

**COMPARATIVE ANALYSIS OF NUMERICAL AND NUMERICAL-ANALYTICAL MODELING OF PERMANENT MAGNET GENERATORS****I.P. Kondratenko\*, V.V. Grebenikov\*\*, R.V. Gamaliia\*\*\*, R.S. Kryshchuk\*\*\*\*****Institute of Electrodynamics National Academy of Sciences of Ukraine,****56, Beresteiskyi Ave., Kyiv, 03057, Ukraine. E-mail: [dep7ied@ukr.net](mailto:dep7ied@ukr.net); [elm1153@gmail.com](mailto:elm1153@gmail.com).**

*The paper investigates the adequacy of a simplified mathematical model of a numerical-analytical method for calculating low-power permanent magnet generators for autonomous power systems operating with a rectifier and active load. The object of study is generators with a radial arrangement of permanent magnets on the rotor surface. The model considered is based on an idealized representation of the air gap between the rotor and stator and does not take into account the tooth-and-groove structure, the actual geometric dimensions of the permanent magnets, and the nonlinear magnetic properties of ferromagnetic materials. To assess the adequacy of the model, a comparative analysis of the results of numerical-analytical calculations with the data of high-precision modeling using the finite element method in the Ansys Maxwell software package was performed. A series of FEM models of generators with varying levels of detail and complexity were constructed. The paper presents mathematical expressions of the numerical-analytical method based on the Fourier transform, oriented towards computer implementation. The modeling was performed taking into account the electrical circuit of the generator, which includes a three-phase rectifier and an active load. The time dependencies of the magnetic field distribution and electromagnetic torque were calculated. The errors in the calculation of the electromagnetic torque obtained by the numerical-analytical method were determined in comparison with FEM models of varying degrees of detail. The results demonstrate the capabilities and limitations of the simplified mathematical model and confirm the feasibility of its use for preliminary calculations and optimization of the parameters of permanent magnet generators for autonomous renewable energy systems. References 12, figures 6, tables 3.*

**Keywords:** modeling; electric generator; renewable energy; permanent magnets; energy characteristics; electromagnetic torque; numerical-analytical method; finite element method.

**Introduction.** The efficient use of renewable energy sources, in particular wind energy, is one of the priority areas for the development of modern energy systems. Low-power permanent magnet (PM) generators operating with a rectifier and active load have become widely used in autonomous energy systems. Such generators are characterized by high efficiency, reliability, and the ability to operate under variable rotational speeds, which makes them promising for autonomous wind power installations.

Effective design of PM generators requires accurate calculation of the magnetic field and energy characteristics, which allows optimizing the geometric parameters of the machine and ensuring stable operation under variable load conditions. The most accurate results are provided by finite element modeling (FEM) in modern software packages. At the same time, the development of finite element models requires considerable time, high qualifications, and careful validation of the results obtained.

In this regard, for preliminary engineering calculations and the selection of design parameters for generators, it is advisable to use analytical and numerical-analytical methods that allow for a quick assessment of the main characteristics of the machine without performing a complete FEM simulation. Similar to traction electric drive design tasks [1], such methods make it possible to quickly determine key indicators - in particular, electromagnetic characteristics, torque density, and efficiency - in a wide range of operating modes and to perform preliminary optimization of geometric dimensions.

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A number of approaches to the analytical and numerical-analytical calculation of the electromagnetic field of permanent magnet generators have been proposed in the scientific literature. These methods take into account the dependence of the magnetic field, currents, and voltage of the stator windings on the geometric parameters of the machine and the electromagnetic properties of the materials. Thus, in [2], a method was developed based on the rotor magnetic potential model in combination with a magnetic equivalent circuit, which allows taking into account the saturation effects of ferromagnetic materials. In [3], a two-dimensional analytical calculation of the magnetic field in the air gap of a generator is presented using vector magnetic potential and the method of separation of variables. Work [4] is devoted to a calculation method based on scalar magnetic potential, taking into account the interaction of the stator poles and slots, which provides an accurate determination of the radial and tangential components of magnetic flux density induction. In [5], an analytical method based on a magnetic equivalent circuit with iterative determination of magnetic resistances, taking into account saturation, is proposed.

A significant contribution to the development of analytical methods for calculating electric machines has been made by employees of the Electromagnetic Systems Department of the Institute of Electrodynamics of the National Academy of Sciences of Ukraine. In particular, monograph [6] discusses the application of the integral transformation method for the analysis of electromechanical systems, including linear machines and radial magnetic flux machines with permanent magnets. Further development of analytical methods for calculating radial magnetic flux machines with permanent magnets is also presented in works [7, 8].

The simplest analytical approach to modeling electric machines with permanent magnets is to use a two-dimensional model of an air gap with idealized surfaces of the rotor and stator magnetic conductors. Within such a model, materials are considered linear, and the magnetic field of permanent magnets and stator currents are represented as equivalent current loops [6–8]. Although analytical methods allow calculations to be performed in several areas with boundary conditions and take into account the volumetric distribution of field sources, the inevitable simplification of geometry and linearization of material properties lead to certain errors.

