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DETERMINATION OF THE PONDEROMOTIVE MAGNETIC FORCE WHEN CALCULATING THE FIELD BY THE CONFORMAL TRANSFORMATION METHOD

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The method of conformal transformation has been widely applied to the research of devices using ponderomotive action of the magnetic field. This method enables the analysis and calculation of stationary 2D electric and magnetic fields meeting the Laplace equation. It allows essential simplification of the problem of the field calculation. However, its basic drawback consists in the absence of the general method for the determination of the complex potential. Consequently, it is usually impossible to obtain expressions for the magnetic field strength and its ponderomotive force in the explicit form as a function of coordinates in the initial domain. This paper deals with the solution to the problem of direct determination of the specific ponderomotive force of the magnetic field with the use of complex potential with the known function of conformal transformation. The analyzed examples of the calculation of the ponderomotive force in the working zone of the poles of different shapes can be a model for the research of the traction performance of the electromagnetic systems of magnetic separating devices. Besides, the obtained expressions can also be used in the calculation of the systems working on the principle of the ponderomotive action of the electrostatic field. References 12, figures 2.

Keywords: ponderomotor force, magnetic field strength, conformal transformation method, magnetic systems

Introduction. In the generalized analysis of devices using the ponderomotive action of the magnetic field (separators, magnetic transport units, etc.), the method of conformal transformation is widely used [1−6], because the fields of such units may be considered plane-parallel. The specific (per volume unit) ponderomotive force of the magnetic field in the working inter-polar space is the required parameter defined by the expression [7–9]

$$f_{\mathrm{m}} = \frac{1}{2}\mu_0\chi \cdot \mathrm{grad}H^2, \tag{1}$$

where μ_0 is the magnetic permeability of air; χ is the the specific magnetic susceptibility of the substance of a body placed in a magnetic field; H is the module of magnetic field strength in the working area.

Problem statement. During the use of this method of research, the strength is usually not explicitly obtained as a function of coordinate z of the initial domain [3, 10] (the strength is found as the function of coordinate t of the mapped domain). Therefore, the calculation of H requires the determination of the correspondence between the coordinates of the initial plane z and the mapped domain t. Besides, in (1) sign "grad" stands for the gradient in plane z, and H, according to the above said, is found in the form of a function of t. Thus, the use of (1) for generalized analysis of fields in the air gaps of magnetic devices is significantly complicated. The difficulties arising in this case can be avoided in determining the value of the force at the characteristic points (on the symmetry axes) of the magnetic systems [1], when the gradient in (1) is replaced by a derivative of one of the coordinates. However, in the case of the general formulation of the problem, the calculation is complicated due to the need to take the gradient in (1) over z from function t.

In this paper, the task is to transform (1) to the form that allows to directly calculate forces, having only the function of conformal transformation of pole configuration in initial plane z on mapped domain t.

Solving of the problem. To solve the posed problem the complex potential function will be introduced into the analysis

$$W = \varphi + j\phi,$$

where φ is the function of the scalar potential; ϕ is the function of the flux; j is an imaginary unit.

In this case, as the field outside the sources is considered, functions φ and ϕ meet the Laplace equation and are interrelated by the Cauchy-Riemann conditions [10−11]. Besides,

$$H = -\mathrm{grad}\varphi. \tag{2}$$

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Then the following transformations can be written down:

$$\mathrm{grad}H^2=\mathrm{grad}\left[\left(\frac{\partial\varphi}{\partial x}\right)^2+\left(\frac{\partial\varphi}{\partial y}\right)^2\right]=2\left[\frac{\partial\varphi}{\partial x}-j\frac{\partial\varphi}{\partial y}\right]\left[\frac{\partial^2\varphi}{\partial x^2}+j\frac{\partial^2\varphi}{\partial x\partial y}\right]. \tag{3}$$

As φ and ϕ are related to each other by the Cauchy-Riemann conditions,

$$\frac{\partial\varphi}{\partial x}-j\frac{\partial\varphi}{\partial y}=\frac{\partial}{\partial x}\left(\varphi+j\phi\right)=\frac{\partial W}{\partial z}. \tag{4}$$

In (4) the derivative by x is substituted with the derivative by $z=x+jy$, as W is an analytical function [11].

