

## ASYNCHRONOUS MOTOR DRIVE INTERHARMONICS CALCULATION BASED ON GENERALIZED FOURIER SERIES OF SEVERAL VARIABLES

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*In the paper impact of low-frequency interharmonics on AC devices, in particular asynchronous motors is described. It is shown that because of the indefinite time interval of measurement, interharmonics detection and calculation is complicated. To improve the method of interharmonics calculating, we propose to use a generalized Fourier series of several variables and outline the basic theoretical principles for its use. The example of an adjustable electric drive of an asynchronous motor based on developed theoretical method shows the influence of interharmonics on the motor magnetization. A model of asynchronous electric drive in MatLab Simulink® environment confirms that the error of calculation of interharmonics based on the generalized Fourier series does not exceed 5%. References 10, figures 5.*

**Key words:** interharmonics; asynchronous motor drive; generalized Fourier series of several variables.

**Introduction.** Periodic changes in the electrical parameters of AC power grid systems or individual loads with frequency  $f_{per}$ , not a multiple of the power grid frequency  $f_g$  cause the appearance of interharmonics [1] with frequencies  $f_i$

$$f_i = m_1 f_g \pm m_2 f_{per}, \quad (1)$$

where  $m_1, m_2$  are integer numbers.

Particular danger is posed by interharmonics, whose frequency  $\omega_i$  is less than the frequency of the power grid,  $f_i < f_g$ , since they are not suppressed by the power grid input filters. The appearance of low-frequency interharmonics is especially critical for electric machines and causes overheating, vibration, which impair the stability of their operation and can disrupt them [2].

Detection of interharmonics requires analysis of grid voltage waveform over a longer time interval than one period. This time period is not fixed and depends on the frequency of the harmonics, which complicates the spectrum analysis.

In most countries, various techniques are available for the detection of interharmonics, such as in EU standards IEC 61000-4-7 [3], EN 50160 and the Ukrainian standard DSTU EN 50160: 2014 [4] derived from it, measurement time interval duration is 10 periods of grid voltage, i.e. 0.2 sec. This approach theoretically allows to determine interharmonics whose frequency is not less than 5 Hz. In IEC 61000-4-30 [5], it is recommended to perform short-term time intervals (up to 3 sec), short (up to 10 min) and long (up to 2 hours) to determine interharmonics over a wide frequency range.

However, due to the uncertainty of the measurement period and the effect of "harmonic absorption" [3], the use of numerical methods for the determination of interharmonics leads to significant calculation errors. Therefore, in order to identify of interharmonic components and analyze their impact on electric machines, it is advisable to simulate the operation of electric drives of electric machines. For this purpose, a number of methods are proposed, among which it is advisable to distinguish the window Fourier transform, the wavelet transform [6], the Fourier series of two variables [7]. The vast majority of electric machines are asynchronous motors powered by an adjustable electric drive based on semiconductor converters of electrical energy. Therefore, the Fourier series of two variables, which allows to divide the influence on the voltage spectrum of the modulating and carrier functions, allows to describe the supply voltage of the engine in a compact analytical form and effectively search for interharmonic components.

However, for the more efficient detection of interharmonics that magnetize the induction motor [8], it is necessary to generalize the Fourier series and to add variables corresponding to disturbances in the power supply system and lead to the appearance of the interharmonics.

In the paper, based on the generalized Fourier series, the effect of low-frequency interharmonics on the asynchronous motor magnetization is analyzed.

**Generalized Fourier series of several variables.** Let consider the Fourier series of  $M$  variables  $x_1, x_2, \dots, x_M$ , where the variable  $x_1=2\pi f_H$  is phase of carrier function with frequency  $f_H$ ; variable  $x_2=2\pi f_M$  is phase

of the modulating function with frequency  $f_M$ ; the variables  $x_3, \dots, x_M$  are phases of the components of the voltage caused by the load switching, with frequencies  $f_{per(1)}, \dots, f_{per(M-2)}$ . The coefficients of the series  $A_{(m_1)(m_2)\dots(m_M)}$  and  $B_{(m_1)(m_2)\dots(m_M)}$  allow to describe the signal in the time domain

