \to \infty} u_0 = \lim_{\rho_n \to \infty} \frac{\sum_{k=1}^{m-1} u_k \rho_k}{1 + \sum_{k=1}^{m-1} \rho_k} = u_n; u_{\rho k}\Big|_{k \neq n} = (u_k - u_n)\rho_k;$$

$$u_{\rho n} = \lim_{\rho_n \to \infty}\left(u_n - \frac{\sum_{k=1}^{m-1} u_k \rho_k}{1 + \sum_{k=1}^{m-1} \rho_k}\right)\rho_n = u_n + \sum_{k=1; k \neq n}^{m-1} (u_n - u_k)\rho_k.$$

Thus, under the condition $r_n = 0$ the equations of the apparent power and active current will be as follows

$$S = \sqrt{(\mathbf{i} \circ \bar{\mathbf{R}}\mathbf{i}) \times (\mathbf{u} \circ \mathbf{u}_\rho^{r_n=0})}; \mathbf{i}_A(t) = \frac{P}{\mathbf{u} \circ \mathbf{u}_\rho^{r_n=0}}\mathbf{u}_\rho^{r_n=0}(t), \quad (16)$$

where $\mathbf{u}_\rho^{r_n=0}(t) = \begin{Vmatrix} (u_1 - u_n)\rho_1 & ... & u_n + \sum_{k=1, k \neq n}^{m-1} (u_n - u_k)\rho_k & ... & (u_{m-1} - u_n)\rho_{m-1} \end{Vmatrix}^{\wedge}$.

**The formulas of apparent power and active current which meet the requirements of modern IEEE standard.** The current standard [10] defines apparent power as the maximum active power that can be transmitted under sinusoidal and balanced conditions with the same RMS values of voltage and current. To fulfill this requirement for apparent power definition, equation (2), which includes periodic voltages and currents of arbitrary shape, needs to be corrected, but is useful as a basis for comparison.

Let the direct sequence detector [22] select a vector of symmetrical sinusoidal voltages $\mathbf{u}_+(t)$ from the voltage vector of general periodic shape $\mathbf{u}(t)$, then the active current providing the power $P$ at the points of the general coupling under the additional condition of the same form of consumed currents is determined as follows [15]

$$\mathbf{i}_{A+}(t) = \frac{P}{\mathbf{u} \circ \mathbf{u}_+}\mathbf{u}_+(t) = \frac{P}{\mathbf{u}_+ \circ \mathbf{u}_+}\mathbf{u}_+(t). \quad (17)$$

This current differs from the active current (9), which consists of asymmetrical currents at different values of transmission line impedances and causes power loss in the transmission line

$$P_{LS+} = \mathbf{i}_{A+} \circ \mathbf{R}\mathbf{i}_{A+} = \left(\frac{P}{\mathbf{u}_+ \circ \mathbf{u}_+}\right)^2 \times (\mathbf{u}_+ \circ \mathbf{R}\mathbf{u}_+) = \frac{P^2(\mathbf{u}_+ \circ \mathbf{R}\mathbf{u}_+)}{(\mathbf{u}_+ \circ \mathbf{u}_+)^2}. \quad (18)$$

The resulting value differs from the minimum value by (10). Let's find the ratio of the corresponding power losses

$$k_{LS+} = \frac{P_{LS}^{MIN}}{P_{LS+}} = \frac{P^2}{\mathbf{u} \circ \mathbf{R}^{-1}\mathbf{u}} \Big/ \frac{P^2(\mathbf{u}_+ \circ \mathbf{R}\mathbf{u}_+)}{(\mathbf{u}_+ \circ \mathbf{u}_+)^2} = \frac{(\mathbf{u}_+ \circ \mathbf{u}_+)^2}{(\mathbf{u}_+ \circ \mathbf{R}\mathbf{u}_+) \times (\mathbf{u} \circ \mathbf{R}^{-1}\mathbf{u})}. \quad (19)$$

This coefficient does not exceed a unit value, since $(\mathbf{u}_+ \circ \mathbf{R}^{-1}\mathbf{u}_+) \le (\mathbf{u} \circ \mathbf{R}^{-1}\mathbf{u})$, therefore

$$k_{LS+} \le \frac{(\mathbf{u}_+ \circ \mathbf{u}_+)^2}{(\mathbf{u}_+ \circ \mathbf{R}\mathbf{u}_+) \times (\mathbf{u}_+ \circ \mathbf{R}^{-1}\mathbf{u}_+)},$$

and the right side of the last inequality does not exceed the unity in according with the Cauchy-Schwartz inequality.