A systematic comparison of the results of simplified analytical models [8] with data from high-precision FEM modeling in software packages such as Ansys Maxwell [9] makes it possible to determine the limits of applicability of the analytical approach and evaluate its accuracy at various stages of designing permanent magnet generators for autonomous renewable energy systems.

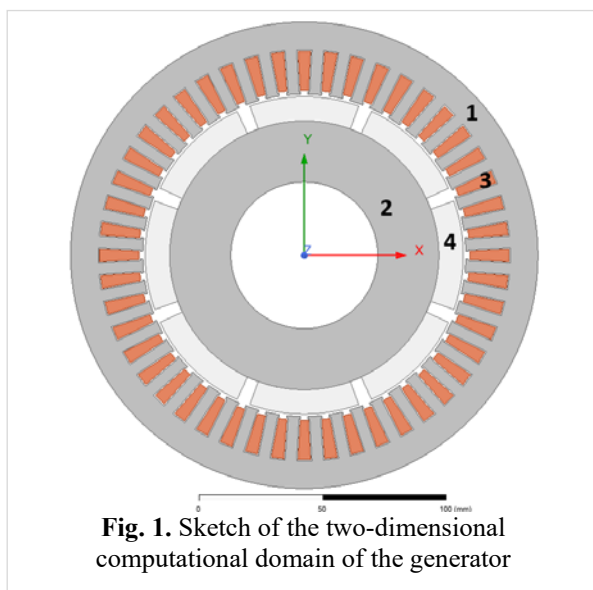
**The purpose of this work is** to validate a mathematical model and a numerical-analytical method for calculating the magnetic field and energy characteristics of electric generators with surface-mounted permanent magnets (SMPM) on the rotor for renewable energy systems (with a rectifier and active load) based on the example of a low-power generator.

To achieve this goal, a comparative analysis of the results of numerical and analytical calculations was performed using high-precision finite element modeling data in the Ansys Maxwell software package. A series of FEM models with varying levels of detail was constructed to determine the effect of geometric simplifications on the accuracy of calculating the electromagnetic characteristics of the generator.

Electromagnetic torque was chosen as the main parameter for comparison, since it is an integral characteristic that comprehensively reflects the interaction between the rotor magnetic field and stator currents. Unlike local magnetic field parameters, the electromagnetic torque provides a more accurate assessment of the adequacy of the model and is sensitive to the influence of higher harmonics arising from the cogging structure of the stator and the peculiarities of the rectifier operation.

**Description of the FEM model.** Fig. 1 shows a sketch of a 2D cross-section computational domain of a generator with a three-phase stator winding and SMPM on the rotor. This model is used for finite element modeling in the Ansys Maxwell program [9]. Figure 1 denotes: 1 is the stator core, 2 is the rotor core, 3 is the slots with stator winding, 4 is the permanent magnets.

The model has the following basic geometric parameters:



**Fig. 1.** Sketch of the two-dimensional computational domain of the generator

- radius of the active surface of the stator is 66 mm, outer radius of the stator yoke is 95,5 mm;
- active surface radius of the rotor is 55 mm, inner radius of the rotor core is 30 mm;
- magnet height is 10 mm, ratio of magnet width to pole width along the azimuth coordinate is 40/45;
- height and width of stator teeth is 17 mm and 4,7 mm, respectively;
- height and width of wedge-shaped tooth tip protrusions of teeth is 1,0 mm and 5,7 mm, respectively.

Rotor rotation frequency is  $\nu_r(t) = \text{const} = 3000$  rpm. For the stator and rotor core, 1008 steel with standard magnetization characteristics from the Ansys Maxwell material library was used, which ensures correct reproduction of the properties of ferromagnetic materials in the model. Electromagnetic parameters of the winding: number of conductors in the slots is 7, number of pole pairs is 4, number of slots per pole and phase is 2, pitch reduction factor is 1.

The model does not contain electrically conductive elements, losses in steel are not taken into account. In FEM models with a stator tooth-slot structure, the slot leakage inductance is automatically taken into account by the Ansys Maxwell software package. The active resistance of the winding is calculated using analytical methods [10] and taken into account in the electrical circuit, which includes the electric machine, Fig. 2. The scattering inductance and active resistance of the end parts of the winding are not taken into account in this study.

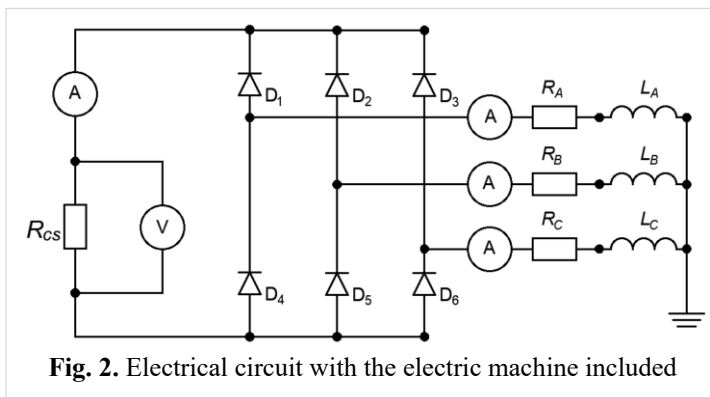


Fig. 2. Electrical circuit with the electric machine included

To simulate the actual operating conditions of the generator, a typical circuit for autonomous wind power systems with a six-diode rectifier [11] and an active load (Fig. 2) was used in the Ansys Circuit program, which, together with Ansys Maxwell, is part of the Ansys Electronics software suite [9]. The rectifier ( $D_1 \dots D_6$ ) creates a non-sinusoidal load that affects the shape of the phase currents and electromagnetic torque. The active resistance  $R_{cs}$  simulates the load (Table 1). The inductances  $L_A, L_B, L_C$  and active resistances  $R_w = R_A = R_B = R_C$  represent the parameters of the

stator winding. The circuit accounts for the rectifier and resistive load effects on phase winding currents, and consequently on the magnetic field and electromagnetic torque.