$$\begin{aligned} \Phi(x_1, x_2, \dots, x_M) = & \frac{1}{2} A_{(0)\dots(0)} + \sum_{m_1=1}^{\infty} (A_{m_1..0} \cos(m_1 x_1) + B_{m_1..0} \sin(m_1 x_1)) + \\ & \sum_{m_2=1}^{\infty} (A_{(0)\dots(m_2)(0)} \cos(m_2 x_2) + B_{(0)\dots(m_2)(0)} \sin(m_2 x_2)) + \dots + \\ & + \sum_{m_M=1}^{\infty} (A_{(0)\dots(m_M)} \cos(m_M x_M) + B_{(0)\dots(m_M)} \sin(m_M x_M)) + \\ & \sum_{m_2=1}^{\infty} \sum_{m_1=1}^{\infty} (A_{(m_1)(m_2)\dots(0)} \cos(m_2 x_2 + m_1 x_1) + B_{(m_1)(m_2)\dots(0)} \sin(m_2 x_2 + m_1 x_1)) + \\ & + \sum_{m_3=1}^{\infty} \sum_{m_1=1}^{\infty} (A_{m_1 0 m_3..0} \cos(m_3 x_3 + m_1 x_1) + B_{m_1 0 m_3..0} \sin(m_3 x_3 + m_1 x_1)) + \dots + \\ & + \sum_{m_M=1}^{\infty} \sum_{m_1=1}^{\infty} (A_{m_1..m_M} \cos(m_M x_M + m_1 x_1) + B_{m_1..m_M} \sin(m_M x_M + m_1 x_1)) + \dots + \\ & + \sum_{m_M=1}^{\infty} \dots \sum_{m_2=1}^{\infty} \sum_{m_1=1}^{\infty} \left( A_{(m_1)(m_2)\dots(m_M)} \cos\left(\sum_{i=1}^M m_i x_i\right) + B_{(m_1)(m_2)\dots(m_M)} \sin\left(\sum_{i=1}^M m_i x_i\right) \right). \end{aligned} \quad (2)$$

The spectral components of  $C_{(m_1)(m_2)\dots(m_M)} = A_{(m_1)(m_2)\dots(m_M)} + jB_{(m_1)(m_2)\dots(m_M)}$  of the Fourier series of  $M$  variables are calculated by the formula

$$C_{(m_1)(m_2)\dots(m_M)} = \frac{1}{2\pi^M} \int_0^{2\pi V} \dots \int_0^{2\pi} y(x_1, x_2, \dots, x_M) e^{j \sum_{i=1}^M m_i x_i} \prod_{i=1}^M dx_i, \quad (3)$$

where  $V$  is determined on the basis of the smallest multiple periods of the  $M$  signals that take part in the formation of the modulated signal.

In this case, the variable  $x_1$  corresponds to the carrier function  $y_M$ ,  $x_2$  to the modulating function  $y_H$ ,  $x_3 \dots x_M$  to the perturbing factors that form the interharmonic components of  $y_{per(1)} \dots y_{per(M-2)}$ .

The set of spectral components  $C_{(m_1)(m_2)\dots(m_M)}$  contains complete information about the modulated signal at an arbitrary ratio of the frequencies of  $M$  components that take part in the formation of the modulated signal. If the relationship between the variables  $x_1, x_2, \dots, x_M$  of the modulated signal  $y$  in the  $M$ -dimensional space is known, it is possible to go to the time domain and calculate the spectrum consisting of the harmonics  $C_k$  of the given modulated signal.

If frequency of one or more components of the modulated signal is not a multiple of the frequency of the modulating function  $\omega_2$ , it is necessary to search for the smallest common multiple of periods  $T_1 \dots T_M$ ,  $T_{Lcm} = Lcm(T_1, T_2, \dots, T_M)$  such that for any  $i$  the condition that the multiplicity of the modulation  $P_{Lcm(i)}$  of any component of the modulated signal relative to the period  $T_{Lcm}$ ,  $P_{Lcm(i)} = T_{Lcm} / T_i$ , is an integer. In this case, the number  $V$  in formula (3) is calculated as

$$V = T_{Lcm} / T_2, \quad (4)$$

where  $T_2$  is modulating function period.

In this case, the first harmonic of the modulated signal  $C_1$  have a frequency  $\omega_{Lcm} = 2\pi / T_{Lcm}$ , and the frequency of the modulating function corresponds to the harmonic  $C_k$

$$C_k = \sum_{m_1=0}^{\infty} \sum_{m_3=-\infty}^{\infty} \dots \sum_{m_M=-\infty}^{\infty} C_{(m_1)(k-V(m_1 \cdot P + m_3 P_3 \dots + m_M P_M))(m_3)(m_4)\dots(m_M)}. \quad (5)$$

The ratio of the number of independent variables  $M$  of the Fourier series and the ratio of the periods  $T_1 \dots T_M$  is given by the principle of operation of the power supply system, the model of which is discussed in the next section.