Gain in power loss when generating active current (17) in the transmission line

$$W_+ = \frac{P_{LS}}{P_{LS+}} = \frac{P_{LS}}{P_{LS}^{MIN}} \times \frac{P_{LS}^{MIN}}{P_{LS+}} = Wk_{LS+} = \lambda^{-2}k_{LS+} \quad (20)$$

differs from (11) by a factor $k_{LS+}$. The value of the apparent power $S_+$ subject to the limitation of the consumed currents is also subject to correction, since the minimum achievable value of losses $P_{LS+}$ under this condition differs from (10). In this case, the formula for modified apparent power definition follows from (8), (11), (19)

$$S_+ = P\sqrt{P_{LS} / P_{LS+}} = P\sqrt{\lambda^{-2}k_{LS+}} = S\sqrt{k_{LS+}} = \frac{\mathbf{u}_+ \circ \mathbf{u}_+}{\sqrt{(\mathbf{u}_+ \circ \mathbf{R}\mathbf{u}_+) \times (\mathbf{u} \circ \mathbf{R}^{-1}\mathbf{u})}}S, \quad (21)$$

which allows experimental verification by measuring of the corresponding voltages and powers. Thus, the value $k_{LS+}$ is a correction factor for calculating the apparent power and power loss gains when implementing limits on consumed currents to improve the quality of electrical energy at the points of common coupling.

**Virtual experiment.** The purpose of the experiment is to verify the formulas of apparent power (15) and (21) for the following ratios between the wires of a three-phase four-wire transmission line $r_A = r; r_B = r/2; r_C = r/3; r_N = dr$. Parameter $d$ varies discretely within $0.1 \le d \le 5$, the voltage range of a three-phase four-wire network adopted symmetrical sinusoidal with an effective value $U$=220 V. Nonlinear load is a rectifier according to the three-phase half-wave rectifier circuit (Fig. 1), operating on active resistance $R$=1 Ω. Normalizing resistor resistance $r$ is assumed to be equal to 1 mΩ to ensure a ratio $P_{LS} \ll P \ll P_{SC}$ of at least 100 times for each inequality to ensure one percent instability of the load power under the action of current active filtering compensations and changes in the parameter $d$. The value of the transmission line neutral resistance $r_N$ is set by the parameters of the units d and r. Without Filter and Short Circuit units calculate the load power in the absence of a filter and the short-circuit power, respectively, depending on the parameter $d$, which are then used for the operation of the Control System unit, which calculates the values of compensation currents depending on the chosen strategy. The Control System unit

(Fig. 2) works as follows: Up to time 0.1s there is no compensation, at time 0.1 s the compensation is turned on, which provides active current in the transmission line, at time 0.2 s compensation is turned on, which minimizes the power loss at symmetrical sinusoidal currents in the transmission line.

