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SPECIFICATION OF DETERMINATION THE INDUCTION MOTOR TORQUE BY THE MAGNETIC CORE MODES IN THE SATURATED AREA

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*The error value of the formula for calculating the electromagnetic moment under the assumption of equality of magnetic energies and coenergies has been investigated under the operation conditions of induction motor (IM) the magnetic core in the saturated region. For serial motors, under nominal conditions, the error does not exceed 5%, with a voltage increase of 75%, the error increases to 40%. The specified value of the electromagnetic moment is determined by calculating the value of the integral dependences of the magnetic coenergy on the angle of the rotor rotation in the process of calculating the dynamic mode of attenuation the currents in the windings. To improve the efficiency of refined calculation method of the electromagnetic moment, the dependence of the correction coefficient in the Magneto-Motive Forse (MMF) function of the motor is proposed for use. The use of this pre-determined dependence significantly speeds up refined calculations. The substantiation of the results reliability of the refined definition of electromagnetic moment without the assumption of the equality of magnetic energies and coenergies is carried out by comparing the results of the nominal operating modes caculation of serial motors with two and six pole*s, *which have a significant difference in the degree of saturation.* References 17, tables 2, figures 2.

Keywords: induction motor, saturated mode, electromagnetic moment, coenergy.

Traditional IM designs have reached a high level of perfection thanks to accumulated design experience and manufacturing technologies using traditional electrotechnical engineering materials. In particular, electrotechnical steels are used in the form of thin sheets in laminated magnetic cores to minimize losses in the steel. At the same time, in the majority of cases, the operating condition corresponds to the near region behind the bend of the magnetization curve of this steel. This allows to reasonably apply a number of simplifying assumptions when building mathematical models of IM research and design. In particular, use a simplified expression to determine the magnitude of the IM electromagnetic moment under the condition of equality the magnetic energy and coenergy [1].

The development of adjustable electric drive systems, the involvement of new electrotechnical materials contribute to the expansion of the traditional limits of IM operating conditions. This is connected with modes of voltage forcing, frequency regulation, dynamic modes of switching processes. In addition, the application of one of the modern trends in the development of technologies for the magnetic cores manufacture of electric motors, powder metallurgy (for obtaining without a stamp technology for the manufacture of magnetic cores, including new complex forms), creates conditions for a significant reduction of losses in steel and a shift in the parameters of the optimal operating conditions of IM magnetic cores to great saturations.

The tasks of increasing the electric drives efficiency with the wide application of saturated operation conditions determine the need for precise consideration of this factor in mathematical modeling for their effective design synthesis. Mathematical modeling of IM in saturated modes is carried out taking into account the nonlinear properties of magnetic cores, both by circular [2] and field methods of analysis [3, 4]. At the same time, a refinded definition of the currents of the operating conditions is provided in the iterative process of field and circular calculations. In the latter case, nonlinear dependences of the IM electromagnetic parameters are used for this purpose. Field methods provide greater accuracy, but they are also more expensive in terms of calculation time. This limits their use in design synthesis tasks. Using the advantages of field methods in terms of accuracy and circular methods in terms of speed is provided by the parameterization of field models [5].

For refined take into account saturation, in addition to improving the definition of currents, there is a need to develop refined mathematical tools of determining the torque based on the magnitude of these currents. For this purpose, the expression of the derivative of magnetic coenergy reserve by displacement is often used [1, 6-11].This expression is obtained from the energy balance equation in the electromechanical system with taking into account the change in the energy reserve in the magnetic field. The energy of the magnetic field can be determined by calculating the processes of attenuation the currents in electric circuits

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with the release of magnetic field energy in the form of electrical losses. The expression of this energy reserve is significantly simplified for a linear medium and represents half the product of the flux linkage by the current. Due to the non-linearity of the magnetic cores characteristics, the use of such a simplification leads to significant errors because in the expression of electromagnetic torques [1], instead of half the product of the flux coupling on the current, the expression of the integral dependence of the flux coupling on the current appears, the deviation of which from half the product of the flux coupling on the current increases with increasing saturation.