**Power supply model of asynchronous motor electric drive.** The asynchronous motor electric drive is usually powered by a three-phase 3x380 V voltage. It consists of three phase uncontrolled bridge rectifier, a capacitive filter and a three-phase bridge inverter.

With the use of specialized modulation methods vector PWM or PWM with pre-modulation by the third harmonic [7] RMS value of the voltage at the output of the drive also reaches 380 V, which allows to maximize the use of motor torque.

Because of the ripple of the rectified voltage, the frequency of which is  $f_p=300$  Hz, in the three-phase voltage of the drive, interharmonics arise, the frequency and values of which depend on the frequency of the voltage generated by the inverter. To study the parameters of the interharmonics, we determine the voltage form on the filter capacitor. The model for determining the voltage on the filter capacitor is shown in Fig. 1.

The input link of the drive consists of a three-phase system of voltages  $e_A, e_B, e_C$ , three-phase rectifier on the diodes  $VD_1-VD_6$ , capacitor  $C$  of the DC link and a current source  $J(t)$  whose current corresponds to the input current of three-phase inverter. Since the high-frequency component of the drive phase current is almost completely eliminated by capacity  $C$ , the drive input current can be written as follows:

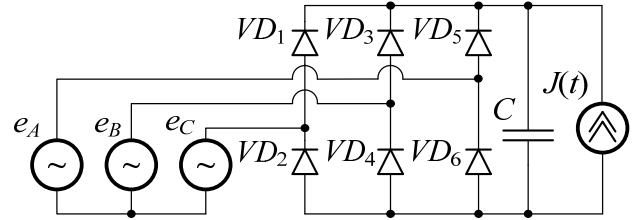


Fig. 1

$$J(t) = |i_A(t)| + |i_B(t)| + |i_C(t)| = I_m (|\sin(2\pi f_r t)| + |\sin(2\pi f_r t - 2\pi/3)| + |\sin(2\pi f_r t - 4\pi/3)|), \quad (6)$$

where  $f_r$  is the motor rotation frequency,  $I_m$  is the amplitude of the motor phase current.

Eliminating the module in expression (5), we obtain

$$J(t) = 2I_m \cos(2\pi f_r t - \pi/6), \quad (n-1)\pi/3 \leq 2\pi f_r t < n\pi/3, \quad n=1,2,3,4,5,6, \quad (7)$$

i.e. the current consumed by the drive is within  $J(t) \in [\sqrt{3}I_m; 2I_m]$  and has a ripple about 6.7%. Since the frequency of ripple of the rectified voltage  $f_p = 6f_g$  does not generally coincide with the frequency of ripple of the drive current  $f_c = 6f_r$ , the voltage period on the capacitor  $T_{uc}$  is calculated as the least common multiple of the ripple periods,  $T_{uc} = Lcm(T_p; T_c)$ . Therefore, the expression for the voltage on the capacitor can be directly written only if it coincides with the voltage of the grid

$$u_c(x_3) = U_m \cos((x_3 - \pi)/6), \quad \varphi_1 \leq x_3 < \varphi_2, \quad (8)$$

where  $x_3 = 2\pi f_p t$ ,  $\varphi_1, \varphi_2$  are angles that determine the open state interval of the rectifier diodes.

The angle  $\varphi_1$  also determines the ripple coefficient of the rectified voltage  $K_p$

$$K_p = 0,5 [1 - \cos((\varphi_1 - \pi)/6)]. \quad (9)$$

The voltage dependence on the interval  $\varphi_2 \leq \varphi < 2\pi + \varphi_1$ , calculated from the differential equation

$$C \frac{du_c}{dx_3} = -J(x_2 + \varphi_0), \quad (10)$$

where  $x_2 = 2\pi f_r t$ ,  $\varphi_0$  is initial phase of the current relative to the period beginning of rectified voltage.

The expression for describing the current of the drive (6) is piecewise continuous, so when the current phase output of the continuity range, it becomes incorrect. Given this, for the correct solution of the differential equation, it is advisable to set the expression to calculate the current in the form

$$J(x_2 + \varphi_0) = 2I_m \cos \left( x_2 + \varphi_0 - \pi/6 - \pi/3 \left[ \frac{x_2 + \varphi_0 - \pi/6}{\pi/3} \right] \right), \quad (11)$$

that allows to correctly set the current form at an arbitrary interval.