**Description of the analytical model.** Figure 3 shows a sketch of the generator model simulation domain of the generator air gap (within one a single pair of poles) used for numerical and analytical calculations using based on the integral transformation method [6, 8]. In the model, two current loops  $j_p$  are

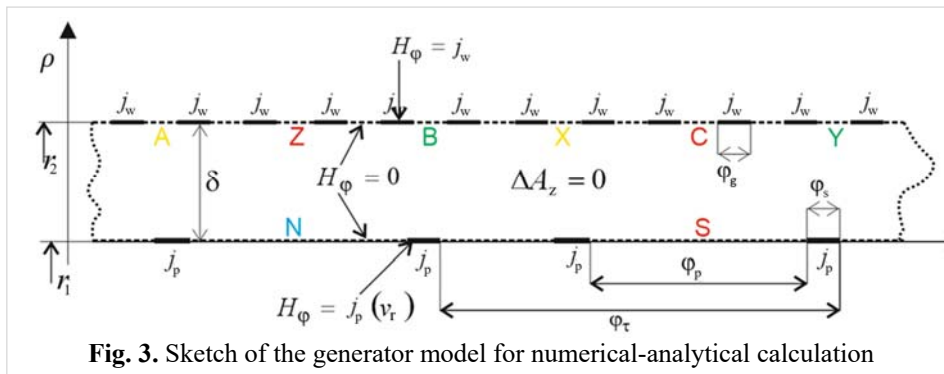


Fig. 3. Sketch of the generator model for numerical-analytical calculation

used instead of each permanent magnet at a radius  $r_1$ . The position of the current loops  $j_p$  along the azimuth coordinate depends on the specified rotation frequency  $\nu_r$  (rpm), which can be set either as constant or variable over time. In this case, the angular width between two such current

contours is equivalent to the angular width of a permanent magnet  $\phi_p$ , and the length of the contours  $\phi_s$  along the axis  $\phi$  does not depend on the height of the permanent magnets, but is chosen to be as small as possible to localize the equivalent field source only at the edges of the imaginary magnet:

$$\phi_s \ll (\phi_\tau - \phi_p). \quad (1)$$

This ensures the most accurate reproduction of the magnetic flux density induction distribution in the air gap in the model (Fig. 3), where the magnetic field source is specified by the boundary condition [8].

The representation of the stator's tooth-slot zone is performed by boundary conditions at the radius  $r_2$ . The total current of each slot is represented by a linear current contour  $j_w$  on the surface of the ideal stator magnetic conductor.

**Numerical-analytical calculation method.** To calculate the inductance and induced EMF in the stator phase windings, a program was written in the Wolfram computer algebra system [12] based on the following expressions (2)...(9) of the calculation method described in [8]. The calculation is performed in three stages: determining the current density distribution in the stator and rotor contours according to expressions (2)–(4); calculating the induced EMF and inductances according to expressions (5)–(7); calculating the electromagnetic torque according to expression (8). The input data for the calculation are given in Table 1.

**Table 1**

Parameter	Value	Units	Description
$\mu_0$	$4\pi \times 10^{-7}$	Gn/m	magnetic constant
p	4	-	number of pole pairs
q	2	-	slots for pole and phase
m	3	-	number of stator phases
$r_2$	0,066	m	stator radius
$\delta_1$	0,001	m	air gap
$h_p$	0,010	m	magnet height
$k_{\phi p}$	40/45	-	magnet overlap of rotor pole
$b_g$	0,0052	m	slot width
$l_d$	0,130	m	air gap length
$M_p$	1003441	A/m	magnetization of magnet
$\mu_{rp}$	1,045	-	relative magnetic permeability of magnets
$\beta$	1	-	pole pitch reduction coil pitch factor
$w_c$	3,5	-	number of effective turns in 2-layer winding coils, in half a slot
$v_r(t)$	3000	rpm	the rotor speed is set constant in time to simulate the steady state operation of the generator
$L_g$	0,00117	Gn	slot leakage inductance dispersion in slots per phase
$L_e$	0	Gn	frontal end-winding inductance dispersion per phase
$L_{cs}$	0	Gn	inductance of the load
$R_w$	0,03	Om	active resistance of the winding per phase
$R_{ce}$	50	Om	active resistance of the load

