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MATHEMATICAL MODEL TO CALCULATE THE TRAJECTORIES OF ELECTROMAGNETIC MILL OPERATING ELEMENTS

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The purpose of the research under consideration is to develop a mathematical model to calculate the trajectories of the ferromagnetic operating elements (millstones) of an electromagnetic mill, moving in a rotating magnetic field under electrodynamic and hydrodynamic resistance forces being limited by the space of the mill's working chamber. The millstone motion is described through the equations of plane motion of arbitrary-shaped two-dimensional body. The driving forces of this motion are determined on the basis of the approximation of the tabulated functions connecting the module and the orientation of the equivalent force applied to the millstone, with its position in the working chamber and composite MMF phase of mill inductor winding. These tabulated functions are derived from the estimation of the magnetic field inside a working chamber with millstones, in two-dimensional quasi-stationary approximation, using FEM analysis. The publication contains the approximation algorithm for these tabulated vector functions of a vector argument, mathematical statement of millstones trajectories calculating, and analysis of mathematical experiments results that make it possible to evaluate the adequacy of the model. The developed tool enables conducting quantitative analysis of grinding/mixing process and will help to establish relationships between the electromagnetic mill design parameters and its performance. References 21, figures 6.
Keywords: electromagnetic mill, grinding, mixing, tabulated functions interpolation, tabulated functions approximation, plane motion, FEM analysis.

1. Introduction. Electromagnetic mill (EMM) is a production equipment designed to grind or mix various mixtures of non-corrosive or corrosive substances. The technological process takes place inside the working chamber (WCh) of the mill, owing to the force interaction (collisions, vibrations, shocks, friction, etc.) of its operating elements with the substance for grinding or mixing. This substance will hereinafter be referred to as the working medium (WM), and the operating elements – the millstones. The ferromagnetic millstones move under the influence of electromagnetic forces arising in WCh rotating magnetic field generated by the mill inductor.

The development of issues related to EMMs creation, optimization or operation has numerous problems. In particular, there are no design techniques combining EMM's performance with its structural elements dimensions, inductor magnetic field power characteristics and other design indicators. On the other hand, the topicality of the research is confirmed by numerous publications and current trends related higher requirements for technological processes in terms of their environmental friendliness, energy efficiency, operational reliability, etc.

2. Recent research analysis. The problems arising while dealing with the issues of creating technological lines designed to grind/mix various substances are so complex in scientific and technical aspects, that their solution requires combined efforts of specialists in different fields. For example, energy-saving issues and minimizing the impact on the power grid are considered jointly by not only electricians but also by mining professionals [1, 2, 3].

The research is being conducted in specific areas, such as EMM inductors design [4, 5], parametric influences and relationships of characteristic physical processes [3, 6], control systems development [2, 7]. A comprehensive study of the whole technological line, including, apart from EMM, pneumatic and screw feeder systems of grinding substance, several separators of different purpose and operation principle, a number of guide pipelines, inlet and outlet collectors, valves, cyclone, etc., is presented in [1]. This equipment is also used to adjust and optimize an effective control system [3, 8] and to conduct experimental studies [7, 9].

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The surveys [9-12] are dedicated to the effectiveness of dry and wet methods of copper ore grinding, the influence of WM humidity on this process and its value measuring.

Among the urgent issues is the problem of average particle size quantitative evaluation of the milling substance, in other words, the problem of WM grain structure estimation before and immediately after the grinding process. For example, the studies [2, 13] use an intellectual optical image processing system.

Separately, we want to address the issues of determining EMM performance. Using one of the grinding quality indicators, such as its grain structure, the researchers determine the size of particles in WM samples, depending on millstones size, material feeding volume, grinding process time, WCh filling and compare it with a given value [1, 7, 9, 14-16]. At the same time, WM volume or mass is determined, which meets the quality index. In almost all these studies, the analysis is performed on the basis of experimental data.

The disadvantages of this approach are obvious. Investigating the impact on mill performance of any of its design indicators requires either modifying the existing design, with possible additional elements installation, or making a completely new prototype. It is clear that under this approach the solution of design or of EMM construction optimization issues is problematic.

A relatively small number of publications focuses on theoretical solution of the relationship between EMM design index and its performance. In [6], the authors, emphasizing the relevance of this problem, propose a mathematical model of the metal polishing, based on a genetic algorithm and artificial neural network, and in [7] a similar model is created on the basis of physical experiments results statistical processing. It is mathematical modeling that has been chosen as a priority as a way to solve the above problem.