Besides,

$$\frac{\partial^2\varphi}{\partial x^2}+j\frac{\partial^2\varphi}{\partial x\partial y}=-\frac{\partial}{\partial x}\left(H_x+jH_y\right)=\frac{\partial}{\partial x}\left(\frac{\partial\overset{*}{W}}{\partial z}\right)=\frac{\partial^2\overset{*}{W}}{\partial z^2}, \tag{5}$$

where H_x and H_y are the components of the magnetic field strength vector respectively by axis *x* and *y*; $\overset{*}{W}$ is the complex conjugate potential of the field.

Here, also, the transition to derivatives by z is made due to the fact that $\partial\overset{*}{W}/\partial z$ is an analytical function [11].

Taking into account (3) – (5), initial expression (1) is transformed to the form

$$f_{\mathrm{m}}=\mu_0\chi\frac{\partial W}{\partial z}\cdot\frac{\partial^2\overset{*}{W}}{\partial z^2}. \tag{6}$$

Thus, we were able to reduce the expression for the ponderomotive force to an expression that depends only on the complex potential W and complex conjugate potential $\overset{*}{W}$.

In the study of magnetic systems, one of the most effective ways of finding the complex potential of a field function is the method of conformal transformation, when the points of the initial plane z are aligned with the points of the mapped plane t:

$$z=z\left(t\right). \tag{7}$$

To solve the above posed problem we substitute in (6) the derivatives by z with derivatives by t. There will be

$$\left.\begin{aligned}&\frac{\partial W}{\partial z}=\frac{\partial W}{\partial t}\cdot\frac{\partial t}{\partial z};\\&\frac{\partial^2\overset{*}{W}}{\partial z^2}=\frac{\partial^2\overset{*}{W}}{\partial t^2}\cdot\left(\frac{\partial t}{\partial z}\right)^2+\frac{\partial\overset{*}{W}}{\partial t}\cdot\frac{\partial t}{\partial z}\cdot\frac{\partial^2 t}{\partial z^2}.\end{aligned}\right\} \tag{8}$$

Substituting (8) into (6), we obtain

$$f_{\mathrm{m}}=\mu_0\chi\left[\frac{\partial W}{\partial t}\cdot\frac{\partial^2\overset{*}{W}}{\partial t^2}\left(\frac{\partial t}{\partial z}\right)^3+\frac{\partial W}{\partial t}\cdot\frac{\partial\overset{*}{W}}{\partial t}\cdot\left(\frac{\partial t}{\partial z}\right)^2\cdot\frac{\partial^2 t}{\partial z^2}\right]. \tag{9}$$

The latter expression is a complex function whose real and imaginary parts are the force components, correspondingly by axes *x* and *y*.

Thus, the problem posed above for expressing the ponderomotive force of a magnetic field through the parameters of conformal transformation is solved.

Expression (9) becomes essentially simpler when the initial domain can be mapped on a band of a uniform field with strength H_0. Then $\partial W/\partial t=\partial\overset{*}{W}/\partial t=H_0$; $\partial^2\overset{*}{W}/\partial t^2=0$ and (9) transforms to the form

$$f_{\mathrm{m}}=\mu_0\chi H_0^2\left(\frac{\partial t}{\partial z}\right)^2\cdot\frac{\partial^2 t}{\partial z^2}. \tag{10}$$

In cases where researchers are only interested in the value of the ponderomotive force module, expression (9) can also be transformed to a form more convenient to use. With this purpose in view, (6) will be written as

$$\left|f_{\mathrm{m}}\right| = \mu_0\chi\left|\frac{\partial W}{\partial z}\right|\cdot\left|\frac{\partial^2 \overset{*}{W}}{\partial z^2}\right| = \mu_0\chi\left|H_z\right|\cdot\left|\frac{\partial H_z}{\partial z}\right|, \tag{11}$$

where H_z is the magnetic field strength in plane z.