The current density in the stator and rotor current contours is calculated using the expressions:

$$j_{pn}(t) = k_{jp} I_p k_{pn}(t), \quad j_{wn}(t) = k_{wj} \sum_{K=1}^m i_K(t) k_{wn}(K), \quad (2)$$

where  $t$  is time;  $m=3$  is the number of stator phases;  $i_K(t)$  is current of phase  $K$  ( $K = A, B, C$ ),  $k_{wn}(K)$ ,  $k_{pn}(t)$  and  $k_{pn}(t)$  are coefficients of distribution of current contours by azimuth coordinate (winding coefficients [8]):

$$k_{wn}(K) = \frac{\sin(0,5n\varphi_g)}{n\pi} \sum_{q_1=0}^{q-1} \sum_{p_1=0}^{p-1} \left( e^{in\theta_{w1}(K)} + e^{in\theta_{w2}(K)} \right) \left( 1 - e^{in\varphi_\tau} \right), \quad (3)$$

$$k_{pn}(K) = \frac{\sin(0,5n\varphi_s)}{n\pi} \sum_{p_1=0}^{p-1} e^{in\varphi_r(t)} \left( e^{in\theta_{p1}} + e^{in\theta_{p2}} \right) \left( 1 - e^{in\varphi_\tau} \right), \quad (4)$$

where  $n$  is the Fourier transform parameter [6];  $i$  is the imaginary unit. Other components of expressions (2)...(7) are presented in Tables 1 and 2. The voltage induced by permanent magnets in the phase  $K$  of the stator winding is calculated using the expression:

$$u_{pK}(t) = \sum_n \frac{l_d w_{ph} \pi}{pq\varphi_g} E_{pn}(t) k_{wn}^*(K), \quad E_{pn}(t) := -in\omega_r A_{pn}(t), \quad (5)$$

where  $k_{wn}^*(K)$  is the complex conjugate of the distribution coefficient (3);  $E_{pn}(t)$  is the EMF due to the rotation of permanent magnets at coordinate  $r_2$ . To calculate the self ( $K' = K''$ ) and mutual ( $K' \neq K''$ ) inductance, the following expression is used:

$$L_{K'K''} = -\sum_n \frac{l_d w_{ph} \pi}{pq \varphi_g} \cdot \left( k_{jw} k_{wn}(K') \frac{r_2 (r_1^{2n} + r_2^{2n}) \mu_0}{n (r_1^{2n} - r_2^{2n})} \right) \cdot k_{wn}^*(K''). \quad (6)$$

The coupling coefficients between the stator winding phases are:

$$k_K = \frac{L_{K'K''}}{\sqrt{L_{K'K'} \cdot L_{K''K''}}}. \quad (7)$$

Electromagnetic torque of the rotor relative to the stator:

$$M(t) = \sum_{K=1}^m i_K(t) u_{pK}(t) / \omega_r. \quad (8)$$

**Table 2**

Formula	Description
$\varphi_z = \varphi_\tau / (mq)$	tooth pitch angle
$\varphi_g = \varphi_z k_{gt}$	slot angle
$\varphi_p = \varphi_\tau k_{\varphi p}$	magnet angle
$r_1 = r_2 - \delta$	rotor core radius
$\delta = \delta_1 + h_p / \mu_{rp}$	equivalent non-magnetic gap between magnetic conductors
$k_{gt} = b_g / (\varphi_z r_2)$	slot/tooth pitch ratio
$\varphi_{sm} = \varphi_\tau (1 - k_{\varphi p}) / 2$	angle of maximum width of the rotor linear current contour
$\varphi_s = \varphi_{sm} \cdot 10^{-1}$	angle of the width of the linear current contour
$w_{ph} = 2pqw_c$	number of turns per phase
$\varphi_\tau = \pi / p$	pole angle
$\omega_r(t) = 2\pi v_r(t) / 60$	angular frequency of the rotor
$\varphi_r(t) = \int \omega_r(t) dt$	position of the rotor relative to the stator
$\varphi_{ph}(K) = (2\varphi_r(K - m)) / m$	initial coordinates of linear current contours
$\theta_{w1}(K) = \varphi_{ph}(K) + 2p_1 \varphi_\tau + q_1 \varphi_z$	coordinate of the current contour of the upper layer of the two-layer stator winding
$\theta_{w2}(K) = \theta_{w1}(K) - \varphi_\tau (1 - \beta)$	coordinate of the current contour of the lower layer
$\theta_{p1} = 2\varphi_\tau p_1$	coordinate of the first contour
$\theta_{p2} = \theta_{p1} + \varphi_\tau - \varphi_p - \varphi_s$	coordinate of the second contour
$k_{jw} = Wc / (r_2 \varphi_g)$	current density coefficient in the stator contours
$k_{jp} = 1 / r_1 \varphi_s$	current density coefficient in the rotor contours
$I_p = M_c h_p$	current in the current contours
$K = 1, 2, \dots, m$	phase sequence number
$A_n(t) = A_{pn}(t) + A_{wn}(t)$	magnetic potential at the coordinate $r_2$
$A_{pn}(t) = j_{pn}(t) \frac{2r_1^{1+n} r_2^n \mu_0}{n (r_1^{2n} - r_2^{2n})}$	magnetic potential from the magnetic field of permanent magnets
$A_{wn}(t) = j_{wn}(t) \frac{r_2 (r_1^{2n} + r_2^{2n}) \mu_0}{n (r_1^{2n} - r_2^{2n})}$	magnetic potential from the magnetic field of stator currents

