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ANALYSIS OF TRANSIENT AND STEADY-STATE MODES IN THE POWER SUPPLY SYSTEM OF ELECTRIC VEHICLES, WHICH CONTAINS A BATTERY AND SUPERCAPACITORS CONNECTED IN PARALLEL

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The paper analyzes transient and steady-state processes under the condition of pulse-width regulation of modes in the power supply system of an electric transport device (ETD) containing a battery and parallel-connected supercapacitors. Numerical experiments were carried out on a mathematical model of the electrical equivalent circuit of such a power supply system using the Matlab Simulink application software package. The dependences of the current values in the electrical circuits of the battery, supercapacitors and load on the frequency of pulse currents, their relative pulse duration, as well as the parameters of the battery and supercapacitors were determined. It has been established that the use of supercapacitors, which have a much lower internal resistance compared to the internal resistance of the battery, makes it possible to significantly reduce pulse currents in the electrical circuit of the battery of an ETD due to the operation of the integrated power system, in which the currents in the electrical discharge circuit of the supercapacitors will be much greater than the currents in the battery circuit. References 12, figures 6, tables 3.
Key words: supercapacitor, battery, internal resistance, integrated power system.

Introduction. At present, companies from Ukraine, USA, Japan, and EU countries have significantly improved the electrical energy characteristics of capacitive energy storage devices called supercapacitors (ultracapacitors, ionistors, and double-layer electrochemical capacitors). This improvement has caused a new round in the development of research aimed at the practical use of supercapacitors (SCs) in autonomous power supplies of electronic and computing devices, pulse technology and traction electric drives [1–3]. Supercapacitors, which surpass widely used electrolytic capacitors in specific energy and weight-dimensional parameters, can be used in parallel connection to the battery (B) of electric transport device [4–6] in order to for the implementation of short-term forced modes (i.e. high-power modes).

Connecting SCs in parallel with B virtually eliminates or significantly reduces pulse currents in the battery electric circuit during short-term acceleration of electric vehicles, their uphill movement and other short-term forced modes, which significantly increases the energy efficiency and service life of ETDs batteries [7–9]. The electrical equivalent circuit of the electric vehicle power supply system with this type of movement can be considered as a circuit, one of the parameters of which (load resistance) is characterized by a random variable. Consequently, all electrical characteristics of the circuit that depend on this parameter also become stochastic. The analysis of the probabilistic properties of electrical characteristics in circuits containing stochastic load was carried out in [10]. At the same time, in [11] it was shown that the declared advantages of supercapacitors in power supply systems of autonomous electric vehicles are not always realized. The experimental power supply modes of *R-L* load, represented by series-connected active resistance R_{load} = 0.1 Ohm and inductance L_{load} = 5–2000 μH, are described in [11]. It has been shown that currents with a frequency of 1–10 kHz and relative pulse duration of 1.75 – 4 in the battery circuit remain much larger than currents in the circuit of SCs even with their capacitance up to 60 F.

It is known that the energy characteristics of sources of constant electromotive force (voltage sources) and the ratio of currents in the circuits of parallel-connected B and SCs depend significantly on the value of their internal resistances. Therefore, it became advisable to conduct additional studies of currents in electrical circuits of B, SC and load in steady-state and transient modes of power supply of load, taking into

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account the mentioned internal resistances. Modes of increased power consumption in load, simulating the acceleration of experimental ETUs (i.e. forced modes), in particular when they move uphill, are especially interesting. It was also important to study the dependence of the electrical and energy modes of the power supply system on the value of the SCs capacitance, as well as frequency and relative pulse duration of the current pulses in the load.

Such experiments, which involve wide-range changes in many parameters of the power supply system and studies of its critical modes, should be carried out using numerical modeling methods. Currently, the Matlab Simulink application software package is widely used to develop mathematical models of power supply systems of this type.