We are convinced that the adequate mathematical model development designed to estimate the ferromagnetic millstones movement indices in mill's inductor rotating magnetic field, taking into account the mechanical interaction with the particles of the milled or mixed substance, will give the reliability of EMM designing results a new level, corresponding to modern requirements of information and computer support for creating complex technical systems.

3. Research objective. The analysis of current publications suggests that existing EMM design techniques are based, at best, on experimental data statistical processing and do not provide clear recommendations correlating the mill's expected performance with the combination of its design indicators. Therefore, there is a need to create a virtual tool that will allow to conduct quantitative analysis of a grinding/mixing process as a determinate physical process. It will help to identify the factors affecting the mill's performance.

Goal of research. To develop a mathematical model to estimate the working medium millstones and particles mass centers trajectories moving in a rotating magnetic field under electrodynamic and hydrodynamic resistance forces and being limited by the space of the mill's working chamber.

Object of research. Processes of dynamic interaction between ferromagnetic millstones and working medium in a rotating magnetic field inside the working chamber, in transient and quasi-stable modes.

4. Methods of research. A complexity of the problem, a large number of dependent variables, and lack of information not only about attempts to solve such problems, but also about similar formulations, led us to adopt the following strategy. First of all, on the basis of the results [17], we calculate tabulated functions (TF) reflecting the relationship between the vector of total electromagnetic force $\overline{F_m}$ applied to the center of the millstone mass, and a number of specific parameters – arguments of this function. It should be noted that after analysing the results of the above mentioned study, it is assumed that $\overline{F_m}$ is a function of only MMF resultant phase of the mill inductor φ, the angular γ_m and radial r_m coordinates of the mill's position in the working chamber and millstone's dimensions – its diameter d_m and length l_m.

Fig. 1, in polar coordinates system, presents two succession of force vector F_m hodographs, which often reflect these correspondencies. Depending on Fig. 1, *a* the angular coordinate of the millstone position *1* – γ_m= 0°; *2* – γ_m= 48°; *3* – γ_m= 60°; *4* – γ_m= 108°; *5* – γ_m= 120°; *6* – γ_m= 168° (r_m = 83 mm); Fig. 1, *b* the radial coordinate of the millstone position *1* – r_m= 83 mm; *2* – r_m= 80 mm; *3* – r_m= 75 mm; *4* – r_m= 68 mm; *5* – r_m= 60 mm; *6* – r_m= 50 mm; *7* – r_m= 40 mm (γ_m = 0°). It should be noticed that, horizontal axis on the graphs denotes force scale (N), and circular one – millstone position angular coordinate γ_m (°).

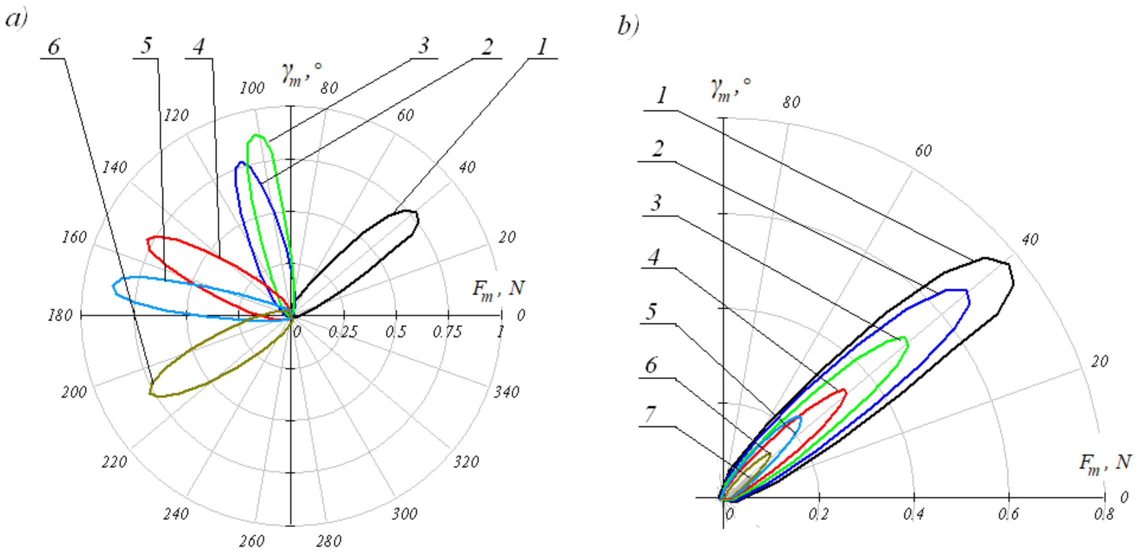

Fig. 1

The calculation was performed for the EMM with WCh capacity of 6300 cm^3. Its main dimensions are: estimated core length - 200 mm, WCh diameter - 200 mm. The average magnetic induction value in WCh – 0.153 T. Millstones dimensions – d_m = 2.5 mm and l_m = 10 mm. The MMF phase φ varied in the range of 0 ÷ 180°. The complete results of this study can be found in [17].