To calculate the angle  $\varphi_2$  at which the rectifier diodes are closed, it is sufficient to calculate the derivative of the rectified voltage (8) and substitute in formula (10)

$$\varphi_2 = 6 \arcsin \left( \frac{6J(x_2 + \varphi_0)}{U_m C} \right) + \pi. \quad (12)$$

The value of the parameter  $\varphi_0$  in the calculation of the interharmonics can be considered as the accumulation of the phase difference between two quantities - the frequency of the pulsation of the rectified voltage and the angular frequency of the motor relative to the variable  $x_2$

$$\varphi_0 = x_2 - x_2 P_2, \quad (13)$$

where  $P_2$  is the multiplicity of the frequency of grid voltage ripple relative to the motor angular frequency,  $P_2 = x_3/x_2$ .

The analytical expression of the voltage at the interval  $\varphi_2 \leq \varphi < 2\pi + \varphi_1$  is a solution of the differential equation (10)

$$\begin{aligned}
u_c(x_3) = & -\frac{1}{C} \int_{\varphi_2}^{x_3} J(x_2 + \varphi_0) dx_3' = -\frac{1}{C} \int_{\varphi_2}^{x_3} J\left(2 \frac{x_3'}{P_2} - x_3'\right) dx_3' = U_m \cos((\varphi_2 - \pi) / 6) + \\
& \frac{12I_m P_2}{C(2 - P_2)} \left( \sin\left(\left(x_3 \frac{2 - P_2}{6P_2} - \pi / 3 \left[ \frac{x_3}{\pi / 3} \frac{2 - P_2}{6P_2} \right] \right) - \pi / 6 \right) - \right. \\
& \left. - \sin\left(\left(\varphi_2 \frac{2 - P_2}{6P_2} - \pi / 3 \left[ \frac{\varphi_2}{\pi / 3} \frac{2 - P_2}{6P_2} \right] \right) - \pi / 6 \right) + \left[ \lfloor x_3 - \varphi_2 \rfloor \frac{2 - P_2}{6P_2} \right] \right).
\end{aligned} \tag{14}$$

These calculations are used to calculate the interharmonics based on a Fourier series.

**Calculation of spectral components of the Fourier series.** According to the considered the electric drive model, the electric drive current interharmonics are formed because of the interaction of three processes: the ripple of the rectified and filtered voltage with frequency  $f_p$ , the ripple of the total current of the actuator with frequency  $f_c$  and the frequency of the modulation carrier function of the electric drive current. Therefore, it is advisable to use three variables,  $M=3$ , in the Fourier series, namely  $x_1=2\pi f_H t$ ,  $x_2=2\pi f_r t$ ,  $x_3=2\pi f_p t$ , to determine the interharmonics that occur in the system. In this case, the expression for the calculation of the spectral components  $C_{(m_1)(m_2)(m_3)}$  under the condition of voltage modulation by unipolar PWM with the pre-modulation of the third harmonic is equal to

$$\begin{aligned}
C_{(m_1)(m_2)(m_3)} = & \frac{1}{2\pi^3} \int_0^{2\pi V} e^{jm_2 x_2} \left( \int_{\pi(1-\mu^*(\sin(x_2') + a^* \sin(3x_2'))) }^{\pi(1+\mu^*(\sin(x_2') + a^* \sin(3x_2'))) } e^{jm_1 x_1} dx_1 \right) \times \\
& \times \left( \int_{\varphi_1(x_2)}^{\varphi_2(x_2)} U_m \cos((x_3 - \pi) / 6) e^{jm_3 x_3} dx_3 + \int_{\varphi_2(x_2)}^{2\pi + \varphi_1(x_2)} u_c(x_3) e^{jm_3 x_3} dx_3 \right) dx_2,
\end{aligned} \tag{15}$$

where  $x_2'$  is the fixed value of variable  $x_2$  at the beginning of each subsequent period.