Therefore, **the aim** of the work was to develop a Matlab Simulink-model and analyze transient and steady-state processes under the condition of pulse-width regulation of the modes of experimental ETD power supply from B and SCs connected in parallel. To achieve this aim, it was necessary to study the dependence of the current values in the electrical circuits of the B, SCs and load on the frequency of pulse currents, their relative pulse duration, as well as the electrical parameters of the B and SCs.

Discussion of the obtained results. Fig. 1 shows the electrical equivalent circuit of the power supply system that was used in the numerical experiments. This equivalent circuit differs from the circuit studied in [11] only by the introduced resistances R_B and R_C, which correspond to the resistances of the battery (which is represented in the circuit by the constant voltage source *B*) and SCs (which are represented in the circuit by the equivalent capacitance *C*). When modeling and analyzing transient processes in the equivalent circuit in Fig. 1, the parameters of its elements corresponded to the average parameters of standard batteries used in cars and the parameters of supercapacitors from Yunasko (Ukraine), Maxwell Technologies (USA), and NEC Tocin (Japan). The load parameters are selected for an experimental low-power electric vehicle. When using this model to obtain and analyze the results of transient processes in such a combined power supply system, it is necessary to take into account that the parameters of the circuit elements must be in the ranges of correctly selected values (i.e. within the limits of application of this model). The internal resistances of the battery and supercapacitors are especially important to consider. To realize the advantages of such a system, it is necessary that these resistances differ by an order of magnitude or more. If the ratio of internal resistances of the battery and supercapacitors decreases, such a combined system will lose its advantages. Thus, in the equivalent circuit of the combined power supply system in [11], the internal resistances of the battery and supercapacitors were not taken into account (they were assumed to be equal to zero), which did not correspond to the described experimental studies conducted by the authors. In the experiment, the supercapacitors were connected in such a way that their combined internal resistance significantly exceeded the internal resistance of the battery, which led to the loss of the advantage of such a combined system and, as a consequence, to the incorrect conclusion about the inefficiency of parallel connection of SC to B in ETD power systems.

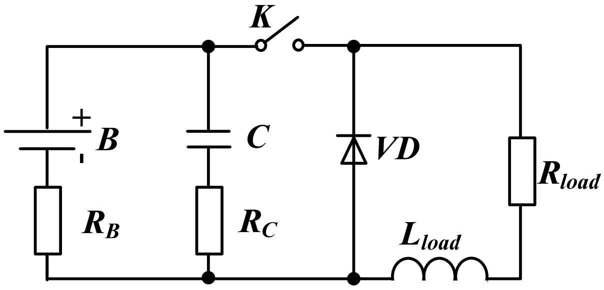


Fig.1

Fig. 2 shows the developed Simulink Matlab-model corresponding to the electrical equivalent circuit of the ETD power supply system shown in Fig. 1.