In this study, the load is represented by active resistance and is connected to the generator winding terminals via a 6-diode bridge rectifier [11] (Fig. 2). A combined approach is used: the generator parameters (induced voltage, inductance, and coupling coefficients) in the Wolfram environment are preliminarily determined by an analytical method using expressions (5)–(7), and additionally, the calculation of the slot leakage inductance  $L_g$ , the end parts inductance  $L_e$ , and the active resistance of the winding  $R_w$  is performed according to textbooks for the design of electrical machines [10]. All results ( $u_{pK}(t)$ ,  $L_{K'K''}$ ,  $L_g$ ,  $L_e$ ,  $R_w$ ,  $k_K$

and  $R_{cs}$ ) are exported to the Micro-Cap program to find the phase currents  $i_A(t)$ ,  $i_B(t)$ ,  $i_C(t)$  of the three-phase generator winding, as well as to find the current through the resistive load  $R_{cs}$ . In this study, the rotational speed is assumed constant (according to Table 1), while the time dependencies of the currents are obtained from the transient analysis of the electrical circuit.

The currents in Micro-Cap are calculated using transient analysis over a given time interval and imported back into the analytical model in the Wolfram environment for further calculation of the electromagnetic torque (8). At each step of the transient analysis, the machine parameters are used for the given (constant) speed, and the time variation manifests through the circuit currents, not through variable  $\omega_r(t)$ .

For an electrical circuit without a voltage rectifier, for example, with an active-inductive load on each phase ( $R_{eq}$  and  $L_{cs}$ ), electrical currents can be calculated without using Micro-Cap, according to the system of equations [8]:

$$\begin{cases} (L_{AA} + L_e + L_g + L_{cs})i_A + L_{AB} \frac{di_B}{dt} + L_{AC} \frac{di_C}{dt} + (R_{eq} + R_w)i_A + u_{pA} = 0 \\ (L_{BB} + L_e + L_g + L_{cs})i_B + L_{BA} \frac{di_A}{dt} + L_{BC} \frac{di_C}{dt} + (R_{eq} + R_w)i_B + u_{pB} = 0; \\ (L_{CC} + L_e + L_g + L_{cs})i_C + L_{CA} \frac{di_A}{dt} + L_{CB} \frac{di_B}{dt} + (R_{eq} + R_w)i_C + u_{pC} = 0. \end{cases} \quad (9)$$

The resistance of three resistive loads  $R_{eq}$  (9), which is equivalent to one resistance with a rectifier (Fig. 2), is determined by the expression:

$$R_{eq} = R_{cs} \left( \frac{\pi^2}{18} \right), \quad (10)$$

which is established based on the ratio between the input phase voltage and the rectified voltage of the Larionov circuit [11].

To verify the numerical-analytical method, a finite element method (FEM) model (Fig. 1) was implemented in Ansys Maxwell software. The numerical-analytical calculations were performed using Wolfram Mathematica/Wolfram System, while the analysis of the complex electrical circuit (Fig. 2) for the numerical-analytical method was conducted using Micro-Cap software. The FEM results are compared with the numerical-analytical results in the following section to validate the accuracy of the numerical-analytical method by comparing the electromagnetic torque.

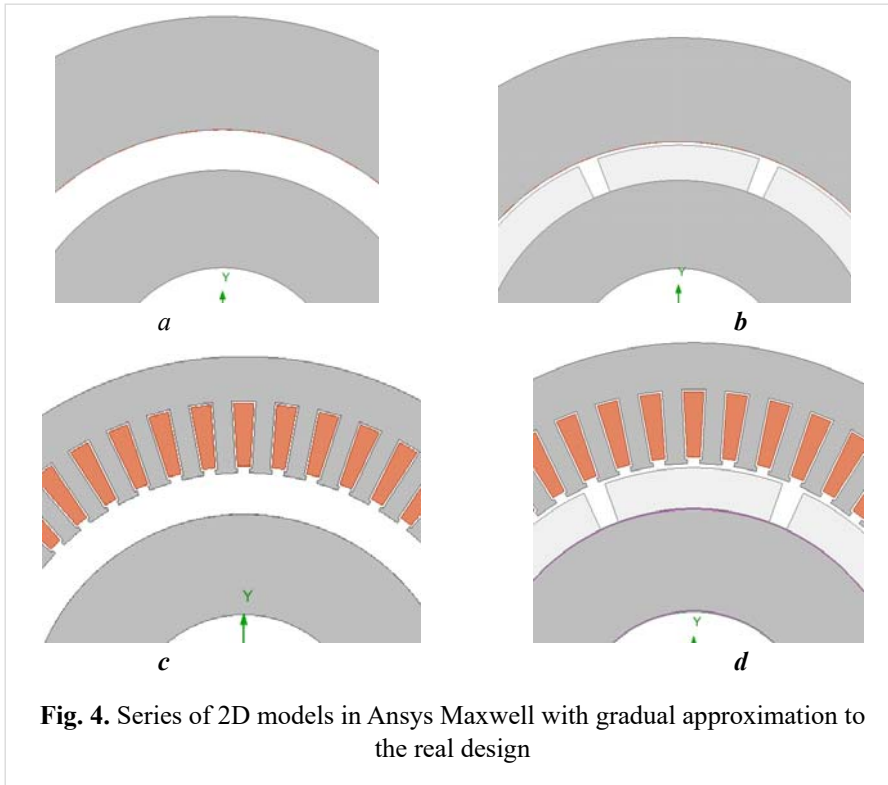
**Comparison of the results of the numerical-analytical method with the results of the FEM (Ansys Maxwell).** To verify the numerical-analytical method for calculating the electromagnetic torque of the generator, a comparison was made with the results of numerical modeling in the Ansys Maxwell software package. Since the numerical-analytical method is based on simplifications (linearization of the magnetization characteristics of ferromagnetic materials, replacement of permanent magnets with equivalent surface currents, representation of the slot winding with thin current contours), for an adequate comparison in Ansys Maxwell, a series of 2D models with a gradual approximation to the real design was created (Fig. 4). Fig. 4 shows two-dimensional models in Ansys Maxwell 2D: *a* – slotless model with current contours and slotless model with thin equivalent magnets; *b* – slotless model with real magnets; *c* – model with slots and equivalent thin magnets; *d* – complete models with linear iron and nonlinear iron.