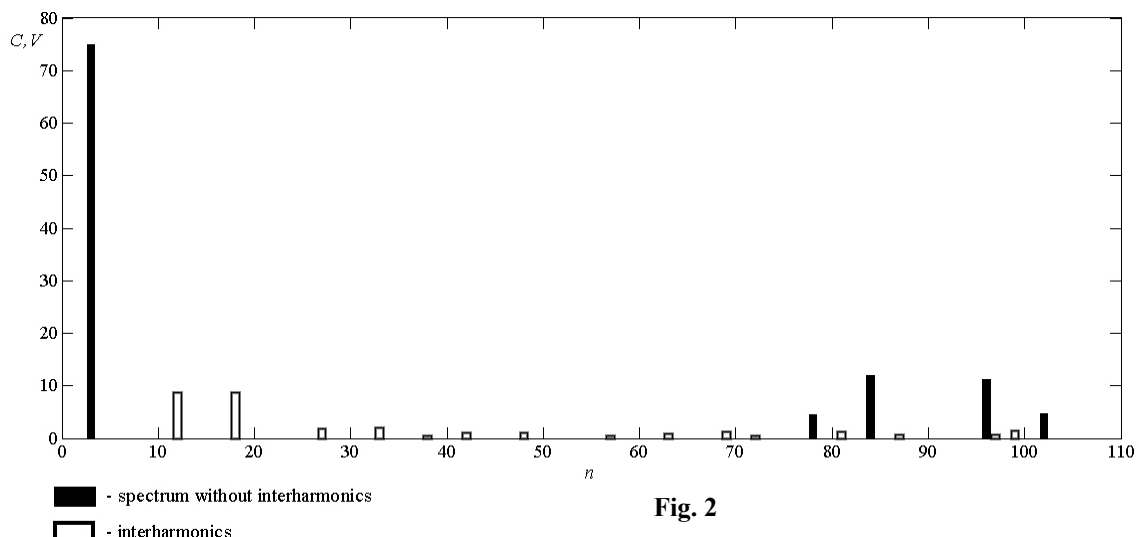
When calculating the spectrum of voltage, phase with a given ratio of PWM frequencies of the voltage and electric drive, to the formula (5) substitutes the corresponding values of the modulation multiplicities  $P=x_1/x_2$  and  $P_3=x_3/x_2$  taking into account the phase shift of phases  $B$  and  $C$  relative to the phase  $A$ . The total value of the voltage harmonic with number  $k$ ,  $C_{\Sigma(k)}$  is determined by the formula

$$C_{\Sigma(k)} = C_{A(k)} + C_{B(k)} + C_{C(k)}, \tag{16}$$

where  $C_{A(k)}$ ,  $C_{B(k)}$ ,  $C_{C(k)}$  are the harmonic values of the respective inverter phases.

For the case of the modulation ratio  $P=30$ ,  $P_3=5/3$ , the modulation depth  $\mu=0.5$ , the modulated voltage spectrum of the motor phase, calculated by formulas (5) – (15) is shown in fig. 2: for the rectified voltage without pulsations (ideal filter), which are shown black, and with a voltage ripple factor  $K_p=6.7\%$  (without filter), which caused the interharmonics shown in gray.

For the case  $P_3=5/3$ , "basic harmonics", multiple motor rotation frequency, have multiple numbers of three,  $n=3, 9, 15, 2k + 1$ , the harmonic that sets the motor rotation frequency  $n=3$ . Odd harmonics, not multiples of three are interharmonics formed as a result of the modulation of the motor phase voltages by the pulsation of the rectified voltage.



It is clear that, in practice, the voltage ripple coefficient and, as a consequence, the values of the interharmonics are much smaller, but even they can cause significant motor magnetization. The following section analyzes the quantitative indicators of the negative influence of interharmonics, depending on the value of parameter  $P_3$  and the capacity of the rectifier filter.

**Analysis of the influence of interharmonics on the asynchronous motor.** The interharmonics create additional magnetization for the induction motor, which reduces its output power. Unlike higher harmonics caused by voltage modulation, interharmonics caused by rectified voltage ripple can have a frequency lower or in the same order as the main harmonic and are therefore not suppressed by the filter. Therefore, interharmonics can significantly magnetize the asynchronous motor, which can be estimated with magnetization coefficient  $B$  calculated by the formula

$$B = n_1 \sqrt{\sum_{k=1}^q \frac{C_{i(k)}^2}{k^2}} \cdot (I_{\max})^{-1}, \quad (17)$$

where  $n_1$  is the fundamental harmonic number,  $C_{i(k)}$  is interharmonic RMS value with number  $k$ ,  $I_{\max}$  is the nominal value of the motor phase current.