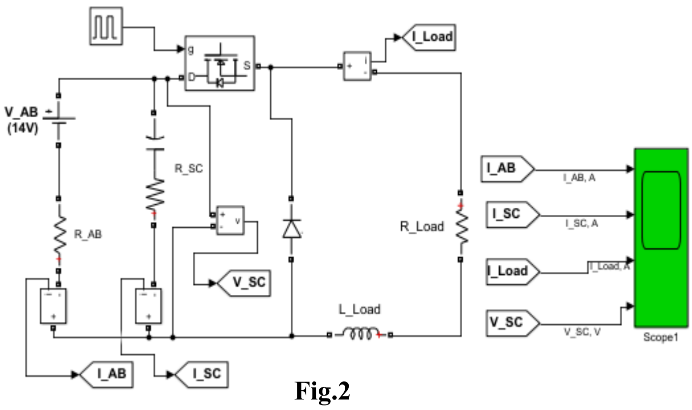


Fig.2

It was assumed that when the *R-L* load was disconnected, the battery voltage was $E = 14$ V and its internal resistance $R_B = 0.02$ Ohm. The capacitance *C* was 60–400 F, and internal resistance $R_C = 0.002$ Ohm = const. It was assumed that *C* was always charged to a voltage of $U_{C0} = E = 14$ V before connecting the *R-L* load. The active load resistance R_{load} was considered equal to 0.1 Ohm, and the load inductance L_{load} – equal to 100 µF. Switch *K* was a fully controlled switch that carried out pulse-width regulation of the output current of the power supply system, switching with a frequency f in the range from 0.4 kHz to 10 kHz, and relative pulse duration s in the range from 4/3 to 4. When the switch K is broken, the supercapacitor is charged by the battery, and when the K is closed, the supercapacitors and battery are discharged to the R-L load. It should be noted that at the moment of breaking the K, the current in the inductance I_L, according to the switching law, does not change instantly, but flows in the same direction in the electrical circuit L_{load}-VD-R_{load}, dissipating the energy accumulated in the inductance (W_L=0.5$L_{load}I_L^2$) in the active resistance R_{load}.

Let us note that at C = 60–400 F, the voltage on C changes insignificantly during the one load connection, therefore the subsequent charging of C from battery is carried out with an initial positive voltage on C, which differs insignificantly from the voltage on the battery terminal. Under such conditions, the losses of electrical energy in the active resistances of the charging circuit of the capacitor C are insignificant and the efficiency is close to 100%. [12].

When an ETD starts moving, accelerates, or moves uphill, the power of its engine (that is, the current in the load) briefly increases several times compared to the steady-state driving mode at a constant speed. The highest current in the load (in the electric vehicle's motor) is usually needed during the first few seconds when the ETD starts moving. According to the authors of [7–9], it is these short-term high-amplitude current pulses that reduce the battery's service life and determine the need to connect supercapacitors. Therefore, in this paper, the analysis of the indicated modes was carried out first.

Fig. 3 represents the calculated oscillograms of the currents in the electrical circuits of the battery $I_{AB}(t)$, supercapacitor $I_{SC}(t)$ and R-L load $I_{load}(t)$, as well as the voltage $U_{SC}(t)$ at the terminals of the supercapacitor when it is connected at the moment simulating the start of the ETD's movement. In this case, switch K created current pulses with a frequency f = 1 kHz and relative pulse duration s = 2.