The next step to achieve this goal was creating mathematical model to calculate the millstones trajectories in space bounded by WCh walls, under the above-mentioned electromagnetic forces. The value and direction of these forces, at each integration step in time of the differential equation system describing the millstones movement, are determined on the basis of the results of the tabulated function approximation $F_m[\varphi, r_m, \gamma_m]$.

It is obvious that approximation accuracy of this tabulated function directly affects the results of millstones dynamics calculation, so we will consider the method of its approximation in more detail. This tabulated function is essentially a vector function of a vector argument. Let us introduce the following symbol

$$\vec{F}_m = \begin{Vmatrix} F_m \\ \alpha_m \end{Vmatrix}; \qquad \vec{s} = \begin{Vmatrix} \varphi \\ r_m \\ \gamma_m \end{Vmatrix}, \tag{1}$$

where F_m and α_m are the modulus and the electrodynamic force $\overline{F}_m$ inclination vector in a Cartesian rectangular coordinate system, which beginning coincides with the working chamber centerline.

Suppose that with a constant value r_m, each of the components of a vector $\vec{F}_m$ approximates with two-variable φ and γ_m quadrate Taylor polynomial. The force module F_m will be calculated as

$$F_m[\varphi,\gamma_m] = c_1 + c_2\varphi + c_3\gamma_m + c_3\frac{\varphi^2}{2!} + c_5\varphi\gamma_m + c_6\frac{\gamma_m^2}{2!} = \vec{T}[\varphi,\gamma_m]\vec{c}, \tag{2}$$

where $$\vec{T}[\varphi,\gamma_m] = \begin{Vmatrix} 1 & \varphi & \gamma_m & \frac{\varphi^2}{2!} & \varphi\gamma_m & \frac{\gamma_m^2}{2!} \end{Vmatrix} \tag{3}$$

is the two-variable quadrate Taylor polynomial;

$$\vec{c} = \begin{Vmatrix} c_1 & c_2 & c_3 & c_4 & c_5 & c_6 \end{Vmatrix}_* \tag{4}$$

Is the polynomial coefficient column (2).

We define them on the basis of the knot values of the scalar tabulated function $F_m[\varphi, \gamma_m]$, selected from the range, filling the tabulated function vector $\overrightarrow{F_m}[\vec{s}]$ on the basis of min$|r_{mt} - r_m|$, where $|r_{mt} - r_m|$ is difference modulus between the tabulated value of the radius vector length of the millstone mass center r_{mt} and its current value r_m.

Now, let us form tabulated function $F_m[\varphi, \gamma_m]$ pattern. The number of its knots should be equal to six, which corresponds to the number of unknown interpolation coefficients in (2). The very formation of a pattern $F_m[\varphi, \gamma_m]$ starts with the search for the knot closest to the values of the interpolant argument. This knot will be referred to as "central". In Fig. 2 the tabulated function $F_m[\varphi, \gamma_m]$ is schematically presented as a table, and the tabulated function pattern, depending on the "central" knot position, – as the shaded boxes.

Let us create a Taylor matrix T by the following rule: the row j of this matrix is equal to the value of the Taylor row (3) calculated at the knot j of the tabulated function. It looks as following