The basic model *a* (Fig. 4) in Ansys Maxwell corresponds as closely as possible to the assumptions of the analytical method: a stator without slots with current loops on the surface of the stator iron with a thickness of 0,05 mm, the relative magnetic permeability of ferromagnetic materials is assumed to be constant ( $\mu_r = 1000$ ). The same geometry (Fig. 4, *a*) is used for two variants of the rotor magnetic system representation: with thin equivalent permanent magnets 0,05 mm thick with equivalent magnetization

$M'_p = M_p \left( \frac{h_p}{0,05} \right)$  (Table 1), or with equivalent current loops on the rotor surface instead of magnets (as in

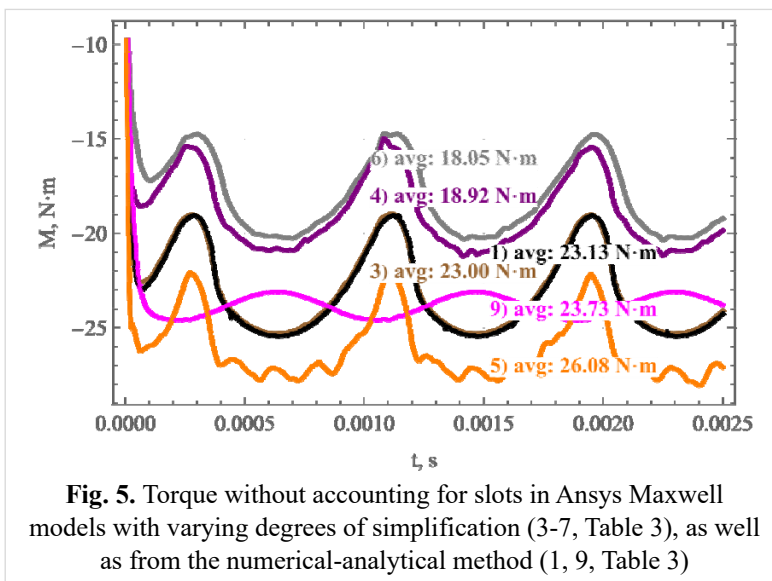
Fig. 3) with a given current  $I_p$  (Table 2). In this case, the equivalent non-magnetic gap  $\delta$  is calculated according to Table 2.

Model *b* (Fig. 4) restores the real geometry of permanent magnets of nominal thickness (10 mm) and the real non-magnetic gap ( $\delta = \delta_1 + h_p$ ) while maintaining the slotless structure of the stator and the linear magnetic properties of iron. Model *c* (Fig. 4) adds the actual slot structure of the stator with winding, while maintaining the linear magnetic properties of iron. This geometry is used with thin equivalent permanent magnets with a thickness of 0,05 mm. The complete model *d* (Fig. 4) includes all geometric features of the



**Fig. 4.** Series of 2D models in Ansys Maxwell with gradual approximation to the real design

design (stator slots with winding and magnets of nominal thickness). The same geometry is used for two calculation options: preserving the linear magnetic properties of iron and taking into account the nonlinear  $B-H$  magnetization curve of the ferromagnetic material of the magnetic conductors (1008 steel from the Ansys Maxwell library), which allows the influence of the magnetic saturation effect on the calculation of the electromagnetic torque of the generator to be taken into account. This consistent approach allows not only to evaluate the accuracy of the analytical method, but also to quantitatively determine the influence of each



**Fig. 5.** Torque without accounting for slots in Ansys Maxwell models with varying degrees of simplification (3-7, Table 3), as well as from the numerical-analytical method (1, 9, Table 3)

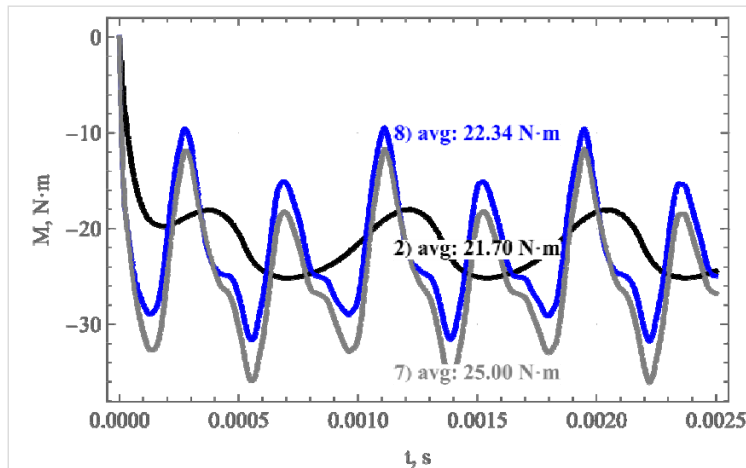
simplification on the results of the calculation of the electromagnetic torque of the generator.