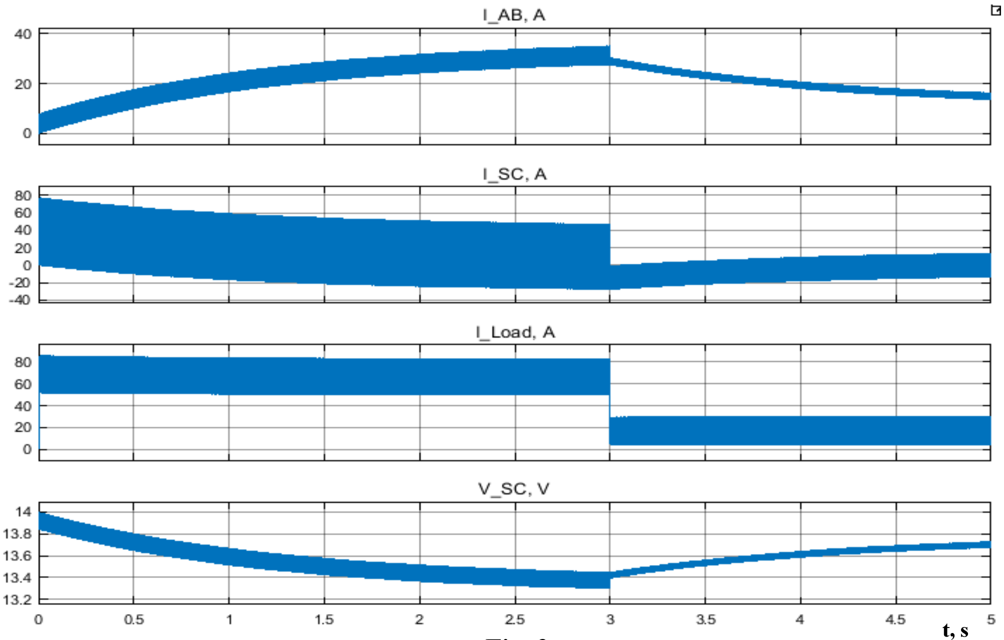


Fig. 3

The active load resistance R_{load} increased abruptly by 4 times after Δt = 3 s, simulating a decrease in the acceleration of the ETD. As can be seen from Fig. 3, the current in the battery circuit was several times less than the current in the supercapacitor circuit in the time range from 0 to 3 s. The greatest differences between the currents were observed at t < 0.5 s, that is, immediately after connecting the load simulating the start of the ETD's movement. It should be noted that the current in the battery circuit increased smoothly, starting from almost zero values, and not abruptly, as in the case without a supercapacitor.

When the load resistance increases abruptly by 4 times (t = 3 s), the current in the SC's electrical circuit decreases, and the voltage at its terminals increases (SC charges). As SC charges, the current in B's circuit begins to decrease.

Thus, it can be concluded that if the internal resistance of SC is an order of magnitude less than the internal resistance of B, then connecting SC to B virtually eliminates the abrupt increase in currents in the B's circuit when the ETD starts, accelerates, and moves uphill. In all the listed cases of increasing engine power, the current in the battery's electrical circuit will increase smoothly, starting from values close to zero.

Connecting a supercapacitor with low internal resistance to the battery also reduces the magnitude of pulse currents in the battery's circuit under conditions of constant power consumption of electricity in the load, that is, when the ETD moves at a constant speed (time range $0.5 < t < 3$ s in Fig. 3).

The influence of the switching frequency of the semiconductor switch *K*, the relative pulse duration of the pulse currents and the capacitance of the SC on the currents in the circuits of the power supply system of the ETD was also studied.

Table 1 presents the results of the numerical calculation of the dependencies of the highest values of currents in the electrical circuits of the B and SC, as well as the ranges of change of the load current and the average power (characterizing the electrical energy consumption in the load during one period) at different switching frequencies of the switch *K* and capacitances of SC, but with a constant the relative pulse duration of pulse currents in the switch *K* ($s = 2$).

Fig. 4 shows the dependence of the load power on the switching frequency of the switch *K* for different values of the supercapacitor capacitance.

Table 1

Experiment #	f, kHz	C, F	$I_{B\ max}$, A	$I_{SC\ max}$, A	$I_{B\ max} / I_{SC\ max}$	$[I_{load\ min}; I_{load\ max}]$, A	P, W
1	0.4	60	42.6	62	0.69	[24; 105]	463.7
2	1	60	37	45	0.82	[47; 83]	424.7
3	10	60	32	35	0.91	[63; 67]	423.5
4	0.4	100	42	62	0.67	[24; 104]	463.3
5	1	100	37	45	0.82	[46; 82]	426.8
6	10	100	35	31	1.12	[63; 66]	425.0
7	0.4	400	39	66	0.59	[22; 105]	478.8
8	1	400	35	48	0.73	[48; 83]	439.3
9	10	400	32	35	0.91	[63; 67]	435.0

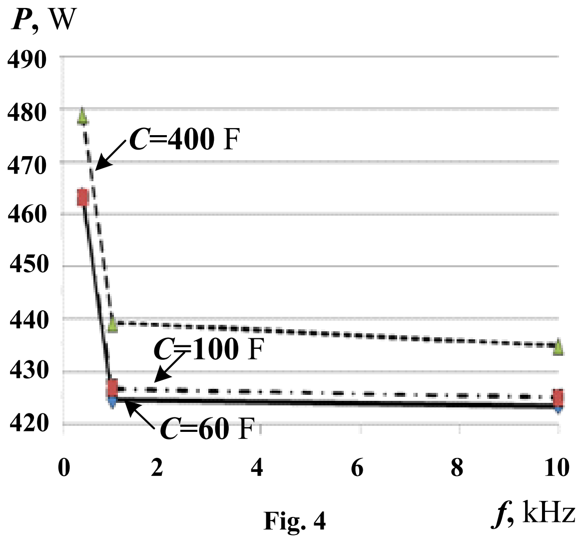


Fig. 4

As can be seen in Fig. 4, when the relative pulse duration $s = 2$, an increase in the switching frequency causes a decrease in the load power, and the greater the frequency, the less significant this decrease becomes. At a frequency f >1 kHz, such a decrease in load power practically disappears.