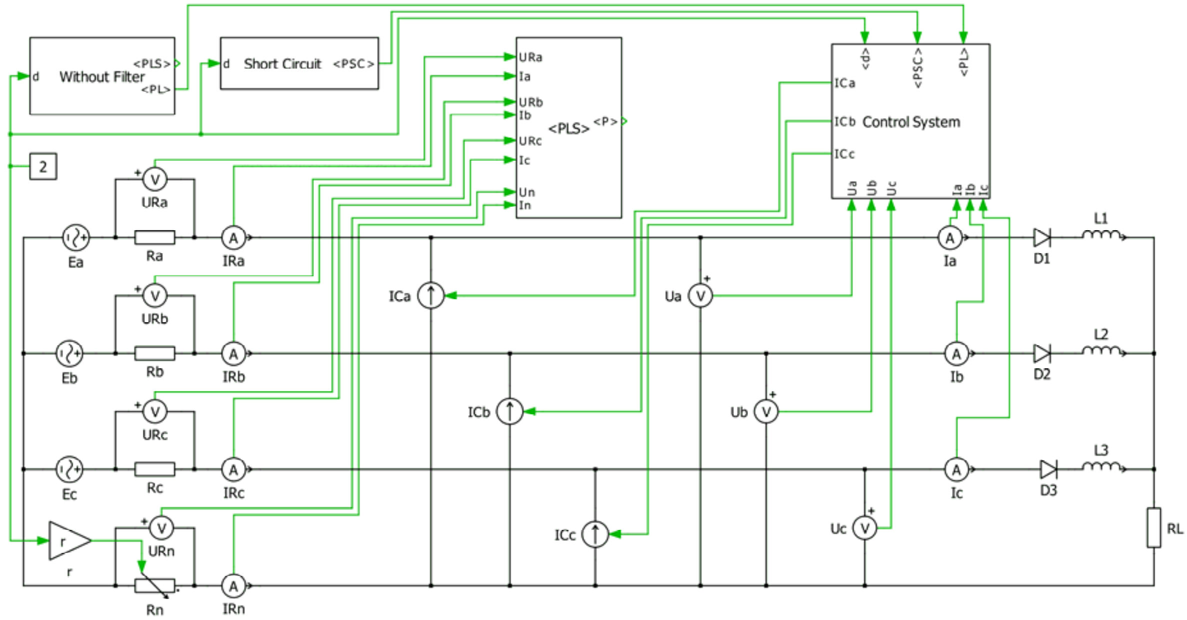
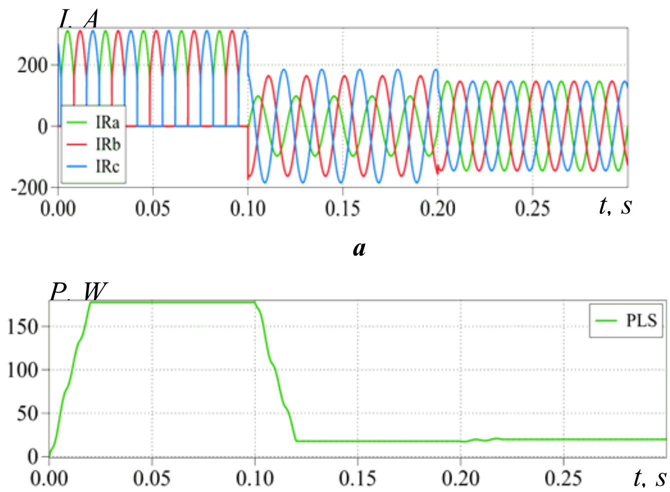



**Fig. 2**. Computer model of a three-phase four-wire power supply system




***a***

***b***

**Fig. 3** Time diagrams of current consumption ($a$) and loss power ($b$) under different compensation modes

The adopted computer model of the network makes it possible to obtain the analytical dependence of the apparent power at (15) on the value of the parameter $d$. The matrices $\mathbf{\bar{R}}, \mathbf{\bar{G}}$ for the given resistive parameters of the transmission line will be as follows

$$\mathbf{\bar{R}} = \frac{1}{r_N}\mathbf{R} = \begin{Vmatrix} \rho_A^{-1} & 0 & 0 \\ 0 & \rho_B^{-1} & 0 \\ 0 & 0 & \rho_C^{-1} \end{Vmatrix} + \begin{Vmatrix} 1 \\ 1 \\ 1 \end{Vmatrix}\begin{Vmatrix} 1 \\ 1 \\ 1 \end{Vmatrix}^{\wedge};$$

$$\rho_A = \frac{r_N}{r_A} = \frac{dr}{r} = d; \quad \rho_B = \frac{r_N}{r_B} = \frac{dr}{r/2} = 2d;$$

$$\rho_C = \frac{r_N}{r_C} = \frac{dr}{r/3} = 3d;$$

$$\mathbf{\bar{G}} = \mathbf{\bar{R}}^{-1} = \begin{Vmatrix} \rho_A & 0 & 0 \\ 0 & \rho_B & 0 \\ 0 & 0 & \rho_C \end{Vmatrix} - \frac{1}{\rho_+}\begin{Vmatrix} \rho_A \\ \rho_B \\ \rho_C \end{Vmatrix}\begin{Vmatrix} \rho_A \\ \rho_B \\ \rho_C \end{Vmatrix}^{\wedge}; \qquad \rho_+ = 1 + \rho_A + \rho_B + \rho_C = 6d + 1.$$