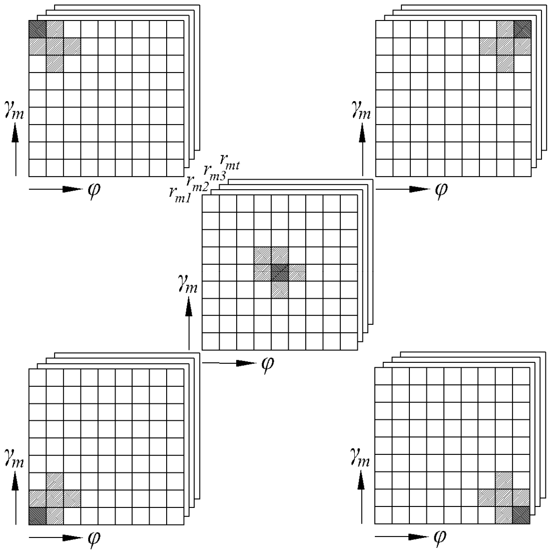

Fig. 2

$$T = \begin{Vmatrix} 1 & \varphi_1 & \gamma_{m1} & \frac{\varphi_1^2}{2!} & \varphi_1\gamma_{m1} & \frac{\gamma_{m1}^2}{2!} \\ \vdots & & & & & \\ 1 & \varphi_6 & \gamma_{m6} & \frac{\varphi_6^2}{2!} & \varphi_6\gamma_{m6} & \frac{\gamma_{m6}^2}{2!} \end{Vmatrix}. \qquad (5)$$

Let us combine the tabulated function (discretes) into a column-vector

$$\vec{F}_{mt} = \|F_{m1} \quad \ldots \quad F_{m6}\|_* . \qquad (6)$$

Having solved the vector equation

$$T \cdot \vec{c} = \vec{F}_{mt} \qquad (7)$$

relatively unknown variable $\vec{c}$, we obtain the column-polynomial coefficients (2).

Thus, to estimate the tabulated function interpolated value $F_m[\varphi, \gamma_m]$ for arbitrary values and independent variables φ_z and γ_{mz}, it is necessary to:

- estimate the Taylor matrix (5) for a given tabulated function;
- create a discrete column (6) for a given tabulated function;
- solve the linear vector equation (7);
- estimate the value of the Taylor row (3) following $\varphi = \varphi_z$, $\gamma_m = \gamma_{mz}$;
- estimate the interpolated value of tabulated function as a product $\vec{T}[\varphi_z, \gamma_{mz}] \cdot \vec{c}$.

Note that the direction of force $\overline{F}_m$ is estimated similarly from the tabulated function interpolant $\alpha_m[\varphi, \gamma_m]$, which is obtained simultaneously with the tabulated function. The described algorithm is applicable for any number of tabulated function knots and arbitrary relative arrangement of knots in the pattern.

To test the efficiency of the proposed algorithm for tabulated function approximation and to evaluate the accuracy of the electrodynamics forces field interpolation affecting the millstone in a rotating magnetic field, it has been tested on the tabulated function $F_m[\varphi, \gamma_m]$, which was obtained under the same conditions as the hodographs in Fig. 1.

The graph of the electrodynamic force modulus interpolation F_m dependence on the phase of the inductor composite MMF φ is shown in Fig. 3, *a* (curve *2*).

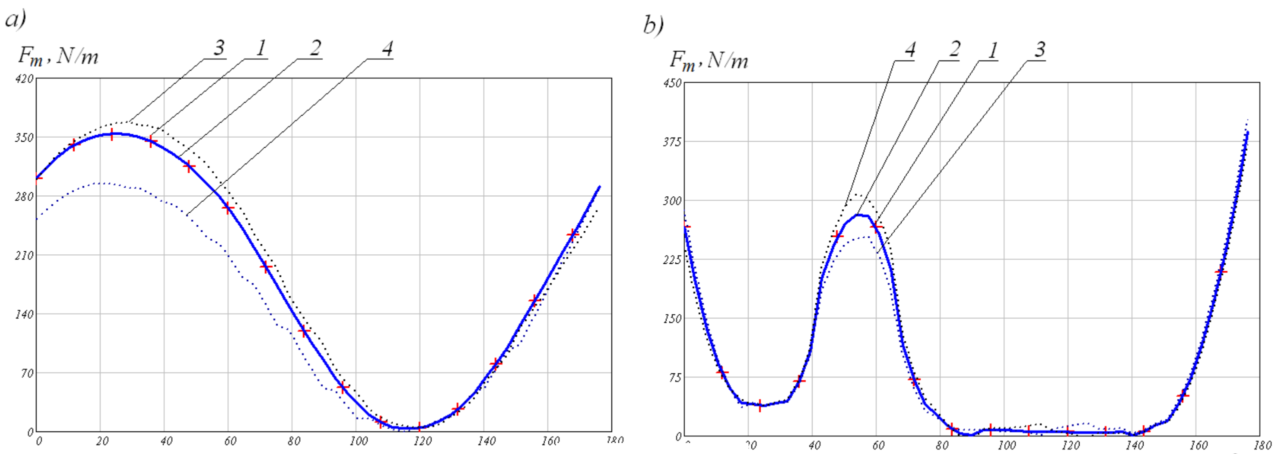

Fig. 3

The inclination angle of the radius vector to the millstone mass center is γ_m = 60º. Also, this graph shows similar dependences corresponding to the millstone position angular deviation within 60±5º (Fig. 3, *a*, curves *3*, *4*). Note that the value of this force is related to the millstone unit length, perpendicular to the estimated plane. This is because of the two-dimensional formulation of a field problem of its estimation, and that is why the dimension of this force [N/m].