An increase in the frequency f causes a narrowing of the range of current change in the load (see Table 1), but this current does not change direction in the inductance L_{load} and its average value changes insignificantly.

Table 2 presents the results of calculating the highest values of currents in the battery and supercapacitor, changes in the average load power and ranges of load current depending on the relative pulse duration *s*, when the supercapacitor capacitance $C = 100$ F and the current switching frequency f was equal to 0.4, 1 and 10 kHz.

Fig. 5 shows the dependences of the ratio of the maximum values of pulse currents in the battery and supercapacitor on the relative pulse duration of the currents at different switching frequencies.

From Fig. 5 and Table 2 it is evident that by changing the relative pulse duration of the pulse currents in the electrical circuit of the switch, it is possible to significantly change the ratio between the currents in the circuits of the battery and supercapacitor, as well as regulate the power consumption of electricity in the load.

At low relative pulse duration of pulse currents in the switch *K* circuit, the currents in the B circuit are approximately 2 times higher than the currents in the SC circuit. With increasing relative pulse duration, the currents in the SC circuit begin to exceed the currents in the B circuit, since the SC charging time increases and its discharge time decreases, which increases the voltage on the SC. This causes a decrease in the voltage difference between B and SC and the SC charge currents from B. In this case, the power consumption of electricity in the load also decreases.

Table 2

Experiment #	*s* r.u.	*f*, kHz	$I_{B\ max}$, A	$I_{SC\ max}$, A	$I_{B\ max}/I_{SC\ max}$	$[I_{load\ min}; I_{load\ max}]$, A	P, W
1	1.33	0.4	77	41	1.87	[55; 118]	927.5
2	2	0.4	42	62	0.67	[24; 104]	463,3
3	4	0.4	15	53	0.28	[4; 68]	124
4	1.33	1	74	32	2.31	[77; 106]	896
5	2	1	37	45	0.82	[46; 82]	426,8
6	4	1	11	34	0.32	[17; 45]	94.8
7	1.33	10	74	23	3.21	[94; 97.5]	930
8	2	10	35	31	1.12	[63; 66]	425
9	4	10	19	22	0.86	[28; 31]	88.6