According to (12), we determine the complex effective value

$$\dot{U}_0 = U(\rho_A + \tilde{a}\rho_B + \dot{a}\rho_C)/\rho_+ = U(d + 2d\tilde{a} + 3d\dot{a})/\rho_+ = Ud(\tilde{a} + 2\dot{a})/\rho_+ = \sqrt{3}Ude^{j5\pi/6}/\rho_+.$$

The relative voltage power loss does not depend on the selected load and, according to (14), is determined as follows

$$\mathbf{u}\circ\bar{\mathbf{G}}\mathbf{u} = \rho_A U_A^2 + \rho_B U_B^2 + \rho_C U_C^2 - \rho_+ U_0^2 = d(1+2+3)U^2 - U^2\frac{3d^2}{1+6d} =$$

$$= \frac{dU^2(6+36d-3d)}{1+6d} = \frac{3dU^2(2+11d)}{1+6d} = U^2\frac{1+5.5d}{1+1/6d}.$$

The currents of the transmission line for the selected load are symmetrical sinusoidal within each third of the mains voltage period (Fig. 3 in the range 0-0.1s), so the normalized power of current losses at (15)

$$\mathbf{i}\circ\bar{\mathbf{R}}\mathbf{i} = I_A^2\rho_A^{-1} + I_A^2\rho_B^{-1} + I_C^2\rho_B^{-1} + I_N^2 = \frac{3+\rho_A^{-1}+\rho_B^{-1}+\rho_C^{-1}}{T}\int_{-T/6}^{T/6}\frac{2U^2}{R^2}\cos^2(\omega t)dt =$$

$$= \frac{U^2[3+d^{-1}(1+1/2+1/3)}{R^2 T}\times\left[\frac{T}{3}+\frac{2\sin(2\omega T/6)}{2\omega}\right] = \frac{U^2(1+11/18d)}{R^2}\times\left(1+\frac{3\sqrt{3}}{4\pi}\right).$$

Substitution of the obtained values of the relative power losses on the transmission line impedances in (15) gives a theoretical formula for the dependence of the apparent power on the parameter *d*:

$$S(d) = \frac{U^2}{R}\sqrt{1+\frac{3\sqrt{3}}{4\pi}}\times\sqrt{\frac{(1+5.5d)(1+11/18d)}{1+1/6d}}. \tag{22}$$

Fig. 4 presents the graph of this dependence, which is fully confirmed by the plotted points, the ordinates *S1* of which are calculated according to (4) as the geometric mean values of the experimentally obtained power losses $P_{LS}, P_{SC}$, summarized in Table 2.

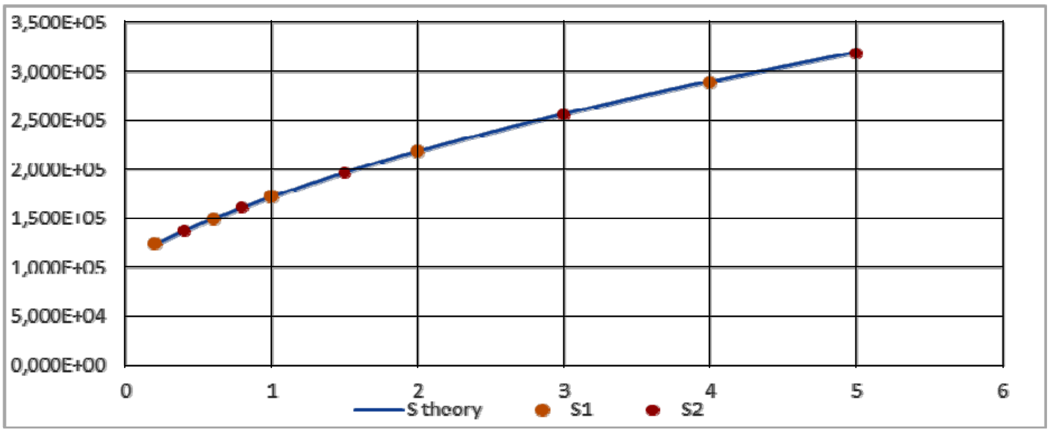



**Fig. 4**. Graph of the apparent power dependence on the parameter $d$ by (15) and experimental points $S_1, S_2$

To verify this apparent power functional dependence from other hand, we use (8) with experimental determination of the apparent power, where the minimum current loss power $P_{LS}^{MIN}$ is created by the active current (13). In this computer model of the power supply system, this current will be as follows