In (11) from a derivative by z we pass to a derivative by t and, additionally, take into account that [10]

$$\left|H_z\right| = \frac{\left|H_t\right|}{\left|\partial z / \partial t\right|}. \tag{12}$$

Then expression (11) for the module of specific ponderomotive force will be of the form

$$\left|f_{\mathrm{m}}\right| = \mu_0\chi\frac{\left|H_t\right|\cdot\left|\partial H_t / \partial t\right|}{\left|\left(\partial z / \partial t\right)^2\right|}, \tag{13}$$

where H_t is the magnetic field strength in mapped plane t.

Thus, specific ponderomotive force f_{m} or its module in all obtained expressions (9), (10), (13) are got as functions of t. Based on this, the calculation is as follows: coordinate z is assigned in the researched domain; solving transcendental equation (7) the corresponding t is found; the required value of the force for the found t is calculated by one of expressions (9), (10), (13).

Consider the examples of calculating ponderomotive force in the working zone of poles of various shapes, which can serve as a model for the research of traction performance of electromagnetic systems of magneto-separating devices [2, 12].

Example 1. Poles in the form of two end plates of the same size (Fig. 1).

The initial domain of the field in plane z will be mapped on plane t as shown in Fig. 1, *b*. In this case the coordinates of domains t and z are interrelated by expression [10]

$$\frac{\partial z}{\partial t} = \frac{p}{2a\,K(k)}\left\{\left[1-\left(\frac{z}{a}\right)^2\right]\left[1-\left(\frac{z}{b}\right)^2\right]\right\}^{-1/2}, \tag{14}$$

where $K(k)$ – a complete elliptic integral of the first kind with module $k = a / b$.

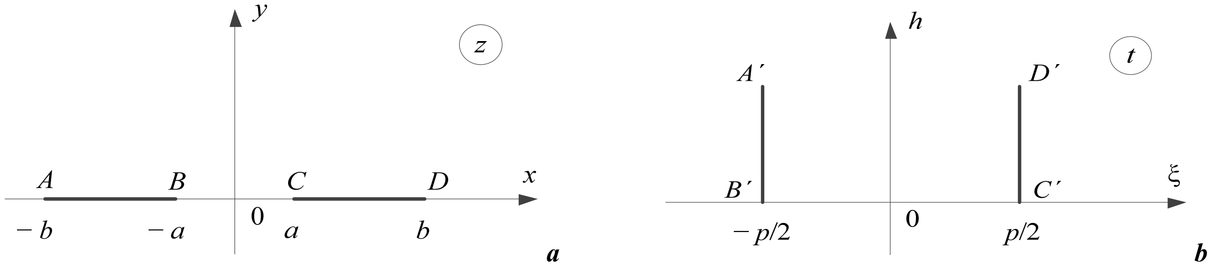


Fig. 1

Since the analyzed domain is mapped in a plane on a band with a uniform field, expression (10) can be used to calculate ponderomotive force. As a result of simple transformations, the required expression for a specific ponderromotive force between two equal bands of finite width has the form