The dependence of the electrodynamic force modulus F_m on the inclination angle of the millstone radius vector γ_m is created for the temporal phase of inductor MMF φ = 60ºel; (Fig. 3, *b*, curve *2*). The effect of the phase shift deviation within 60±5ºel is shown in Fig. 3, *b*, curves *3*, *4*.

In both graphs, the symbol "+" indicates the tabulated function knot values $F_m[\varphi, \gamma_m]$ (Fig. 3, *a, b*, symbol 1).

An additional factor that, in our opinion, influences the dynamic interactions between the millstones and WM particles is hydrodynamic resistance force. This force affects all the components of the mixture fed to the WCh with a flow of compressed air or liquid. Such a flow is also called a traffic flow [8, 12].

Let us establish a number of assumptions allowing mathematical formalization of this interaction. We assume that the hydrodynamic resistance force only affects WM particles, the WM particles shape is spherical, with all these spheres radius r_{pp} being the same. Therefore, if a particle moves in a liquid or gas, it will only be affected by the drag force, with the tractive force equalling to zero. We also assume that in the plane perpendicular to the WCh axis, only a WM particle moves, with the traffic flow being quiet to it. Given the low flow velocities in WCh axial direction and its small fillings, this assumption is quite valid.

Based on experimental studies [18], it was found that fluid or gas movement in the boundary layer is turbulent, and the drag force is recommended to be estimated as follows

$$P_c = C_x S \frac{\rho v^2}{2}, \tag{8}$$

where C_x is the drag coefficient (1.12 for a sphere); S is the body projection area onto a plane perpendicular to that body motion trajectory; v is the body motion velocity projection in a quiet flow.

Note, that for further mathematical experiments we assumed that the WM particles are in a substance with some estimated density, which is 10% of water density. This method allows to reduce the time of the system launching to quasi-steady operating mode, that is, when the average speed of all its elements become equal.

5. Mathematical formulation of the problem. Millstones motion in the mill's working chamber under electrodynamic and resistance forces was regarded as a plane motion. This means that the points in arbitrary planes perpendicular to the working chamber axis and those belonging to the millstone remain in these planes as it moves.

Kinematics states that in order to describe the arbitrary shaped two-dimensional body plane motion, that is, to find the trajectories, velocities, and accelerations of all its points, it is necessary to have the equation of this body mass center motion:

$$x_c = f_1(t); \qquad y_c = f_2(t); \qquad \varphi = f_3(t), \tag{9}$$

where x_c, y_c and φ are the coordinates of mass center and angular position of millstone.

Formulas for bringing inertial forces of a solid are obtained on the basis of d'Alembert's principle

$$m\bar{a} = \sum F_i; \quad -J\bar{\varepsilon} = M_i, \tag{10}$$

where m is the body mass; J is the moment of inertia about axis z; F_i, M_i are the summarized forces and moments; $\bar{a}$, $\bar{\varepsilon}$ are mass center linear and angular acceleration.

The differential equations of body mass center motion and moment equation at relative motion about an axis passing through the center of mass perpendicular to the plane of motion

$$\frac{\mathrm{d}^2 x_c}{\mathrm{d}t^2} = \frac{\sum_{i=1}^{n} F_{ix}}{m}; \qquad \frac{\mathrm{d}^2 y_c}{\mathrm{d}t^2} = \frac{\sum_{i=1}^{n} F_{iy}}{m}; \qquad \frac{\mathrm{d}\omega_c}{\mathrm{d}t} = \frac{1}{J}\sum_{i=1}^{n} M_z(F_i) = \frac{M_z}{J}; \qquad \omega_c = \frac{\mathrm{d}\varphi}{\mathrm{d}t}. \tag{11}$$

where $M_z(F_i)$ is the moment of force F_i relatively to axis $0z$, passing through the mass center; M_z is the principal moment of all external forces about this axis; ω_c is the angular velocity.

