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RESONANT MODES OF A LINEAR PERMANENT MAGNET VIBRATORY MOTOR

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The work considers the resonant operation modes of the linear permanent magnet vibratory motor. On the basis of electrical and mechanical equivalent circuits with lumped parameters, expressions for determining the frequencies of mechanical, electrical, energy and power resonances are obtained. The presence of two frequencies of electrical resonance (when the phases of supply voltage and motor current coincide) in a single-mass electromechanical system and four in a two-mass one is substantiated. Representing, according to the electromechanical analogy approach, the back EMF induced due to the movement of the mover by the corresponding voltage drop, expressions for equivalent mechanical impedances are obtained. The dependences of the energy characteristics of the motor (mechanical work and efficiency) on the equivalent circuit parameters are obtained. Based on the expression for the reluctance electromagnetic force, mechanical work is found and its dependence on the phase difference between displacement and current is shown. The phase difference at which the total mechanical work of the motor is maximal is determined. It is shown that the results of the analysis of resonant modes well agree with results of a numerical field study carried out on the basis of the equations of the quasi-stationary magnetic field in the time domain using the finite element method and the moving type of calculation mesh in the mover region. References 12, figures 6, tables 1.
Keywords: electrical resonance, electromechanical analogy approach, energy characteristics, finite element method, linear permanent magnet motor.

Introduction. Vibration technologies are the basis of many modern technological processes related to the movement and processing of materials, compaction, sorting, granulation, etc. The use of the linear permanent magnet motors in vibrating machines provides a number of advantages, the main of which are a wide frequency operating range, the absence of mechanical transmissions that convert rotational motion into a linear one, and therefore reliability and low noise level, and the ability to control performance in automatic mode [1-4].

To date, there is no clear idea in which mode (in terms of providing the necessary electromechanical performance) a vibrating machine with the permanent magnet motor drive should operate. It is considered generally acceptable to provide a mode close to mechanical resonance, since otherwise the drive efficiency is low [5-7]. It is also known that the frequency of phase mechanical resonance (hereinafter the phase mechanical resonance means the coincidence of the main harmonic components of electromagnetic force and mover speed) does not coincide with the frequency of maximum motor efficiency and maximum amplitudes of acceleration and displacement [8]. If we take into account that in addition to mechanical resonances, there are also electrical resonances, as well as some influence of the salient-pole structure of the magnetic system and harmonics of electromagnetic force on the force-angle characteristics of the motor [9, 10], the problem of choosing the optimal machine parameters to provide the required mode of mechanical oscillations becomes ambiguous and controversial.

Obviously, the simultaneous ensuring of optimal electromechanical characteristics is impossible. We have to give up certain indicators in favor of others, the achievements of which are more important. Therefore, the study of the influence of the motor's equivalent circuit parameters on the electromechanical processes and their interrelationship is essential from a theoretical point of view, and necessary from the point of view of practical implementation of efficient drive modes.

The aim of the work is to determine the operation modes of a linear permanent magnet vibratory motor, which provide the specified electromechanical characteristics.

The basic structure of the permanent magnet vibratory motor is shown in Fig. 1. The stator of the motor contains a laminated ferromagnetic core *1* and a winding *2*, which is powered by a single-phase AC source. The winding has two groups of coils (shown in color), the direction of current in which is opposite. The mover contains ferromagnetic poles *3* and permanent magnets *4* with axial direction of magnetization. The pulsating magnetic field of the winding, interacting with the field of permanent magnets, causes an axial electromagnetic force, the direction of which is determined by the direction of current. The mover motion in

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relation to the stator leads to deformation of the spring system *5*. With a change in the coils current direction, the mover moves in the opposite direction under the influence of electromagnetic force and energy accumulated in the springs.

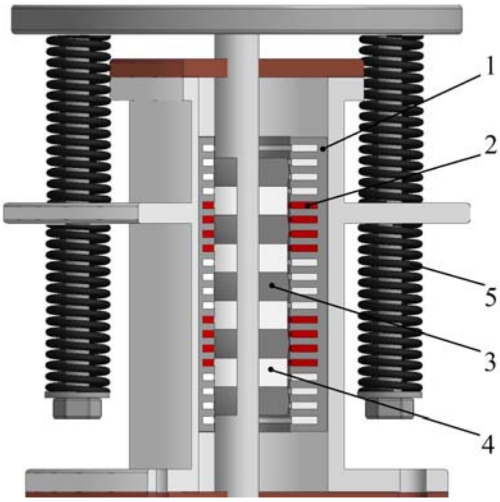


Fig. 1

The nature of the flow of the motor's electromechanical processes is determined by the parameters of its equivalent circuit. Therefore, we will further consider the influence of machine parameters on the provision of resonant operation modes.