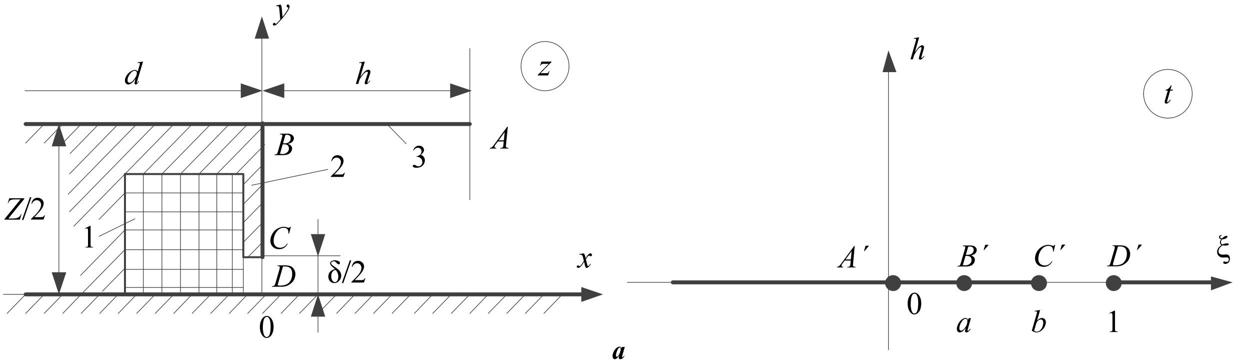
$$f_{\mathrm{m}} = \mu_0\chi\frac{U_0^2}{4K^2(k)}\cdot\frac{z}{a^4}\cdot\frac{\left[1-2\left(z/b\right)^2+\left(a/b\right)^2\right]}{\left[1-\left(z/a\right)^2\right]^2\left[1-\left(z/b\right)^2\right]^2}, \tag{15}$$

where $U_0 = H_0 p$ is the difference of potentials between poles (in the form of two end plates).

From (15) it is easy to find the expression obtained in the traditional way and used in practice [2] to determine the ponderomotive force on the poles vertical axis of symmetry, which in this case (at $z = jy$) has only a component by axis y:

$$f_{my} = \mu_0\chi\frac{U_0^2}{4K^2(k)}\cdot\frac{y}{a^4}\cdot\frac{\left[1+2\left(y/b\right)^2+\left(a/b\right)^2\right]}{\left[1+\left(y/a\right)^2\right]^2\left[1-\left(y/b\right)^2\right]^2}. \tag{16}$$

Example 2. In practice the width Z of the magnetic system of pulley separator and the pulley diameter d is large compared to the airgap δ, magnetic collar (shunt) height h and the depth of extraction, which is height of the material transported. Hence, as it showed in [12], for the determination of fields and forces a pulley electromagnetic separator with magnetic collar may be presented as a planar and two-dimensional structure (Fig. 2, *a*). The configuration of the poles, the correspondence of the points, and the transformation of the initial domain are shown in Fig. 2 (*1* is the magnetizing coil, *2* is the magnetic circuit, *3* is the magnetic collar (shunt)).



a *b*

Fig. 2

In this case the function of conform transformation is of the form [10–12]:

$$z=\frac{Z}{\pi(a+1)}\left\{\left[(t-1)(t-a)\right]^{1/2}+(a+1)\ln\left[(t-1)^{1/2}+(t-a)^{1/2}\right]\right\}-\frac{Z}{2\pi}\ln(1-a), \tag{17}$$

where a is the certain parameter (Fig. 2, *b*), found by equation

$$\frac{\pi h}{Z}=\left\{\frac{a^{1/2}}{a+1}+\ln\left[\frac{a^{1/2}+1}{(1-a)^{1/2}}\right]\right\}. \tag{18}$$

The module of the magnetic field strength in the mapped domain (Fig. 2, *b*) [9]

$$\left|H_t\right|=\frac{U_0}{\pi}\left|\left[t^2-t(1+b)+b\right]^{-1/2}\right|, \tag{19}$$

where U_0 is the difference of the potentials between the poles; b is the certain parameter (Fig. 2, *b*) found by equation

$$\frac{\pi\delta}{Z}=\operatorname{arctg}\left(\frac{1-b}{b-a}\right)^{1/2}+\left[(1-b)(b-a)\right]^{1/2}\Big/(a+1). \tag{20}$$

The module of the magnetic field strength in the initial domain

$$\left|H_z\right|=U_0\frac{a+1}{Z}\left|\frac{1}{t}\left(\frac{t-1}{t-b}\right)^{1/2}\right|. \tag{21}$$

Then, for the module of the ponderomotive force, it is easy to obtain the following expression from (13), using (17) – (21)

$$\left|f_{\mathrm{m}}\right|=\mu_0\chi\pi\frac{U_0^2}{2}\left(\frac{a+1}{Z}\right)^3\frac{\left[(t-a)(t-1)\right]^{1/2}}{t^4(t-b)^2}\left[2t^2-t(3a+b)+2ab\right].$$

The latter expression allows optimizing the geometry of ferromagnetic shunts of electromagnetic pulleys [8, 12].