Thus, the system (11), consisting of four equations and containing four unknowns x_c, y_c, φ and ω_c, is the content of mathematical formulation of the problem of the arbitrary shaped solid body plane motion trajectory estimation, which is affected by external forces and moments. The algorithm for its solving is considered in detail in [19], and the formulation, suitable for the system of bodies and on the basis of MSE - [20]. Basically it solves three problems:

- estimates body mass matrices and their inertial tensors on the basis of a finite element grid;
- defines current values F_i, M_i;
- updates displacement, velocities vectors and inertial tensors so that to avoid deformation of solids.

Processing contact interactions within the system (touch, mutual sliding motion, etc.) is performed through kinematic limitation method [21]. It is superimposed on global equations by converting the main components of knots moving along the contact line. This transformation blocks the freedom degree of the knots responsible for the normal movement relatively the contact line. In order to maintain the efficiency of the algorithm of explicit numerical time integration method, the bodies masses are concentrated (scaled) to such an extent that only the main degrees of freedom of each knot remain bound.

This algorithm is implemented in APDL (Ansys Parametric Design Language) Ansys LS-DYNA.

6. Analysis of the obtained results. In order to evaluate the efficiency and adequacy of the created mathematical model at the qualitative level, let us consider the estimation results of two millstones motion dynamic indices, which are in the mill's working chamber under a rotating magnetic field with an intensity of ≈ 0.1 T. Diameter of the WCh d_{km} = 200 mm, millstone's dimensions is the d_m = 2.5 mm and l_m = 10 mm.

At the beginning of the transition process, both millstones are located on $0.65d_{km}$ diameter, the angular position of the mass center of the first millstone is γ_{m1} =45° with the second one being γ_{m2} =225°. Also, in the middle of the WCh there are 10 WM particles, evenly distributed in a circle with a diameter of $0.92d_{km}$. The estimated millstone and WM particles masses are 0.38 g and 0.49 g, respectively. Their relative position at the beginning of the transition process is shown in Fig. 4, *a*.

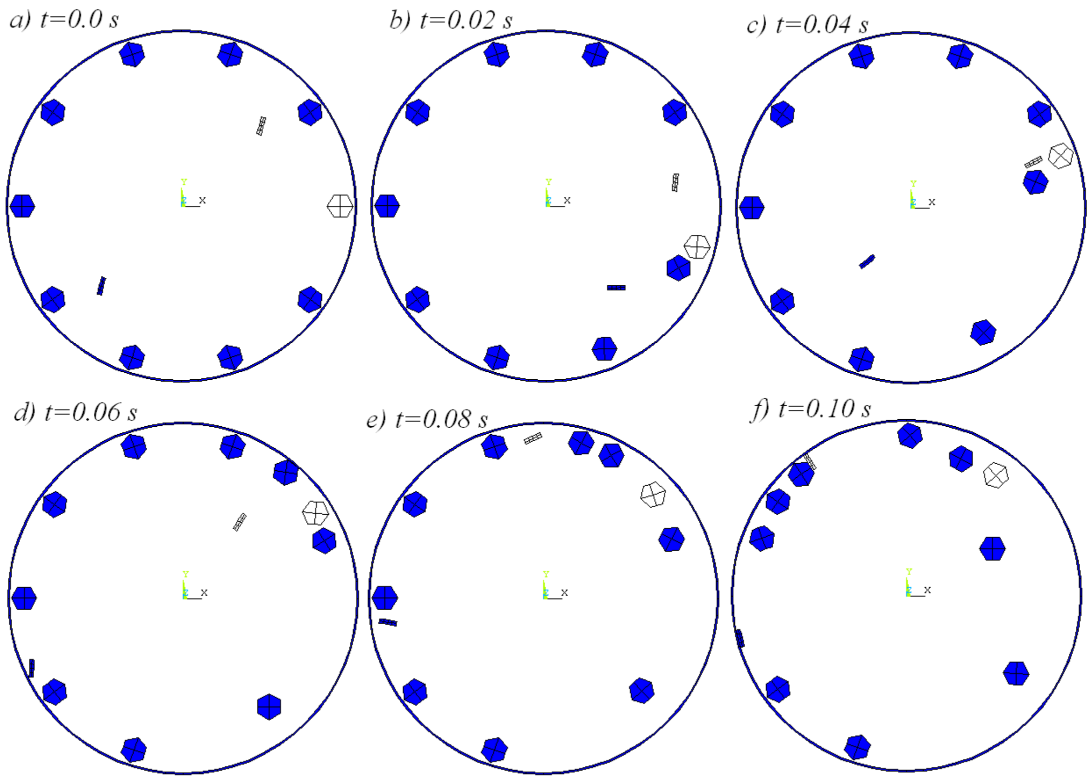

Fig. 4

The initial velocity vector for each millstone, at a time point t= 0, has only a tangential component, and it is $v_{\tau 1}$=-10.21 m/s for the first millstone and $v_{\tau 2}$=10.21 m/s for the second one. The WM particles are at rest.