Mechanical resonance. Here we will consider an electromechanical system with a harmonic electromagnetic force and constant parameters of the equivalent circuit. The behavior of such systems is well known and has been considered in many works. Therefore, we will restrict ourselves to the results of researches that are necessary for further presentation, since the connection of electromagnetic processes with mechanical ones, as will be shown below, is quite significant. Also, due to the great variety of mechanical design, we will consider only single-mass and two-mass, which are most common.

So in the single-mass oscillatory system, the resonance frequency can be determined from the equivalent circuits shown in Fig. 2.

The mechanical equivalent circuit (Fig. 2, *a*) contains the mover *1*, which oscillates under the influence of electromagnetic force F_{ev} relative to the stator *2*. The spring system *3* and the damper *4* are characterized by spring stiffness *k* and viscous friction *b* factors, which take into account the corresponding motor coefficients together with the load. The coordinate system is connected to the stator, with the origin corresponding to the position of mover mechanical equilibrium, when the motor current is zero. Moreover, the system starts with oscillations from the state in which the static equilibrium already exists between gravitational force and the spring forces.

The electrical equivalent circuit (Fig. 2, *b*) is represented by series-connected resistance R_{sv}, inductance L_v and source e_v, which simulate the stator winding resistance, winding inductance and back EMF induced due to the mover motion.

The presented equivalent circuits correspond to the following system of equations written in the frequency domain:

$$\left.\begin{aligned} &\underline{U}_v = \underline{I}_v(R_{sv} + j\omega L_v) + K_{Ev}\underline{V};\\ &-m_a\omega^2\underline{X} = K_{Fv}\underline{I}_v - k\underline{X} - j\omega b\underline{X};\\ &j\omega\underline{X} = \underline{V}, \end{aligned}\right\}, \tag{1}$$

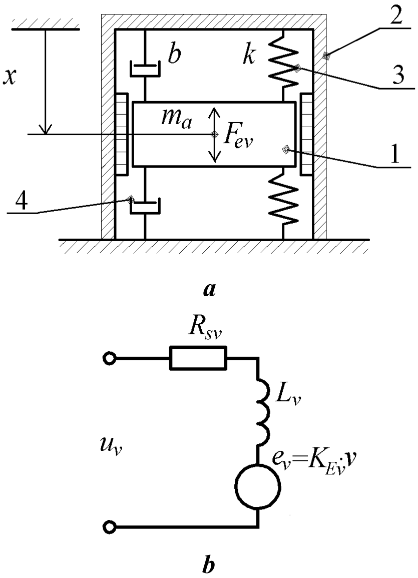


Fig. 2

where $\underline{U}_v, \underline{I}_v$ are the complex supply voltage and stator current, respectively; K_{Ev} is the EMF constant; $\underline{V}, \underline{X}$ are the speed and displacement of mover relative to the stator, respectively; m_a is the mass of the mover; K_{Fv} is the electromagnetic force constant; *k* is the equivalent stiffness factor of the spring system and the load; $b = b_v + b_l$ is the total viscous friction factor of the motor b_v and the load b_l; ω is the supply voltage angular frequency.

The solution of equations (1) makes it possible to obtain the frequency of mechanical resonance at which the amplitude of the mover oscillation will be maximum:

$$\omega_r = \sqrt{\frac{k}{m_a} - \frac{b^2}{2m_a^2}}, \tag{2}$$

as well as the frequency of phase mechanical resonance, at which the phases of electromagnetic force and the mover speed coincide:

$$\omega_0 = \sqrt{k/m_a}. \tag{3}$$

In the two-mass electromechanical system (Fig. 3), the stator of the motor *1* is fixed to the operating element *2*. The mover *3* oscillates under the influence of electromagnetic force F_{ev} relative to the stator on the springs *4*.

The system of equations in the frequency domain will look like this:

$$\left.\begin{aligned} &\underline{U}_v = \underline{I}_v (R_{sv} + j\omega L_v) + j\omega \underline{X} K_{Ev};\\ &-m_a\omega^2 \underline{X} - m_a\omega^2 \underline{X}_b = K_{Fv}\underline{I}_v - k_v\underline{X} - j\omega b_v \underline{X};\\ &-m_b\omega^2 \underline{X}_b = -K_{Fv}\underline{I}_v + k_v\underline{X} + j\omega b_v\underline{X} - k_b\underline{X}_b - j\omega b_b\underline{X}_b, \end{aligned}\right\}, \quad (4)$$

where $\underline{X}_b$ is the displacement of the operating element; m_b is the mass of the operating element; k_v, k_b are the stiffness factors of the motor and operating element, respectively; b_b is the load viscous friction factor.