Dividing by the number of interharmonic  $k$  in formula (17) defines an increase in the magnetizing effect of the steel of the motor with decreasing frequency of the interharmonic. The number of interharmonics  $q$  affecting the motor magnetization is determined depending on the cutoff frequency of the filter  $f_{\text{cutoff}}$ , which is usually chosen so that the group of higher harmonics formed around the harmonic with the number  $n_1 \cdot P$  is completely suppressed [9,10]. Therefore, the cutoff frequency of the filter is set from the condition  $f_{\text{cutoff}} \approx n_1 \cdot P / 2$ . This cutoff frequency  $f_{\text{cutoff}}$  is used to estimate the number of interharmonics  $q$

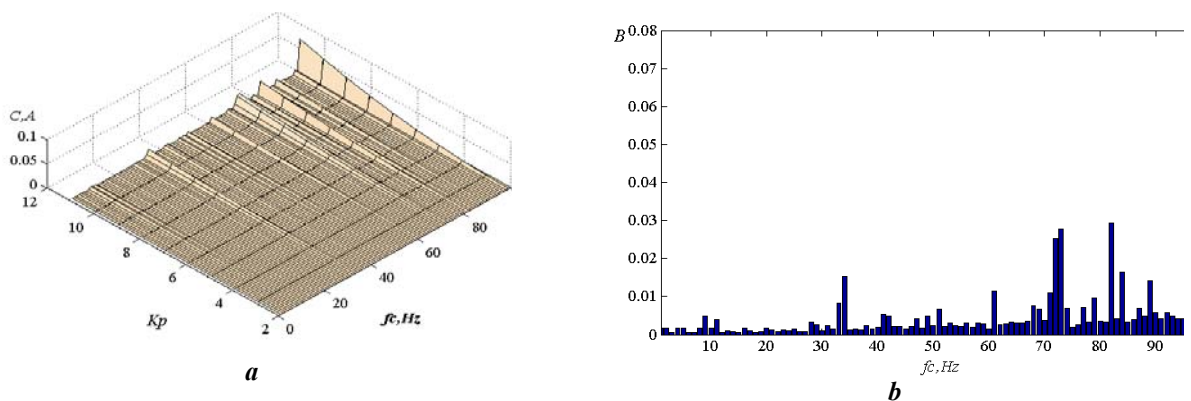
$$q = \lceil f_{\text{cutoff}} / f_1 \rceil, \quad (18)$$

where  $f_1$  is frequency of first harmonic.

As usual, the modulation frequency  $P$  is chosen in such way that the PWM frequency is equal to or greater than 20 kHz to eliminate acoustic noise, so it is advisable to select a 10 kHz filter cutoff frequency,  $f_{\text{cutoff}} = 10$  kHz.

The effect of interharmonics on the magnetization of the motor steel by the formula (17) is estimated for different levels of ripple of the rectified voltage  $K_p$ , starting from the maximum value  $K_{p\max} = 6.7\%$ , to the minimum value  $K_{p\min} = 0\%$ . The coefficients of  $B$  are calculated for the modulation depths  $\mu$  in the range [0.1; 1] for  $\mu = 1$  and motor rotation frequency  $f_c$  in the range [1 Hz; 100 Hz] with step  $\Delta f = 1$  Hz.

The dependence of the coefficient  $B$ , formula (17), on the ripple coefficient  $K_p$  and the motor rotation frequency  $f_c$  for the modulation depth  $\mu = 1$  is shown in aig. 3, *a*, Fig. 3, *b* shows the value of the coefficient  $B$  for case  $K_p = 6.7\%$ .



**Fig. 3**

As can be seen from the figures above, the coefficient  $B$  has the highest value for the rotation frequency with the largest multiple period  $f_{c\max} = 97$  Hz, for which  $B = 0.071$ .

To verify the obtained data, a MatLab Simulink<sup>®</sup> model shown in fig. 4 is used. It consists of models of an input rectifier with diodes VD1-VD6, a capacitive filter with capacitor  $C$  and a three-phase inverter with transistors VT1-VT6. The control system of the three-phase inverter based on the reference voltage sources  $I_{ma}$ ,  $I_{mb}$ ,  $I_{mc}$  containing the third harmonic, generates a PWM control signal of the inverter transistors.