Fig. 4, *b-f* shows the gradual change in millstones and WM particles positions occurring in a magnetic field as a result of their colliding. For better visualization, millstone No. 1 and one of the WM particles in the figure are not shaded. Fig. 5, in its turn, contains the hodographs of the radius-vectors of both millstones mass centers and this WM particle. They are all presented in the polar coordinates system.

These two figures make it possible to trace the direction of millstones motion, the moments of their collision with the WM and the WCh walls. In particular, at a time point t = 7 ms, the first millstone, when moving against the direction of magnetic field rotation, collides with the WM particle, bounces and repeatedly hits the WCh wall, losing the kinetic energy of the initial condition, and then, once under pondemotor forces, starts moving in the direction of the field rotation and close to the WCh inner surface.

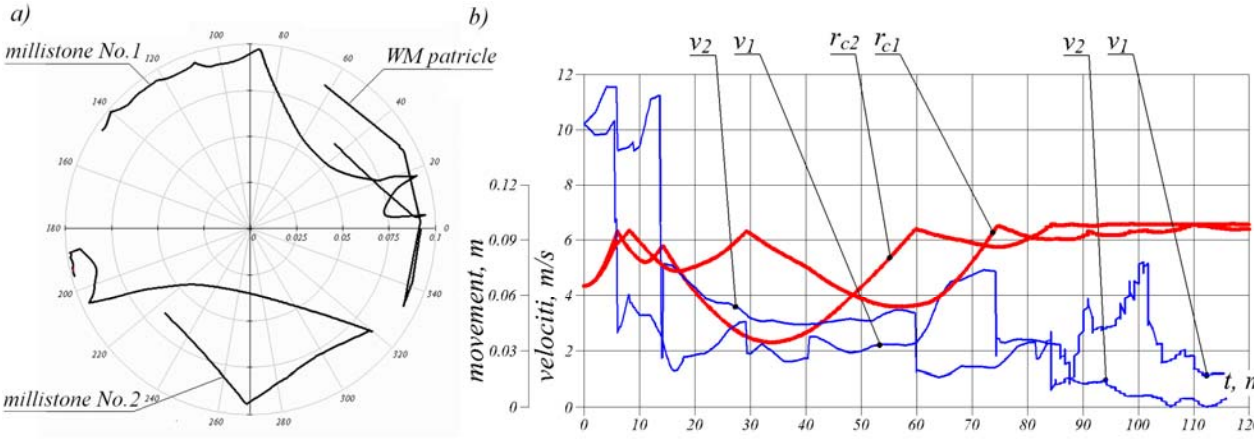

Fig. 5

The second millstone was moving following the direction of the field rotation. After colliding with the WCh wall and further resilience, it bumps into a WM stationary particle at high speed. Repelling from it, the millstone almost loses speed and stops for a while. But then, under the influence of electromagnetic force, it starts moving towards the field.

Fig. 6 shows the modules of movements and velocities of the millstones in one coordinate plane, which allows not only qualitatively but quantitatively to check the obtained modeling results for compliance with the basic physical relation. In particular, breaks in displacement functions correspond to discontinuities in velocity functions. The velocity oscillations at the end of the process indicate that owing to the friction between the elements there is a process of their shock interaction inside the WCh. This can serve as indirect confirmation of the fact of grinding process continuation in the modes characterized by millstones accumulation on WCh periphery and/or millstones “coating” of WM particles with simultaneous filling of the gaps between these particles.

The obtained results do not contradict the physical conception of millstones interaction with the working medium process and allow to state that the mathematical model to estimate the trajectories of ferromagnetic millstones motion in rotating magnetic field of the working chamber is adequate, under the assumtions made, and it can be applied in further researches related to the quantitative evaluation of EMM performance.

To demonstrate the work of the proposed mathematical model with more millstones, Fig. 6 shows WCh sections containing 874 millstones and 54 WM particles. The total filling level of the WCh is ≈ 50%. The millstone diameter in this experiment was 3 mm, its length was also 3 mm.

The estimation was performed on a personal computer with a 2.8 GHz Intel Core™ 2 Quad Q6600 processor; 3.24 GB RAM, and its duration was ≈ 10 hours.