The results of the solution of equations (4) with respect to the oscillation amplitudes are as follows:

– the mover oscillation amplitude relative to the stator:

$$X_m = \frac{K_{Fv}I_{vm}\sqrt{\left(C_1\left(-m_b\omega^2 + k_b - m_a\omega^2\right) + C_2\omega b_b\right)^2 + \left(C_2\left(m_b\omega^2 - k_b + m_a\omega^2\right) + C_1\omega b_b\right)^2}}{C_1^2 + C_2^2}; \quad (5)$$

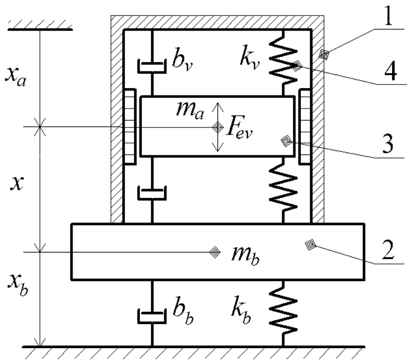


Fig. 3

– the oscillation amplitude of the operating element:

$$X_{bm} = \frac{K_{Fv}I_{vm}m_a\omega^2}{\sqrt{C_1^2 + C_2^2}}, \quad (6)$$

where $C_1 = m_a m_b\omega^4 - m_b\omega^2 k_v - m_a\omega^2 k_b + k_b k_v - \omega^2 b_b b_v - m_a\omega^2 k_v$;

$C_2 = -\omega^3 b_v m_a - b_v m_b\omega^3 + \omega b_v k_b - b_b m_a\omega^3 + \omega b_b k_v$.

As follows from (5, 6), the oscillation amplitudes of the mover and the operating element are maximum when the denominators of the expressions have minimal values, and both masses resonate at the same frequencies.

Electrical resonance. In the equivalent circuit according to Fig. 2, *b* the back EMF induced due to the mover motion can be represented, according to the electromechanical analogy approach, the corresponding voltage drop across the equivalent impedance. Then the first equation of system (1) will take the form:

$$\underline{U}_v = \underline{I}_v\left(R_{sv} + j\omega L_v + \frac{j\omega K_{Ev}K_{Fv}}{-m_a\omega^2 + k + j\omega b}\right), \quad (7)$$

where the mechanical component is defined by the expression $\underline{Z}_{mec} = \dfrac{j\omega K_{Ev}K_{Fv}}{-m_a\omega^2 + k + j\omega b}$.

Analysis of equation (7) shows the presence of two electrical resonance frequencies (when the phases of supply voltage and motor current coincide) in the single-mass system which are defined by the expression:

$$\omega_{1,2} = \sqrt{\frac{K_{Ev}K_{Fv}m_a - L_v b^2 + 2L_v k m_a \pm \sqrt{(-K_{Ev}K_{Fv}m_a + L_v b^2 - 2L_v k m_a)^2 + 4L_v m_a^2(-K_{Ev}K_{Fv}k - L_v k^2)}}{2L_v m_a^2}}. \quad (8)$$

Moreover, the transition between the inductive and capacitive mechanical reactance occurs at the frequency of phase mechanical resonance (3). Since to achieve electrical resonance, the mechanical reactance must be capacitive (to compensate for the inductive component of the winding electrical reactance $X_e = \omega L_v$), both frequencies of electrical resonance will have values greater than ω_0.

Similar considerations are valid for the two-mass electromechanical system, the equivalent mechanical impedance of which will be determined from the system of equations (4), from which:

$$\underline{Z}_{mec} = \frac{K_{Fv}K_{Ev}\left(C_2C_3 - \omega^2 b_b C_1\right) + jK_{Fv}K_{Ev}\left(C_1C_3 + \omega^2 b_b C_2\right)}{C_1^2 + C_2^2}, \quad (9)$$

where $C_3 = -m_b\omega^3 + \omega k_b - m_a\omega^3$.

Analysis of expression (9) shows that the dependence of mechanical reactance on frequency has two areas with are inductive reactance and two areas with capacitive one. It follows that, depending on the mechanical and electrical impedances, electrical resonances at four different frequencies can be observed in such a system. On Fig. 4 is shown the dependences of electrical and mechanical reactance on frequency for the case when the electromechanical system has four resonant frequencies (at points *1*, *2*, *3* and *4*). Here R_{mec}, X_{mec} are the mechanical resistance and reactance, respectively. Obviously, the number of resonances will be determined by the ratio of electrical and mechanical parameters of the motor. In the case of low velocity of moving masses there may be no electrical resonances at all.