$$\mathbf{i}_A(t) = \frac{P}{\mathbf{u}\circ\mathbf{u}_\rho}\mathbf{u}_\rho(t) = \frac{P}{r_N P_{SC}}\begin{Vmatrix}(u_A-u_0)\rho_A\\(u_B-u_0)\rho_B\\(u_C-u_0)\rho_C\end{Vmatrix} = \frac{P}{rP_{SC}}\begin{Vmatrix}u_A-u_0\\2(u_B-u_0)\\3(u_C-u_0)\end{Vmatrix};$$

$$u_0 = \frac{u_A\rho_A+u_B\rho_B+u_C\rho_C}{1+\rho_A+\rho_B+\rho_C} = \frac{(u_A+2u_B+3u_C)d}{1+6d} = \frac{u_A+2u_B+3u_C}{6+1/d}.$$

where $\mathbf{u}(t) = \mathbf{u}_+(t) = \|u_A \quad u_B \quad u_C\|^\wedge$ is the instantaneous voltage vector at the points of common coupling. The measurement results of $P_{LS}^{MIN}$, *P* for each value of *d* are summarized in Table 2, on the basis of which the points of experimental values $S_2$, plotted on the graph (Fig. 2) of the theoretical dependence $S(d)$ according to (22) are calculated. We state the complete coincidence of theoretical and experimental data.

**Table 2**

| $d$ | $P_{SC}$ x $10^8$ | $P_{LS}$ | $P_{LS}^{MIN}$ | $P$ x $10^4$ | $P_{LS+}$ | $S_1$ x $10^5$ | $S_2$ x $10^5$ | $k_{LS+}$ |
|---|---|---|---|---|---|---|---|---|
| 0,2 | 2,772 | 55,348 | 16,818 | 6,830 | 19,617 | 1,239 | 1,239 | 0,857 |
| 0,4 | 2,733 | 68,979 | 17,043 | 6,827 | 19,601 | 1,373 | 1,373 | 0,869 |
| 0,6 | 2,715 | 82,600 | 17,146 | 6,824 | 19,586 | 1,497 | 1,498 | 0,875 |
| 0,8 | 2,704 | 96,209 | 17,201 | 6,821 | 19,570 | 1,613 | 1,613 | 0,879 |
| 1 | 2,697 | 109,808 | 17,233 | 6,819 | 19,554 | 1,721 | 1,721 | 0,881 |
| 1,5 | 2,686 | 143,758 | 17,265 | 6,812 | 19,515 | 1,965 | 1,966 | 0,885 |
| 2 | 2,681 | 177,640 | 17,267 | 6,805 | 19,477 | 2,182 | 2,183 | 0,887 |
| 3 | 2,675 | 245,201 | 17,236 | 6,791 | 19,399 | 2,561 | 2,562 | 0,888 |
| 4 | 2,672 | 312,492 | 17,187 | 6,778 | 19,322 | 2,889 | 2,890 | 0,889 |
| 5 | 2,670 | 379,516 | 17,131 | 6,764 | 19,245 | 3,183 | 3,184 | 0,890 |

To verify the formula of apparent power (21) in the presence of constraints on symmetrical sinusoidal consumed currents, we calculate the theoretical dependence of the coefficient $k_{LS+}$ on the parameter $d$ for a given load, build its graph and plot on it the points calculated from the measurement results $P_{LS+}$. Note that for symmetrical sinusoidal source voltages

$$k_{LS+}=\frac{(\mathbf{u}_+\circ\mathbf{u}_+)^2}{(\mathbf{u}_+\circ\bar{\mathbf{R}}\mathbf{u}_+)\times(\mathbf{u}_+\circ\bar{\mathbf{G}}\mathbf{u}_+)}, \qquad (23)$$

Moreover, for sinusoidal variable vectors, the integral scalar product is equal to the real part of their phasor product $\mathbf{x}\circ\mathbf{y}=T^{-1}\int_T\mathbf{x}^\wedge(t)\mathbf{y}(t)dt=\mathrm{Re}(\mathbf{x}^\wedge\times\mathbf{y}^*)$.