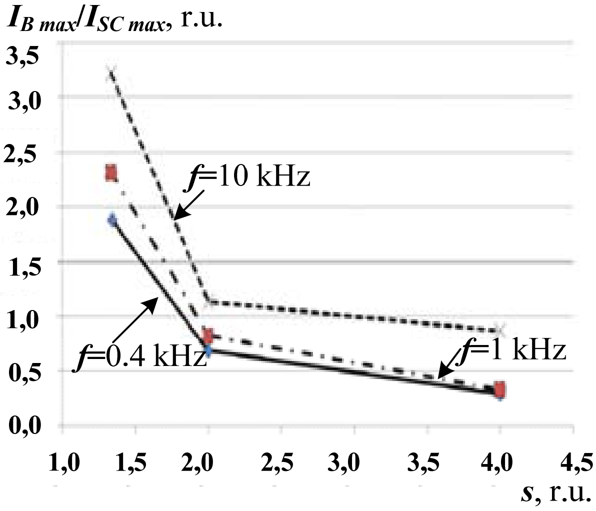


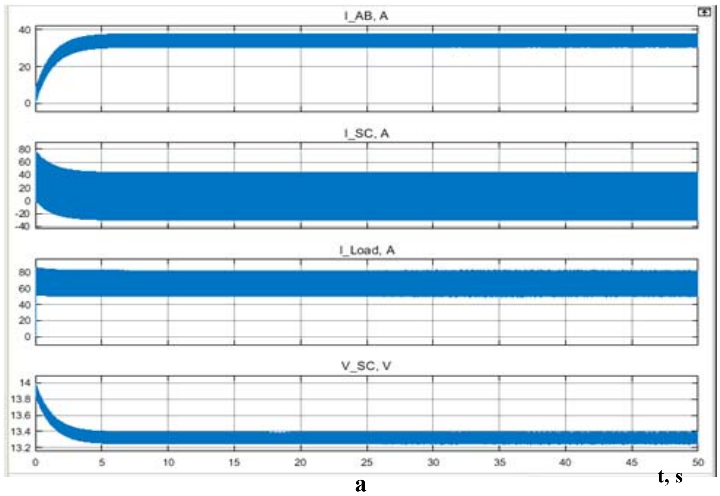
Fig. 5

With a decrease in the relative pulse duration of the pulse currents, the duration of SC discharge increases, and the duration of its charge and consequently the SC voltage decrease, while the currents of its charge from the B increase. The power consumption in the load will increase, but mainly due to the increase in power consumed from B.

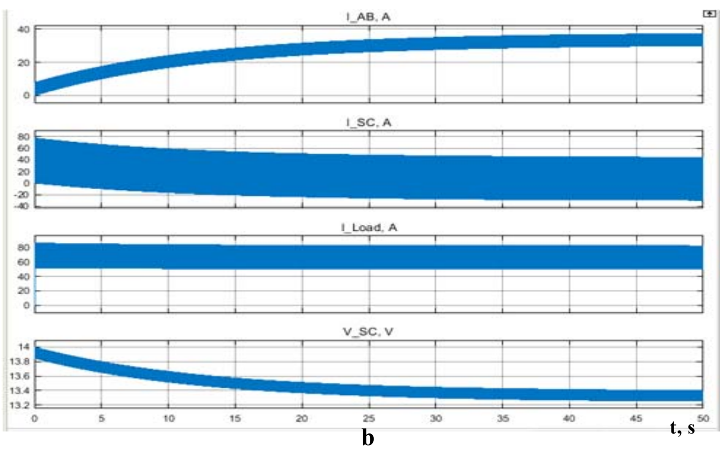
Table 3 shows the calculations of various parameters of a combined power supply system for ETD's engines, consisting of B and SC connected in parallel (as shown in Fig. 1), at different switching frequencies of the switch *K*, capacitances SC and constant relative pulse duration *s* = 2.

Table 3

Experiment #	C, F	f, kHz	I_B max, A	I_{SC} max, A	I_B max / I_{SC} max	$[I_{load\ min}; I_{load\ max}]$, A	P, W
1	60	0,4	42	62	0.67	[24;104]	463
2	100	0,4	42	62	0.67	[24;104]	463
3	400	0,4	39	66	0.59	[22;105]	478,8
4	60	1	37	45	0.82	[47;83]	424,7
5	100	1	37	45	0.82	[46;82]	426,8
6	400	1	35	48	0.73	[48;83]	439,3
7	60	10	32	35	0.91	[63;67]	423,5
8	100	10	35	31	1.12	[63;66]	425
9	400	10	32	35	0.91	[63;67]	435



a



b

Fig. 6

Fig. 6, *a* and 6, *b* show the calculated oscillograms of $I_{AB}(t)$, $I_{SC}(t)$, $I_{load}(t)$ and $U_{SC}(t)$ – currents in the electrical circuits of the B, SC and *R-L* loads, as well as the voltage at the terminals of the supercapacitor for two values of its capacitance: *C* = 60 and 600 F.

The obtained results (Table 3) show

that increasing the SC capacity by almost 7 times slightly increases the power of electrical energy transmitted to the load (the power increases by only 3%). However, as can be seen in Fig. 6 *a*, *b*, with an increase in the SC capacity, the duration of the forced (pulse) mode in the load increases by an order of magnitude due to the increased SC currents compared to the battery currents.