Fig. 5 and Fig. 6 show the time dependence of the electromagnetic torque of the generator within the electrical half-period (0 ms...2,5 ms), calculated using different methods: Ansys Maxwell 2D (for 4 models in Fig. 4) and the numerical-analytical method according to expressions (2)...(8) for the model in Fig. 3 (which corresponds to model *a* in Fig. 4). The results demonstrate a clear correlation between the degree of model simplification and the nature of the torque curves. The analytical method shows excellent agreement with the slotless model with

current contours  $j_p$  in Ansys Maxwell (model *a*, Fig. 4). The results of the comparison of the average values of the electromagnetic torque over the period are given in Table 3.

The analytical method shows excellent agreement with the slotless model in Ansys Maxwell (1 and 3, Table 3). The relative error is only 0,6%, which confirms the correctness of the software implementation of the analytical method according to expressions (2)...(8) with an electrical circuit with 6 diodes [11] (Fig. 2) in the Micro-Cap program.

The representation of magnets by current contours (3, Table 3) shows a more accurate result relative to the complete model (7, Table 3) compared to the use of thin equivalent magnets (4, Table 3). Therefore, the idea of representing the magnetic field of permanent magnets using a boundary condition on the surface of an ideal rotor magnet conductor does not seem appropriate. The error from replacing full-size magnets (5, Table 3) with current contours (3, Table 3) is -11,8 %. The presence of a tooth-slot structure of the stator (6, Table 3) leads to a 4,6% reduction in torque compared to the model without slots (4, Table 3). Not taking into



**Fig. 6.** Torque in Ansys Maxwell with linear (7) and nonlinear (8) iron and stator slots, as well as torque from the numerical-analytical method (2) accounting for slot leakage inductance

account the magnetization curve of the steel of the rotor and stator magnetic conductors increases the torque by 11,9% (7 and 8 in Table 3).

In general, simplification from a complete FEM model with linear magnetic properties (7, Table 3) to a simple analytical model (Fig. 3, Fig. 6, a) results in a calculation error of -8% (comparison of the average torque of 25,00 N·m and 23,00 N·m). This error is the result of the combined effect of all geometric simplifications: replacement of real magnets with current loops, absence of a tooth-and-slot structure in the analytical model, and failure to take into account the scattering leakage inductance of the

slots in the electrical circuit of the simple model. When taking into account the nonlinear magnetization curve of ferromagnetic materials in the complete FEM model, the average torque decreases to 22,34 N·m (Fig. 6, Table 3).

**Table 3**

No, Fig. 5-6	Model	Model characteristics	Torque, N·m	Deviation from full FEM, %
1	Analytical model, Fig. 3	Without consideration $L_g$ (table 1) with the circuit of Fig. 2 in Micro-Cap	23,13	+3,5
2	Analytical model, Fig. 3	With consideration $L_g$ (table 1) with the circuit of Fig. 2 in Micro-Cap	21,70	-2,9
3	FEM model a, Fig. 4	Current contours, without slots	23,00	+3,0
4	FEM model a, Fig. 4	Thin magnets, without slots	18,92	-15,3
5	FEM model b, Fig. 4	Full-size magnets, without slots	26,08	+16,7
6	FEM model c, Fig. 4	Thin magnets with stator slots	18,05	-19,2
7	FEM model d, Fig. 4	Full, linear iron ( $\mu = \text{const}$ )	25,00	+11,9
8	FEM model d, Fig. 4	Full, nonlinear iron $\mu(B)$	22,34	base
9	Analytical model (Fig. 3)	Without taking into account $L_g$ , with the scheme according to expression (9)	23,73	+6,2

If the numerical-analytical calculation method additionally takes into account the diffusion inductance of the slots in the electrical circuit according to the methods [10] (2 in Table 3), the calculated torque value is 21,70 N·m (Fig. 6), which gives an error of less than 3% relative to the complete FEM model with nonlinear material properties (comparisons 2 and 8 in Table 3).

In both figures (Fig. 5 and Fig. 6), the influence of a 6-diode voltage rectifier is noticeable, which causes a threefold decrease in the electromagnetic torque during one electrical half-period. This effect is most pronounced on the curves of models without slots (Fig. 5). In the complete model (7 and 8 in Table 3), the moment curves (Fig. 6) additionally show sawtooth harmonics, whose period is  $2m$  times smaller than the electrical half-period of the current in the stator winding phases.