The study of the mechanical action of electromechanical converters with the determination of the integral dependence of the flux coupling on the current is quite simple to implement using the calculations of motors with one electrical circuit [10]. Such an analysis for multi-circuit systems, for example IM, is much more complicated. An example of its application is given in [11]. The introduction of such clarifying approaches to the practice of IM design synthesis requires the use of special mathematical models and analysis algorithms. Such models are more difficult to realize in software view, they consume more machine time during calculations. Therefore, it is necessary to determine the areas of their appropriate application, to develop a methodology for the refinded determination of the moment of IM saturated with less losses of machine time.

The aim of this work is development and justify the feasibility of using in mathematical models of IM design and the reliability of the refined determinated means of the electromagnetic moment without the assumption of the equality of magnetic energies and coenergies. These models are assigned for the IM improvement with the operation condition of the magnetic core in the saturated region. The article deals with the refinement of the formula for determining the electromagnetic torque of a saturated IM. The correction factor is determined by comparing two options: 1) Calculation of currents for a non-linear formulation of the problem (electromagnetic parameters change in accordance with the change in the saturation coefficient) and determination of the electromagnetic torque by the formula with the assumption of equality the magnetic energies and coenergies; 2) Calculation of currents for a non-linear formulation of the problem and determination of the electromagnetic torque according to a refined formula without the assumption of equality the magnetic energies and coenergies. The article deals with the refinement of the formula for determining the electromagnetic torque of a saturated IM. The correction factor is determined by comparing two options: 1) Calculation of currents for a non-linear formulation of the problem (electromagnetic parameters change according to the change in the saturation factor) and determination of the electromagnetic torque according to the formula with the assumption of equality of magnetic energies and coenergies; 2) Calculation of currents for a non-linear formulation of the problem and determination of the electromagnetic torque according to the refined formula without the assumption of equality of magnetic energy and co-energy.

The research in this paper was performed using the IM mathematical model electromechanotronic systems (EMTS), which is presented in the MATLAB simulation system [12]. This model provides a study of the IM operating conditions with a short-circuited rotor, taking into account a possible arbitrary connection scheme of the stator winding turns, the spectrum of spatial harmonics of the MMF, nonlinearity and asymmetry of the parameters. The differential equations of electrical equilibrium are solved relative to the instantaneous values of the independent currents of the stator windings and the projections of the resulting spatial complexes of currents of the rotor circuits

$$[i_{s\text{н}}];\quad i_{r\nu}^{R}=\mathrm{Re}\left(\sum_{k=1}^{z_2}\bar{i}_{k\nu}\right);\quad i_{r\nu}^{I}=\mathrm{Im}\left(\sum_{k=1}^{z_2}\bar{i}_{k\nu}\right),\tag{1}$$

where $[i_{s\text{н}}]$ is the matrix of independent currents of the stator winding turns, which is related to the currents matrix of all the windings according to Kirchhoff's first law: $[i_s]=[k_{inv}]\cdot[i_{s\text{н}}]$; $[i_s]=[i_{s1}\ldots i_{si}\ldots i_{sV}]$ is the matrix of currents for all stator winding turns; V is the number of the winding turns; $[k_{inv}]$ is the matrix of transformations according to Kirchhoff's first law; $\bar{i}_{k\nu}=i_{k\nu}\cdot e^{j\nu k\delta_k}\cdot e^{j\nu\Theta}$ is the spatial complex of the k-th contour current of the rotor (formed by adjacent rods and sections of rings) according to harmonics ν; $\delta_k=2\pi/z_2$ is the angle between the axes of the rotor teeth in the coordinates of the first harmonic; z_2 – the number of rotor teeth; Θ is the angle between the rotor tooth axis with the number z_2 and the real axis of the complex plane in the coordinates of the first harmonic.

The methodology for obtaining the equations of electrical balance of the IM mathematical model EMTS [12] involves the following steps: formulating the equations in phase coordinates, relative to the instantaneous currents of electrical circuits; definition of the system of new rotor (1) and stator variables (independent currents of the stator windings according to Kirchhoff's first law); conversion of equations to new variables taking into account the unified coordinate system of spatial vectors, the relationship between the currents of the circuits according to Kirchhoff's first law and the connection between the voltages of the circuits and the known voltages of the IM power supply system - according to the second law.