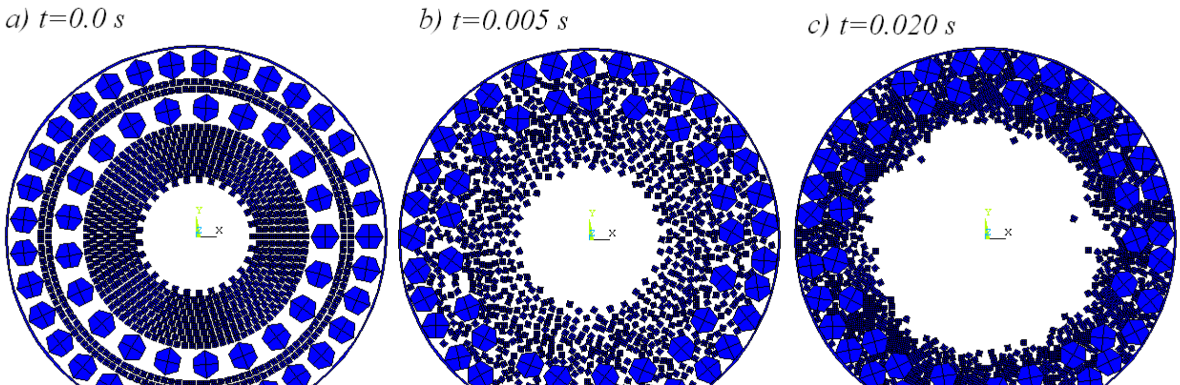

Fig. 6

Conclusions.

1. The method to approximate the tabulated function of electromagnetic force applied to an arbitrary-shaped ferromagnetic element (a millstone) in a magnetic field, depending on its position and the time phase of the MMF of the EMM inductor, via Taylor polynomial, correctly reflects all features of a given vector function.

2. The study has developed a mathematical model to calculate the ferromagnetic millstones motion trajectories, which are in a rotating magnetic field and limited by the space of the working chamber. Its algorithm is applicable for studying working chambers and millstones of arbitrary size and shape. It is created on the basis of equations of bodies motion dynamics by two-dimensional formulation (the so-called plane motion) and takes into account the influence of the contact interaction of millstones with the particles of the material being milled or mixed, hydraulic resistance forces applied to the moving elements in this interaction and dry friction.

3. The estimation results of ferromagnetic millstones transient motion in a rotating magnetic field, obtained with the help of the created mathematical model, confirm the correctness of interaction of interpolation algorithm of tabulated function of electromagnetic force, applied to a millstone, with the algorithm for numerical integration of algebraic differential equations system describing the dynamics of grinding or mixing process components. The research has produced recommendations on the integration step size and necessary computing resources.

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МАТЕМАТИЧНА МОДЕЛЬ РОЗРАХУНКУ ТРАЄКТОРІЙ РОБОЧИХ ЕЛЕМЕНТІВ ЕЛЕКТРОМАГНІТНОГО МЛИНА

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Мета представленого дослідження полягає у розробці математичної моделі розрахунку траєкторій феромагнітних робочих елементів (жорен) електромагнітного млина, що рухаються в обертовому магнітному полі під дією електродинамічних сил і сил гідродинамічного опору та обмежені простором робочої камери млина. Переміщення жорен описується рівняннями динаміки плоского руху двовимірного тіла довільної форми. Вимушуючі сили цього руху визначаються на підставі наближення таблично заданих функцій, що зв'язують модуль та напрям рівнодіючої сили, прикладеної до жорна, з його положенням в робочій камері та фазою результуючої МРС обмотки індуктора млина. Ці табличні функції отримані з результатів розрахунку магнітного поля всередені робочої камери, заповненої жорнами, у двовимірному квазістаціонарному наближенні та з використанням FEM-аналізу. Публікація містить алгоритм наближення цих табличних векторних функцій векторного аргументу, математичне формулювання задачі розрахунку траєкторій жорен та аналіз результатів математичних експериментів, які дають змогу оцінити адекватність моделі. Розроблений інструмент дає можливість кількісного аналізу процесу розмелювання/перемішування та допоможе у встановленні зв'язків між проектними параметрами електромагнітного млина та його продуктивністю. Бібл. 21, рис. 6.

Ключові слова: електромагнітний млин, розмелювання, перемішування, інтерполяція табличних функцій, наближення табличних функцій, плоский рух, FEM-аналіз.

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