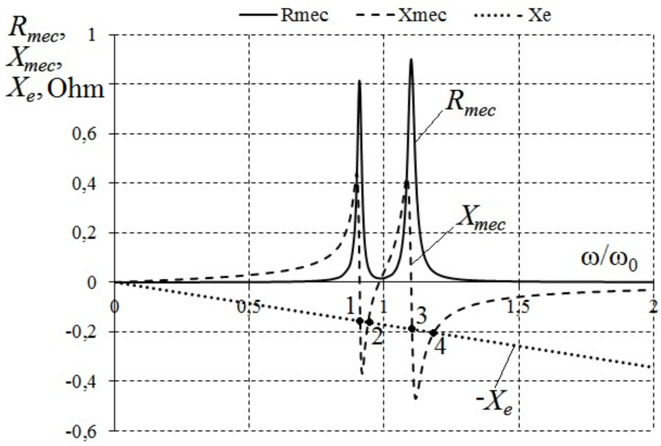


Fig. 4

Energy resonance. Mechanical work which is performed by electromagnetic force during the mover motion from the position $-X_m$ to X_m can be expressed as:

$$W_{mec} = \int_{-X_m}^{X_m} F_{ev} dx = \frac{\Psi_m I_{vm} \pi^2 X_m \sin\theta}{2\tau}, \quad (10)$$

where Ψ_m, I_{vm} are the amplitudes of flux linkage and winding current, respectively; τ is the pole pitch of the machine; θ is the phase difference between displacement and current. And here, as before, the harmonic electromagnetic force with amplitude $F_{em} = K_{Fv} I_{vm} = {I_{vm}\Psi_m \pi}/{\tau}$ is considered.

From expression (10) it follows that for constant equivalent circuit parameters and frequency, the mechanical work is proportional to the oscillation amplitude. In the single-mass electromechanical system, the work will be maximum one at the natural frequency defined by expression (2).

According to the equivalent circuit shown in Fig. 2, *b*, the motor mechanical power can be represented by the following expression:

$$P_{mec} = \frac{1}{T}\int_{t-T}^{t} F_{ev} v dt = F_{ev} V \cos\theta^* = \frac{K_{Fv}}{K_{Ev}} I_v^2 R_{mec},$$

where T is the periodic time; F_{ev}, V are the rms values of electromagnetic force and mover speed, respectively; θ^* is the phase difference between electromagnetic force and mover speed.

Then the expression for the efficiency of the single-mass electromechanical system will be written in the form:

$$\eta = \frac{P_l}{P_v} = \frac{K_{Fv}^2 \omega^2 (b - b_v)}{K_{Fv}^2 b \omega^2 + R_{sv}\left((k - m_a\omega^2)^2 + b^2\omega^2\right)}, \quad (11)$$

where $P_l = P_{mec} - P_{mec.f}$ is the output mechanical power; $P_{mec.f}$ is the power of mechanical losses (losses on viscous friction); $P_v = P_{mec} + P_R$ is the input power; P_R is the power of electric losses.

Analysis of expression (11) shows that the maximum efficiency corresponds to the frequency of phase mechanical resonance (3).

A similar expression for efficiency can be obtained for the two-mass electromechanical system. From the equations (4) we can write:

$$\eta = \frac{P_l}{P_v} = \frac{K_{Fv}^2\left(C_2C_3 - \omega^2 b_b C_1\right)\left(C_1^2 + C_2^2\right) - K_{Fv}^2 b_v \omega^2\left(\left(C_1C_4 + C_2\omega b_b\right)^2 + \left(-C_2C_4 + C_1\omega b_b\right)^2\right)}{\left(K_{Fv}^2\left(C_2C_3 - \omega^2 b_b C_1\right) + R_{sv}\left(C_1^2 + C_2^2\right)\right)\left(C_1^2 + C_2^2\right)}. \quad (12)$$

The cumbersomeness of the obtained expression does not make it possible to determine analytically the resonant frequencies of efficiency, but it is obvious that they are not directly related to either the phase or amplitude resonances of the corresponding masses.

Force resonance. The salient-pole design of the mover (see Fig. 1) determines the difference between the permeance along the longitudinal and transverse axes and the emergence, in addition to synchronous, also reluctance electromagnetic force. That is, the total electromagnetic force will have two components:

$$F_{ev} = F_{es} + F_{er} = \frac{d\Psi_{pm}}{dx} i_v + \frac{1}{2}\frac{dL_v}{dx} i_v^2,$$

where F_{es} is the electromagnetic force due permanent magnets field (synchronous component); F_{er} is the reluctance electromagnetic force that does not depend on the permanent magnets field and is due to the different permeance of the salient-pole mover along the longitudinal and transverse axes; Ψ_{pm} is the flux linkage due to the field of permanent magnets.

Typical static force-angle characteristics, depending on the mover angular position $\theta_a = \pi x / \tau$ and the linear displacement x, are shown in Fig. 5, *a* (similarly with rotating machines, θ_a is the angle between the axes of the mover poles and stator windings).