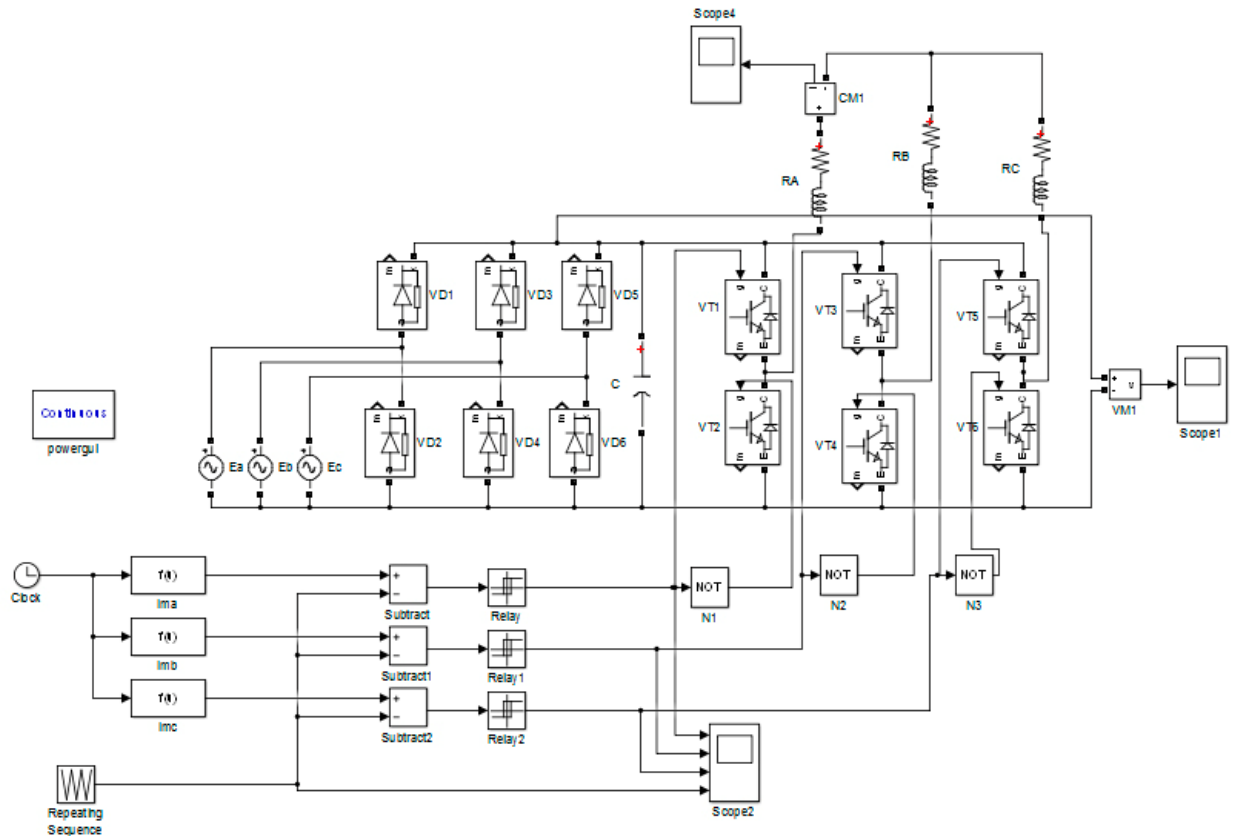


Fig. 4

In fig. 5, *a* a typical diagram of the current of an inverter phase with interharmonics is shown and its spectrum for the case of the rotation frequency of the drive  $f_c=33$  Hz is shown in fig. 5, *b*. In this case the period, over which the spectrum is calculated, is determined as least common multiple of the periods of rectified voltage ripple and the period of rotation of the motor:  $T_{Lcm}=Lcm(T_C, T_p)=1/3$ . Therefore, the harmonic number of the rotating engine is  $k_r=T_{Lcm}/T_C=11$ .