The obtained results show the feasibility of further in-depth studies of the use of SCs to reduce the weight and dimensions of the battery of ETD and hybrid electric vehicles. Since starting the engine of a modern mid-range car requires 4–7 kW of power for several seconds, and SCs are much lighter in weight than currently used Bs, the most optimal power source for electric vehicles may be a hybrid system consisting of a smaller-capacity B and additional lightweight SCs. It should be noted that the cost of SCs, although decreasing every year, exceeds the cost of Bs, but in terms of the number of charge-discharge cycles, SCs exceed Bs by 2–3 orders of magnitude. SCs have also higher resistance to extreme temperatures and loads as well as they have much higher power density (the ability to quickly release and store energy). In turn, batteries have a high energy density (the ability to store more energy per unit volume or mass). The main thing is that, unlike batteries, supercapacitors are a source of powerful, albeit short-term, energy pulses. Accordingly, SCs are used where high power is required for a short period of time. Batteries are a source of constant power, limited only by the discharge current. That is why the use of hybrid power sources (B+SC) is reasonable and promising.

Conclusions. 1. The conducted studies of the modes of a hybrid power supply system for an ETD from a battery and supercapacitors (connected in parallel to the battery) with an order of magnitude lower total internal resistance showed additional advantages of such a system. First of all, the advantages are manifested during periodic implementation of a short-term increase in the power of the ETD (in particular, when starting to move, climbing uphill and other forced modes). In this case, the energy transferred to the ETD's engine is taken mainly from the supercapacitors (since the currents in the supercapacitor discharge circuit significantly exceed the currents in the battery circuit).

2. Regulation of the ETD engine power modes can be achieved by changing the frequency and relative pulse duration of forced switching of the pulse current in the engine. It should be noted that increasing the capacity of supercapacitors has little effect on the power of energy transfer to the load during the time of the processes considered (with an increase in capacity by almost 7 times, the power increases by only 3%). At the same time, such an increase in capacity significantly (by an order of magnitude) increases the duration of the forced (pulse) load mode due to increased supercapacitor currents compared to battery currents.

3. The results of the study of such hybrid power supply system for the electric vehicles show the feasibility of further in-depth research in this direction. The hybrid systems can be an optimal power sources for ETD and electric vehicles, since supercapacitors are a source of powerful short-term, pulses of energy, and batteries are a source of constant power, limited only by the discharge current.

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АНАЛІЗ ПЕРЕХІДНИХ ТА СТАЦІОНАРНИХ РЕЖИМІВ У СИСТЕМІ ЖИВЛЕННЯ ЕЛЕКТРОМОБІЛЬНИХ ТРАНСПОРТНИХ ЗАСОБІВ, ЯКА МІСТИТЬ АКУМУЛЯТОРНУ БАТАРЕЮ ТА СУПЕРКОНДЕНСАТОРИ, З'ЄДНАНІ ПАРАЛЕЛЬНО

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У роботі проаналізовано перехідні та усталені процеси за широтно-імпульсного регулювання режимів системи електроживлення електромобіля від акумуляторної батареї та підключених до неї суперконденсаторів. Проведено чисельні експерименти на математичній моделі заступної електричної схеми такої системи живлення з використанням пакета прикладних програм Matlab Simulink. Отримано залежності величин струмів в електричних ланцюгах акумуляторної батареї, суперконденсаторів та навантаження від частоти імпульсних струмів, їхньої прогальності (відношення періоду повторення імпульсу до його тривалості), параметрів акумуляторної батареї та суперконденсаторів. Встановлено, що використання суперконденсаторів з малим внутрішнім опором дає можливість зменшити імпульсні струми в електричному ланцюзі акумуляторної батареї електромобіля за рахунок встановлення режимів роботи комплексної системи живлення, за яких струми в електричному ланцюзі розряду суперконденсатора будуть набагато більшими за струми в ланцюзі акумуляторної батареї. Бібл. 12, рис. 6, табл. 3.

***Ключові слова**:* суперконденсатор, акумуляторна батарея, внутрішній опір, комплексна система електроживлення.

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