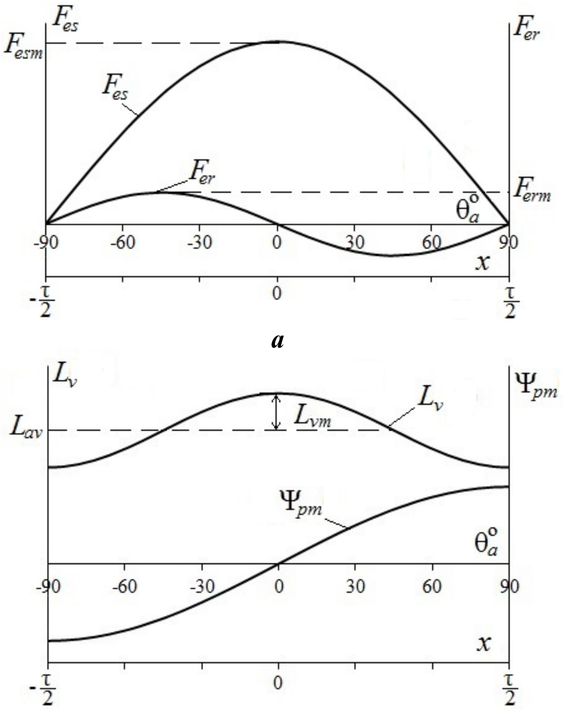
Dependences of the flux linkage and the inductance on the mover position can be approximated by the expressions:

$$\Psi_{pm} = \Psi_m \sin \pi x / \tau ; \quad L_v = L_{av} + L_{vm} \cos 2\pi x / \tau ,$$

where Ψ_m is the flux linkage amplitude; L_{av}, L_{vm} are the mean and amplitude values of the stator winding inductance, respectively (Fig. 5, *b*).

The symmetry of the synchronous electromagnetic force curve F_{es} with respect to the abscissa and ordinate axes makes it possible to represent this dependence on the displacement as a polynomial of the second order:

$$F_{es} = \left(\frac{\Psi_m \pi}{\tau} - \frac{4\Psi_m \pi}{\tau^3} x^2 \right) i_v . \tag{13}$$



b

Fig. 5

Expression (13) takes into account that in the position relative to which the mover oscillates (x = 0), the flux linkage due to the field of magnets is zero (Fig. 5, *b*) and the synchronous component has the maximum value $F_{es} = \Psi_m \pi / \tau \, i_v$. At the positions $x = \pm\tau/2$ the force is zero F_{es} = 0 (see Fig. 5, *a*). Hence, we obtain the corresponding coefficients of the polynomial (13).

The mechanical work of the synchronous electromagnetic force in the range of the mover motion from the position $-X_m$ to X_m is equal to

$$W_{mecs} = \int_{-X_m}^{X_m} F_{es} dx = \frac{\Psi_m I_{vm} \pi^2 X_m \sin\theta \left(\tau^2 - X_m^2\right)}{2\tau^3} . \tag{14}$$

From expression (14) it follows that the work of the synchronous component is always positive, because the phase difference between displacement and current θ in the operating mode is close to 90º and the mover amplitude X_m is always less than the pole pitch τ.

The dependence of the reluctance electromagnetic force on the displacement can be represented by a polynomial of the third order in form:

$$F_{er} = \left(\frac{64\pi L_{vm}}{3\tau^4} x^3 - \frac{16\pi L_{vm}}{3\tau^2} x \right) i_v^2 . \tag{15}$$

Expression (15) takes into account that F_{er} =0, when $x = \pm\tau/2$ (Fig. 5, *a*), and $F_{er} = \pm \frac{\pi L_{vm}}{\tau} i_v^2$ when $x = \pm\tau/4$. Hence, we obtain the corresponding coefficients of the polynomial (15).

Mechanical work of the reluctance component:

$$W_{mecr} = \int_{-X_m}^{X_m} F_{er} dx = \frac{2\pi^2 L_{vm} I_{vm}^2 X_m^2 \sin 2\theta \left(2X_m^2 - \tau^2\right)}{3\tau^4} . \tag{16}$$

Since the expression in parentheses $\left(2X_m^2 - \tau^2\right)$ in the operating mode is always negative, the work will have a positive value if θ > π/2, i.e. when $\omega > \omega_0$, and a negative value if θ < π/2. Therefore, the reluctance electromagnetic force in the oscillating mode performs useful work at frequencies higher than the natural frequency ω_0, and at θ = π/2 its value is zero.

The phase difference at which the work performed by both electromagnetic force components (synchronous and reluctance) is maximum will be determined from the equation:

$$\partial\left(W_{mecs} + W_{macr}\right)\Big/\partial\theta = 0\,,$$

whence

$$\theta = \arccos\left(\frac{-3\Psi_m\tau\left(\tau^2 - X_m^2\right) + \sqrt{9\Psi_m^2\tau^2\left(\tau^2 - X_m^2\right)^2 + 2\cdot 16^2 L_{vm}^2 I_{vm}^2 X_m^2\left(2X_m^2 - \tau^2\right)^2}}{32 L_{vm} I_{vm} X_m\left(2X_m^2 - \tau^2\right)}\right). \qquad (17)$$

Analysis of expression (17) shows that within the operating amplitudes of oscillations ($X_m < \tau/2$) the optimal phase difference increases with increasing winding current I_{vm} and amplitude of inductance L_{vm}. That is, with an increasing reluctance electromagnetic force and oscillation amplitude.