With this in mind, the components of (23) will be as follows

$$\mathbf{u}_+\circ\mathbf{u}_+=3U^2;\bar{\mathbf{R}}\mathbf{u}_+/U=\left(\begin{Vmatrix}\rho_A^{-1}&0&0\\0&\rho_B^{-1}&0\\0&0&\rho_C^{-1}\end{Vmatrix}+\mathbf{j}^\wedge\mathbf{j}\right)\begin{Vmatrix}1\\\tilde{a}\\\dot{a}\end{Vmatrix}=\begin{Vmatrix}\rho_A^{-1}\\\rho_B^{-1}\tilde{a}\\\rho_C^{-1}\dot{a}\end{Vmatrix}=\frac{1}{d}\begin{Vmatrix}1\\\tilde{a}/2\\\dot{a}/3\end{Vmatrix};\mathbf{u}_+\circ\bar{\mathbf{R}}\mathbf{u}_+=\frac{U^2}{d}\begin{Vmatrix}1\\\tilde{a}\\\dot{a}\end{Vmatrix}^\wedge\begin{Vmatrix}1\\\tilde{a}/2\\\dot{a}/3\end{Vmatrix}^*=\frac{11U^2}{6d}.$$

$$\bar{\mathbf{G}}\mathbf{u}_+=\mathbf{u}_\rho(t)=\bar{\mathbf{G}}\mathbf{u}(t)=\begin{Vmatrix}u_A\rho_A\\u_B\rho_B\\u_C\rho_C\end{Vmatrix}-u_0\begin{Vmatrix}\rho_A\\\rho_B\\\rho_C\end{Vmatrix}=d\left(\begin{Vmatrix}u_A\\2u_B\\3u_C\end{Vmatrix}-\frac{u_A+2u_B+3u_C}{6+1/d}\begin{Vmatrix}1\\2\\3\end{Vmatrix}\right);$$

$$\mathbf{u}_+\circ\bar{\mathbf{G}}\mathbf{u}_+=U^2\,\mathrm{Re}\begin{Vmatrix}1&\tilde{a}&\dot{a}\end{Vmatrix}\begin{Vmatrix}1\\2\tilde{a}\\3\dot{a}\end{Vmatrix}^*-U^2\,\mathrm{Re}\left(\frac{1+2\tilde{a}+3\dot{a}}{6+1/d}\right)^*\begin{Vmatrix}1&\tilde{a}&\dot{a}\end{Vmatrix}\begin{Vmatrix}1\\2\\3\end{Vmatrix}^*=U^2\left(6-\frac{1}{2+1/3d}\right).$$

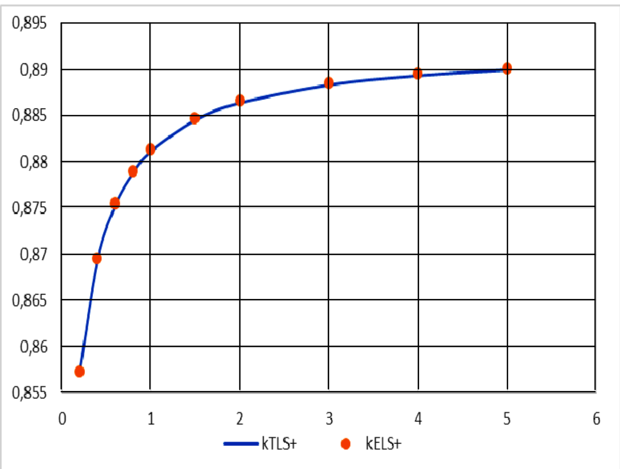



**Fig. 5.** Graph of the dependence of the gross power correction factor $k_{LS+}$ on the parameter $d$ with applied experimental points

Substituting the obtained values in (23) finally gives the theoretical functional dependence of the apparent power correction factor on the parameter $d$:

$$k_{LS+}(d)=\frac{3^2}{\frac{11}{6d}\times\left(6-\frac{1}{2+1/3d}\right)d}=\frac{9/11}{1-\frac{1}{12+2/d}}.$$

The graph of this dependence is presented in Fig. 5 by solid blue line. For a given model of a power supply system with a symmetrical sinusoidal voltages and different values of transmission line resistances, a symmetrical sinusoidal active current according to (17)

$$\mathbf{i}_{A+}(t)=\frac{P}{\mathbf{u}_+\circ\mathbf{u}_+}\mathbf{u}_+(t)=\frac{P}{3U^2}\begin{Vmatrix}u_A\\u_B\\u_C\end{Vmatrix}.$$

The experimental values of the loss rates $P_{LS+}$, created by this current in the transmission line depending on the parameter $d$ are summarized in Table 2. Using them, according to (17), discrete experimental values of the correction coefficients $k_{LS+}$, located in the last column of Table 2 were calculated and plotted on the graph (Fig. 5) with orange dots.