Modeling was performed using an analytical method without a voltage rectifier with three equivalent resistors  $R_{eq}$  (10) using equation (9) to find the currents in the stator winding phases instead of modeling the electrical circuit in the Micro-Cap program. A curve with an average torque of 23,73 N·m (9, Table 3) was established without the influence of harmonics from the voltage rectifier, which is a sine wave with minimum and maximum values of 23,1 and 24,6 N·m, respectively (Fig. 5). Comparing the curves in Fig. 5 with average torques of 23,73 N·m and 23,13 N·m (9 and 1 in Table 3), the deviation is 2,6%. Therefore, to perform preliminary approximate calculations of the generator (Fig. 1) with a small error in the results, it is possible to use a numerical-analytical calculation method and equation (9) with equivalent resistance (10) instead of the electrical circuit in Micro-Cap (Fig. 2).

**Conclusions.** The work validates the numerical-analytical method for calculating the electromagnetic characteristics of permanent magnet generators by systematically comparing it with the results of finite element modeling (FEM) in the Ansys Maxwell software package.

It was found that the numerical-analytical method, based on the method of integral transformations taking into account the complex influence of all applied geometric simplifications, demonstrates sufficiently high accuracy ( $\pm 3,5\%$ ) in calculating the average electromagnetic torque over the period, compared to the complete two-dimensional model in Ansys Maxwell. A comparison with an identical analytical model in Ansys Maxwell showed excellent agreement of the results with a relative error of 0,6%, which confirms the correctness of the analytical solution of the magnetic field equation, the software implementation of numerical calculations together with the Micro-Cap program for performing calculations of a three-phase electrical circuit.

Replacing permanent magnets of real dimensions with equivalent current loops on the surface of the rotor core leads to an error of 11,8%. At the same time, the use of very thin equivalent magnets on the rotor surface in FEM models gives an even greater error (27,5%). This indicates the advantage of representing magnets using boundary conditions with the shortest possible linear current loops at the edges of imaginary magnets.

For preliminary calculations of the generator in the case of the resistive-inductive load at the output of a three-phase rectifier, it is possible to use a simplified electrical circuit without a three-phase rectifier with sufficient accuracy (error of 6,2%), but with three equivalent resistors and inductance on each phase, which will allow not to use the Micro-Cap program to calculate a three-phase electrical circuit.

The research results confirm the adequacy and suitability of the numerical-analytical method for preliminary calculations and selection of design parameters for generators with radially magnetized surface-mounted arranged permanent magnets. The method is easy to use (when implemented as a simple computer program), provides sufficient accuracy with significantly lower computational costs and less time spent on FEM modeling, which makes it an effective tool for optimizing the design of an electric machine at the design stage.

The established limits of applicability of the method: the numerical-analytical method is most effective for machines with a relatively uniform non-magnetic gap and a slotless or low-slot stator with negligible slotting effect structure. For machines with a pronounced tooth-slot structure and significant magnetic saturation, the method provides preliminary estimates with subsequent refinement being necessary using the finite element method. The direction of Further development of the proposed method is the introduction of additional coefficients that can increase the accuracy of the calculation under certain conditions. For example, Carter coefficients and core saturation coefficients to expand the range of applicability.

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**ПОРІВНЯЛЬНИЙ АНАЛІЗ ЧИСЕЛЬНОГО ТА ЧИСЕЛЬНО-АНАЛІТИЧНОГО  
МОДЕЛЮВАННЯ ГЕНЕРАТОРІВ З ПОСТІЙНИМИ МАГНІТАМИ**

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*Досліджено адекватність спрощеної математичної моделі чисельно-аналітичного методу розрахунку генераторів з постійними магнітами малої потужності для автономних енергетичних систем, що працюють із випрямлячем та активним навантаженням. Об'єктом дослідження є генератори з радіальним розташуванням постійних магнітів на поверхні ротора. Розглянута модель ґрунтується на ідеалізованому представленні повітряного проміжку між ротором і статором та не враховує зубцево-пазову структуру, реальні геометричні розміри постійних магнітів і нелінійні магнітні властивості феромагнітних матеріалів. Задля оцінки адекватності моделі виконано порівняльний аналіз результатів чисельно-аналітичного розрахунку з даними високоточного моделювання методом скінченних елементів у програмному комплексі Ansys Maxwell. Побудовано серію FEM-моделей генератора різного рівня деталізації та складності. Наведено математичні вирази чисельно-аналітичного методу на основі перетворення Фур'є, орієнтовані на комп'ютерну реалізацію. Моделювання виконано з урахуванням електричного кола генератора, що включає трифазний випрямляч і активне навантаження. Проведено розрахунок часових залежностей розподілу магнітного поля та електромагнітного моменту. Визначено похибки розрахунку електромагнітного моменту, отриманого чисельно-аналітичним методом, у порівнянні з FEM-моделями різного ступеня деталізації. Отримані результати демонструють можливості та обмеження спрощеної математичної моделі й підтверджують доцільність її використання для попередніх розрахунків та оптимізації параметрів генераторів з постійними магнітами для автономних систем відновлюваної енергетики. Бібл. 12, рис. 6, табл. 3.*

**Ключові слова:** моделювання, електричний генератор, відновлювана енергетика, постійні магніти, енергетичні характеристики, електромагнітний момент, чисельно-аналітичний метод, метод скінченних елементів.

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