The above expressions and dependencies are obtained on the basis of equivalent circuits with lumped constant parameters. Obviously, a number of nonlinear effects associated with fringing fluxes, saturation, electromagnetic force harmonics, etc., can significantly affect the electromechanical behavior of the system. Therefore, the results of the above analysis of resonant modes, which are quite useful for preliminary modeling, must be verified using more accurate models. As the latter, the corresponding field models are usually used, which provide high accuracy of the calculation with the correct formulation of the problem.

Field research. To solve the field problem we will use the results of the study presented in [11], where the design parameters of the motor are obtained in accordance with the specified optimization criteria (costs, force-volume ratio, efficiency) and the mechanical load parameters. The basic parameters of the machine are given in Table.

Here D_{pm}, D_0, τ_{pm} are the outer and inner diameters, as well as the width of the permanent magnet ($SmCo_5$ with a remanent flux density 0.9 T), respectively; $2p$ is the number of mover poles; n_{sp} is the number of stator slots per pole; τ is the pole pitch; h_t is the height of the stator tooth; D_e is the outer diameter of the stator; τ_t, τ_s are the width of the tooth and slot of the stator, respectively. The table also shows the values of efficiency *Eff*, power factor PF and input power P_v, obtained from the results of the preliminary simulation.

Table

D_{pm}-D_0-τ_{pm}, mm	$2p/n_{sp}$	τ, mm	h_t, mm	D_e, mm	τ_f, mm	τ_t, mm	τ_s, mm	*Eff*	PF	P_v, W
50-22-14	5/4	28.2	13.5	91.9	17	3.55	3.5	0.7163	0.9052	1094

Due to the axial symmetry of the machine (see Fig. 1), the problem was solved in axisymmetric formulation, based on the equations of the quasi-stationary magnetic field in the time domain with Comsol Multiphysics simulation software [12]. The mechanical calculation was performed in conjunction with the field calculation, as the problems are interrelated. For this purpose, a moving type of calculation mesh in the mover region was used, the displacement of which is determined by the result of solving the force balance equation. The corresponding system of equations has the form:

$$\left.\begin{aligned} u_v &= i_v R_{sv} + \sum_{n=1}^{n_s}\frac{w_s}{S}\int_{Sn}\frac{\partial 2\pi r A_\varphi}{\partial t}dS_n; \\ m_a\frac{d^2x}{dt^2} &= F_{ev} - kx - b\frac{dx}{dt}, \end{aligned}\right\},$$

where n_s is the number of stator slots; w_s is the number turns per slot; A_φ is the vector magnetic potential (φ component); S is the cross section of the stator slot.

The value of the electromagnetic force was determined by integrating the Maxwell stress tensor $\mathbf{T}$ on the mover surface according to the expression:

$$\mathbf{F}_{ev} = \int_{s_a} 2\pi r\mathbf{n}\mathbf{T}ds_a\,,$$

where $\mathbf{n}$ is the unit vector of the external normal to the mover surface s_a; r is the distance from the axis of symmetry to the integration surface. The axial component of the force was calculated.

The results of the magnetic field calculation are shown in Fig. 6, *a*. The calculations refer to the case of the rms value winding current 2.9 A with a frequency f = 79.6 Hz. The mover position x = 0. The parameters of the mechanical equivalent circuit are as follows: k = 1187511 N/m; b = 110 kg/s; m_a = 4.7 kg.

The motor electromechanical characteristics depending on the relative frequency ω/ω_0 are shown in Fig. 6, *b*. The per unit system and degree angular measure are used to build the characteristics.

The following values are accepted as base: base frequency $\omega_b = \omega_0 = \sqrt{k/m_a}$; oscillation amplitude $X_b = \tau/2$; current $I_b = I_{v.nom}$; electromagnetic force $F_b = K_{Fv} I_b$; base speed $V_b = \omega_b X_b$; power $P_b = 0.5 F_b V_b$.