And in this case, we state the complete coincidence of theoretical and experimental data. The action of symmetrical active filtration (after 0.2 s in Fig. 3) completely equalizes the amplitudes of the consumed currents, slightly inferior to energy-efficient active filtering in terms of the power of losses in the transmission line. For the one presented in Fig. 3 cases $d$ = 2 experimental data of power losses in Table 2 are $P_{LS}^{MIN}$ =17.267 W and $P_{LS}$ =19.477 W corresponding to the experimental correction factor $k_{LS+}^{E}$ = 0.888, the theoretical value of this coefficient

$$k_{LS+}^{T}=\frac{9/11}{1-\dfrac{1}{12+2/2}}=\frac{9\times 13}{11\times 12}=0{,}886$$

corresponds to an error of 0.2%.

**Conclusions.**

1. It is shown that the known formulas of apparent power for multiphase power transmission systems with different transmission line resistances, based on the FBD method, are fully equivalent to the proposed definition of apparent power in the form of the geometric average value of the power losses on the transmission line supports from currents and voltages at the points of common coupling. The advantages of the proposed definition are the reduced coordinate basis, the absence of the problem of organizing the artificial grounding point, the possibility of verifying the formula of the apparent power by measuring the corresponding power losses and its modification under conditions of additional restrictions on the consumed currents.
2. Formulas for the active current of multiphase power system with different transmission line resistances are obtained, which determine energy-efficient active filtration strategies in a reduced coordinate basis with the achievement of a unit power factor value.
3. A virtual experiment has developed and verified an active filtration strategy to improve the quality of electrical energy at the points of common coupling. This strategy ensures the minimum power loss of a multiphase power transmission system with different transmission line resistances at symmetrical sinusoidal consumed currents.
4. The corrective coefficient for the formulas of apparent power and power loss gain in the presence of restrictions on the consumed currents was determined and verified by a virtual experiment. Its value can be used to predict the maximum load of the network in compliance with the existing requirements for the quality of electrical energy at the points of common coupling.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УДК 621.314

**УЗАГАЛЬНЕНЕ ВИЗНАЧЕННЯ ПОВНОЇ ПОТУЖНОСТІ ТА ЕНЕРГОЕФЕКТИВНІ СТРАТЕГІЇ АКТИВНОЇ ФІЛЬТРАЦІЇ В СКОРОЧЕНОМУ КООРДИНАТНОМУ БАЗИСІ БАГАТОФАЗНОЇ СИСТЕМИ ЕЛЕКТРОЖИВЛЕННЯ**

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*У роботі обґрунтовано еквівалентність визначення повної потужності багатофазної системи електроживлення з різними опорами лінії передачі за методом Фриза-Бухгольца-Деппенброка та в скороченому координатному базисі. Запропоновано дві енергоефективні стратегії керування паралельною активною фільтрацією у скороченому координатному базисі. Перша стратегія забезпечує одиничне значення коефіцієнта потужності, а друга – мінімізує потужність втрат в лінії передачі під час дотриманя симетрії та квазісинусоїдної форми споживаних струмів. Перевагами використання скороченого координатного базису є зменшення кількості датчиків і ключових регуляторів активних фільтрів, а також відсутність проблеми організації точки штучного заземлення для вимірювання напруг. Визначено та верифіковано коригувальний коефіцієнт для формул повної потужності та коефіцієнта потужності за наявності обмежень на симетричну та синусоїдну форму споживаних струмів.* Бібл. 22, рис. 5, табл. 2.

***Ключові слова:*** повна потужність, коефіцієнт потужності, мінімізація втрат потужності, стратегія керування активним фільтром, скорочений координатний базис.

Надійшла 09.05.2024
Остаточний варіант 12.09.2024