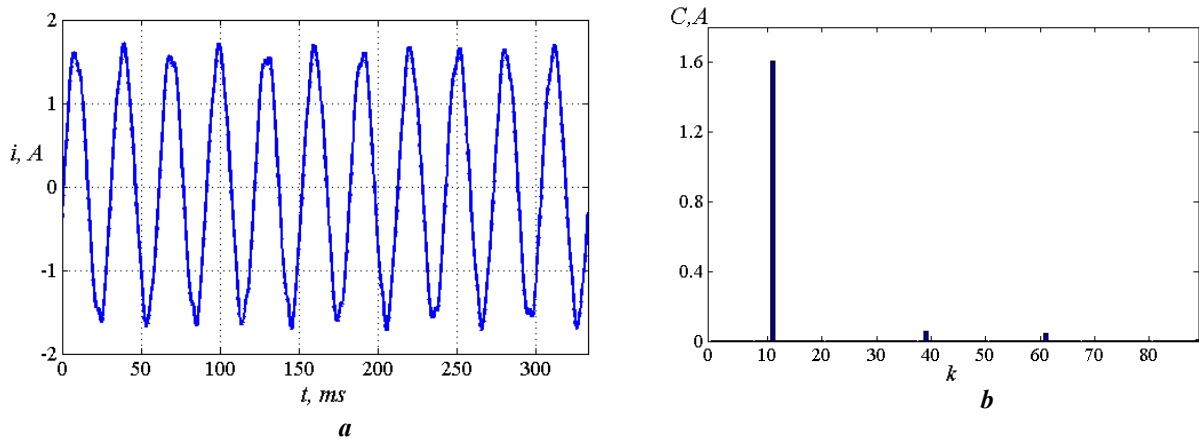


Fig. 5

Due to the presence in the current spectrum of the interharmonics with numbers  $k = 39, k = 61$ , the motor phase currents have a distorted shape, that impairs the motor operating mode and creates additional magnetization. The current spectrum calculated using the generalized Fourier series in previous chapter is tested via Simulink model. Comparison of the obtained results proves that the error of the results obtained on the basis of the Fourier series does not exceed 5%.

**Conclusions.** The interharmonics that arise as result of the modulation of the grid voltage by the processes of changing load resistances impair the operation of electrical devices connected to the power grid, in particular asynchronous motors. In the paper for the interharmonic components analysis we propose the use of a generalized Fourier series of several variables. On the example of a regulated electric drive, the

possibility of interharmonics arise because of presence of a rectified voltage ripple is analyzed. It is shown that the highest influence of the interharmonics is observed at the rotational frequencies of the motor, which are not multiples of the frequency of the grid. In this case the motor magnetization can reach 7.1%. The obtained results were tested on the Simulink model and it is shown that the error of the results does not exceed 5%.

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УДК 621.314

#### **РАСЧЕТ ИНТЕРГАРМОНИК В АСИНХРОННОМ ЭЛЕКТРОПРИВОДЕ НА ОСНОВЕ ОБОБЩЕННОГО РЯДА ФУРЬЕ НЕСКОЛЬКИХ ПЕРЕМЕННЫХ**

**Вербицкий Е.В., Жуйков В.Я.**

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*В статье описано влияние низкочастотных интергармоник на электрические устройства переменного тока, в частности, асинхронные двигатели. Показано, что через неопределенный временной интервал измерения их определение и расчет являются усложненным. Для улучшения методики расчета интергармоник предложено использовать обобщенный ряд Фурье нескольких переменных и описаны теоретические основы его использования. На примере регулируемого электропривода асинхронного двигателя показано влияние интергармоник на подмагничивание двигателя с использованием разработанного теоретического аппарата. Для верификации полученных данных разработана модель асинхронного электропривода в среде MatLab Simulink® и подтверждено, что погрешность расчета интергармоник на основе обобщенного ряда Фурье не превышает 5%. Библ. 10, рис. 5.*

**Ключевые слова:** интергармоники, асинхронный электропривод, обобщенный ряд Фурье нескольких переменных.

#### **РОЗРАХУНОК ІНТЕРГАРМОНІК В АСИНХРОННОМУ ЕЛЕКТРОПРИВОДІ НА ОСНОВІ УЗАГАЛЬНЕНОГО РЯДУ ФУР'Є ДЕКІЛЬКОХ ЗМІННИХ**

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*У статті описано вплив низькочастотних інтергармонік на електричні пристрої змінного струму, зокрема асинхронні двигуни. Показано, що через невизначений часовий інтервал вимірювання, їхні виявлення і розрахунок є ускладненим. Задля покращення методики розрахунку інтергармонік запропоновано використовувати узагальнений ряд Фур'є декількох змінних та наведено основні теоретичні засади його використання. На прикладі регульованого електроприводу асинхронного двигуна показано вплив інтергармонік на підмагничування двигуна з використанням розробленого теоретичного апарату. Для верифікації отриманих даних розроблено модель асинхронного електроприводу в середовищі MatLab Simulink® та підтверджено, що похибка розрахунку інтергармонік на основі узагальненого ряду Фур'є не перевищує 5%. Бібл. 10, рис. 5.*

**Ключові слова:** інтергармоніки, асинхронний електропривод, узагальнений ряд Фур'є декількох змінних.

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