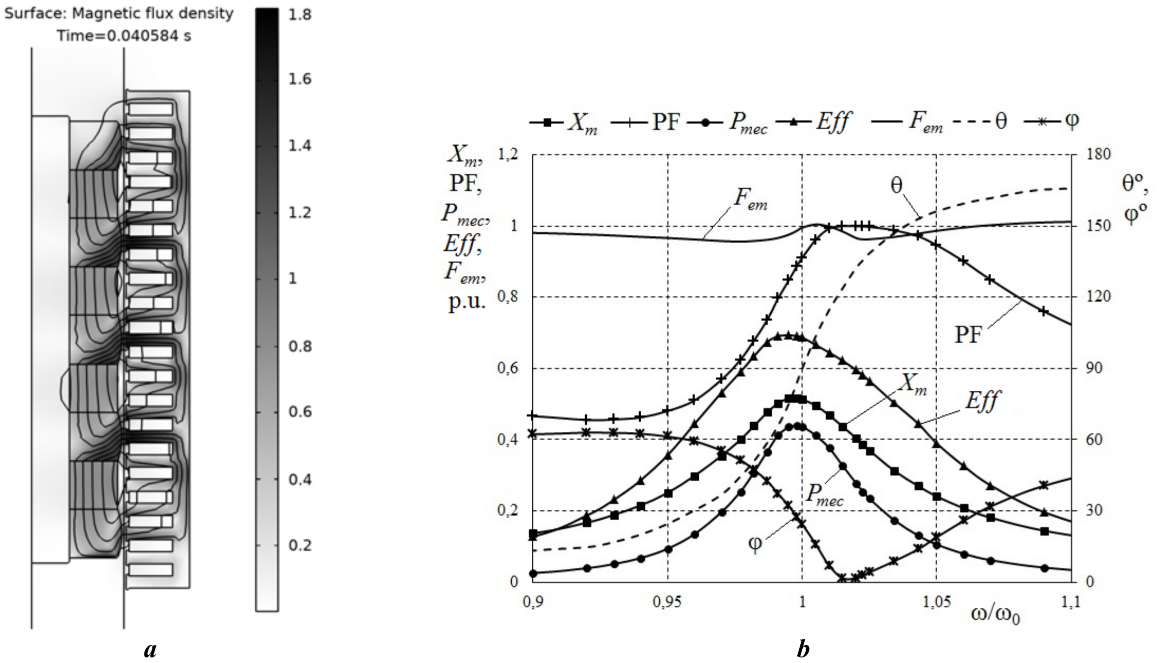


Fig. 6

As follows from the data obtained, the most effective, from the point of view of the implementation of resonant operating modes, is the frequency range within ω/ω_0 = 0.99-1.01. The frequency of mechanical resonance, at which the oscillation amplitude X_m is maximum, is about ω/ω_0 = 0.998. This value is slightly less than that defined by expression (2) which is determined by the presence of electromagnetic damping and the corresponding losses and, as a result, an increase in the factor *b*.

The highest value of the power factor PF is close to 1 and corresponds to the phase shift φ = 1.6º. The electrical resonance, for the given motor parameters, is not observed, but this mode can be achieved by increasing the current and amplitude of oscillation.

The maximum output mechanical power of the motor is P_{mec} = 732.6 W at the frequency ω/ω_0 = 0.998, corresponding to mechanical resonance. Quite close in frequency (ω/ω_0 = 0.995) is the maximum efficiency, which is equal to 0.693.

It can be noted that the amplitude of electromagnetic force F_{em} increases with the approach of the phase difference θ (angle between the phases of current and displacement) to 90º. In this mode, the maximum current coincides with the moment when the mover passes the position $x = 0$, and according to expression (13), the synchronous electromagnetic force is maximum. But the maximum of the total electromagnetic force corresponds to the angle θ = 103.6º, which is explained by the influence of the reluctance component.

In general, the electromechanical system has a predictable behavior and the existent resonances are consistent with the analysis given in the paper. Obviously, the simultaneous ensuring of optimal electromechanical characteristics is impossible. If we consider the maximum efficiency and amplitude of oscillations as the optimal mode, then in this case we should expect a decrease in the power factor and electromagnetic force, the maxima of which are at frequencies > ω_0. It is also necessary to take into account the presence of nonlinear effects inherent in linear permanent magnet vibratory motors, which requires the use of rather accurate, in particular field models during their design.

Conclusions. As a result of the conducted analytical research the electromechanical resonant frequencies of the linear permanent magnet vibratory motor were determined. In the single-mass electromechanical system, there are two frequencies of electrical resonance, when the phases of supply voltage and motor current coincide. An increase in the number of moving masses leads to a corresponding increase in the number of resonant frequencies that could theoretically occur in such systems. The presence of electrical resonances directly depends on the back EMF, and in practice they cannot always be implemented at all possible frequencies, as this requires an increase in the magnetic field and the speed of the mover. In addition, the maximum efficiency and oscillation amplitude correspond to frequencies < ω_0, when it is impossible to achieve electrical resonances without artificial compensation. The amplitude value of the electromagnetic force is maximum when $\omega > \omega_0$, due to the influence of the reluctance component, and does not correspond to the maximum values of efficiency and output mechanical power of the motor.