Conclusion. The problem of transformation of well-known calculated expression of the specific ponderomotive force of the magnetic field in the working zone of the poles was solved. It allows to calculate ponderomotive force directly, having only the function of conformal transformation of pole configuration in initial plane *z* on mapped domain *t*.

It should be mentioned that, since the expressions for the specific ponderomotive force of magnetostatic and electrostatic fields are similar [7], the found expressions can also be used in the calculation of systems operating on the principle of ponderomotive action of the electrostatic field.

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ОПРЕДЕЛЕНИЕ ПОНДЕРОМОТОРНОЙ МАГНИТНОЙ СИЛЫ ПРИ РАСЧЕТЕ ПОЛЯ МЕТОДОМ КОНФОРМНОГО ПРЕОБРАЗОВАНИЯ

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При исследовании устройств, в которых используется пондемоторное действие магнитного поля, широкое применение получил метод конформного преобразования, позволяющий проводить анализ и расчет стационарных двухмерных электрических и магнитных полей, удовлетворяющих уравнению Лапласа, и значительно упростить задачу расчета поля. Основным недостатком этого метода является отсутствие общего способа нахождения комплексного потенциала и, как следствие, невозможность получения выражения для напряженности поля и его пондермоторной силы в явном виде как функции координат в исходной области. В данной работе решается задача непосредственного нахождения выражения для удельной пондеромоторной силы магнитного поля с использованием комплексного потенциала при известной функции конформного преобразования. Рассмотренные примеры расчета пондеромоторной силы в рабочей зоне полюсов различной формы могут служить моделью для исследования тяговых рабочих характеристик электромагнитных систем магнитно-сепарирующих устройств. Кроме того, полученные выражения также могут использоваться при расчете систем, работающих на принципе пондеромоторного действия электростатического поля. Библ. 12, рис. 2.

Ключевые слова: пондеромоторная сила, напряженность магнитного поля, метод конформного преобразования, магнитные системы.

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ВИЗНАЧЕННЯ ПОНДЕРОМОТОРНОЇ МАГНІТНОЇ СИЛИ ПРИ РОЗРАХУНКУ ПОЛЯ МЕТОДОМ КОНФОРМНОГО ПЕРЕТВОРЕННЯ

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У процесі дослідження пристроїв, в яких використовується пондемоторна дія магнітного поля широке застосування отримав метод конформного перетворення, який дає змогу проводити аналіз і розрахунок стаціонарних двовимірних електричних і магнітних полів, що задовольняють рівнянню Лапласа, а також значно спростити задачу розрахунку поля. Але його основним недоліком є відсутність загального способу знаходження комплексного потенціалу, що унеможливлює отримання виразів для напруженості поля і його пондермоторної сили в явному вигляді як функції координат у вихідній області. У даній роботі вирішується завдання безпосереднього знаходження виразу для питомої пондеромоторної сили магнітного поля з використанням комплексного потенціалу при відомій функції конформного перетворення. Розглянуті приклади розрахунку пондеромоторної сили в робочій зоні полюсів різної форми можуть служити моделлю задля дослідження тягових робочих характеристик електромагнітних систем магнітно-сепаруючих пристроїв. Крім того, отримані вирази також можуть використовуватися задля розрахунку систем, що працюють на принципі пондеромоторної дії електростатичного поля. Бібл. 12, рис. 2.

Ключові слова: пондеромоторна сила, напруженість магнітного поля, метод конформного перетворення, магнітні системи.

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