1. Gieras J.F., Piech Z.J., Tomczuk B. Linear synchronous motors. Transportation and automation systems. CRC Press, 2012. 520 p.
2. Won-jong K., Murphy B.C. Development of a novel direct-drive tubular linear brushless permanent-magnet motor. 38th IAS Annual Meeting on Conference *Record of the Industry Applications Conference*. 2003. Vol. 3. Pp. 1664-1671. DOI: https://doi.org/10.1109/IAS.2003.1257779.
3. Lu H., Zhu J., Guo Y. Development of a slotless tubular linear interior permanent magnet micromotor for robotic applications. *IEEE Transactions on Magnetics*. 2005. Vol. 41. No 10. Pp. 3988-3990. DOI: https://doi.org/10.1109/TMAG.2005.855158.
4. Akhondi H., Milimonfared J. Design and optimization of tubular permanent magnet linear motor for electric power steering system. *Journal of Asian Electric Vehicles*. 2009. Vol. 7. No 2. Pp. 1283-1289. DOI: https://doi.org/10.4130/jaev.7.1283.
5. Cherno O.O., Monchenko M.Yu. Energy efficiency of the vibratory device electromagnetic drive system. *Tekhnichna Elektrodynamika*. 2016. No 1. Pp. 20-25. DOI: https://doi.org/10.15407/techned2016.01.020.
6. Bondar R.P., Podoltsev A.D. Complex model with frequency dependent parameters for electrodynamic shaker characteristics. *Tekhnichna Elektrodynamika*. 2017. No 1. Pp. 44-51. (Ukr). DOI: https://doi.org/10.15407/techned2017.01.044 .
7. Tsutomu M., Takuya Y., Masaki T., Makoto U., Hideo Y., Kouyou S., Hajime Y. A novel efficiency measurement of moving-magnet-type linear oscillatory actuator. *International Journal of Applied Electromagnetics and Mechanics*. 2002. Vol. 15. No 1-4. Pp. 163-167.
8. Bondar R.P. Research of the magnetoelectric linear oscillatory motor characteristics during the work on elastoviscous loading. *Electrical engineering & electromechanics*. 2019. No 1. Pp. 9-16. (Ukr). DOI: https://doi.org/10.20998/2074-272X.2019.1.02.
9. Wang J. Performance evaluation of fractional-slot tubular permanent magnet machines with low space harmonics. *Archives of Electrical Engineering*. 2015. Vol. 64. No 4. Pp. 655-668. DOI: https://doi.org/10.1515/aee-2015-0049.
10. Yatchev I., Gueorgiev V., Ivanov R., Hinov K. Simulation of the dynamic behaviour of a permanent magnet linear actuator. *Facta universitatis - series: Electronics and Energetics*. 2010. Vol. 23. No 1. Pp. 37-43. DOI: https://doi.org/10.2298/FUEE1001037Y.
11. Bondar R.P. Optimization approach to determination of constructional parameters of a linear permanent magnet vibratory motor. *Tekhnichna Elektrodynamika*. 2022. No 1. Pp. 33-40. (Ukr). DOI: https://doi.org/10.15407/techned2022.01.033.
12. Comsol Multiphysics. URL: http://www.comsol.com/ (accessed at 06.04.2022).

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РЕЗОНАНСНІ РЕЖИМИ ЛІНІЙНОГО МАГНІТОЕЛЕКТРИЧНОГО ДВИГУНА ВІБРАЦІЙНОЇ ДІЇ

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Розглянуто резонансні режими роботи лінійного магнітоелектричного двигуна вібраційної дії. На підставі електричних та механічних схем заміщення із зосередженими параметрами отримано вирази для визначення частот механічного, електричного, енергетичного та силового резонансів. Представивши згідно методу електромеханічних аналогій індуковану внаслідок руху якоря ЕРС відповідним падінням напруги, отримано вирази для еквівалентних механічних опорів. Обґрунтовано наявність двох частот електричного резонансу (коли збігаються фази напруги живлення та струму двигуна) в одномасовій електромеханічній системі та чотирьох – у двомасовій. Отримано залежності енергетичних характеристик двигуна (механічної роботи та ККД) від параметрів його схеми заміщення. На підставі виразу для реактивної компоненти електромагнітної сили знайдено механічну роботу та показано її залежність від фазового кута коливань. Визначено фазовий кут, за якого сумарна механічна робота двигуна є максимальною. Показано, що результати аналізу резонансних режимів добре узгоджуються з результатами чисельного польового дослідження, проведеного на підставі рівнянь квазістаціонарного магнітного поля в часовій області з використанням методу скінченних елементів та рухомого типу розрахункової сітки в області якоря. Бібл. 12, рис. 6, табл. 1.

Ключові слова: електричний резонанс, енергетичні характеристики, лінійний магнітоелектричний двигун, метод електромеханічних аналогій, метод скінченних елементів.

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