The equation of the electromagnetic moment is defined as the partial derivative of the magnetic coenergy reserve by displacement [1, 11]. The equation of electrical equilibrium of the stator winding and in phase coordinates [12] consider to obtain the expression of the electromagnetic moment

$$u_{si} = r_{si} i_{si} + \frac{d}{dt} \sum_{q=1}^{V} \left(m_{iq} + \sum_{\nu=\nu_1}^{\nu_N} M_{iq\nu} \cos\left(\delta_{i\nu} - \delta_{q\nu}\right) \right) i_{sq} + \frac{d}{dt} \sum_{\nu=\nu_1}^{\nu_N} \sum_{k=1}^{z_2} M_{ik\nu} \cos\left(\nu\Theta - \delta_{i\nu} + k\nu\delta_k\right) i_{k\nu}, \qquad (2)$$

where u_{si}, i_{si}, r_{si} are the instantaneous values of voltage, current and active resistance of the i stator phase; $M_{iq\nu}$, $M_{ik\nu}$, $M_{ki\nu}$ are the maximum mutual inductances according to the main harmonic ν field between the *i* and *q* stator phases, the *i* stator phase and the rotor circuit, the rotor circuit and *i* phase of the stator, respectively, provided their axes are aligned; m_{iq} is a mutual inductance between the *i* and *q* stator phases along the paths of the dissipation flow; $\delta_{i\nu}, \delta_{q\nu}$ are the angular position of the axes of the *i* and *q* stator phases (the position of the maximum MMF of the phase according to the harmonic ν) in the coordinates of the order harmonic ν ; $r_c, r_{\text{ук}}$ are the an active resistance of the rotor rod and the section of the short-circuit ring between the neighboring rods; $M_{kk\nu}, m_{\text{п}}, m_{\text{л}}$ are the a self-inductance of the rotor circuit: according to the main harmonic field ν , according to the paths of the groove and ring dispersion flow.

Let's multiply the left and right parts of (2) by $i_{si}dt$. The value $u_{si}i_{si}dt$ is an energy received by the branch phase of the stator from the power source. The value $r_{si}i_{si}^2dt$ is losses in active resistances.Taking this into account and taking into account that the self-inductance and currents (independent variables) are not functions of the rotor rotation angle Θ and that according to the law of conservation the energy, in addition to active losses, the energy of the source is spent on mechanical work $M_e d\Theta$ and changing the energy reserve of the magnetic field dW_m, we determine the share of the electromagnetic torque M_{esi}, which is caused by energy processes in the stator *i* branch $\left(M_e = \sum_{i=1}^{V} M_{esi}\right.$, $\left.dW_m = \sum_{i=1}^{V} dW_{msi}\right)$

$$\left(M_e = \sum_{i=1}^{V} M_{esi},\ dW_m = \sum_{i=1}^{V} dW_{msi}\right);$$

$$M_{esi} = \frac{d}{d\Theta} \left[i_{si} \sum_{\nu=\nu_1}^{\nu_N} \sum_{k=1}^{z_2} M_{ik\nu} \cos\left(\nu\Theta - \delta_{i\nu} + k\nu\delta_k\right) i_{k\nu} \right] - \frac{dW_{msi}}{d\Theta}, \qquad (3)$$

where M_{esi}, W_{msi} are the particles of the electromagnetic torque and the energy of the magnetic field, which are determined by the stator *i* phase ratios.

To calculate the value (3), it is necessary to obtain an expression for determining the corresponding share of the energy reserve of the magnetic field. The algorithm for obtaining the expression of the magnetic field energy [11] provides drawing up the equations of the electrical balance of all motor circuits for the analysis of the currents attenuation process in case of short-circuited input terminals of phases and dissipation of energy in active resistances. The value $dW_{msi}/d\Theta = dW_{msi.s}/d\Theta + dW_{msi.r}/d\Theta$ is represented by two components, which are due to the processes in the stator and rotor, respectively. In order to obtain the expression $dW_{msi.s}/d\Theta$, at zero voltage we multiply the left and right parts of equation (2) by $i_{si}dt$ and take the integrals from them for the time of complete attenuation of the currents. The energy of the magnetic field is equal to the losses in active resistances. After that, taking into account that changing the limits of integration changes the sign of the integral, the desired expression can be obtained. Further transformations will be performed in two variants: with constant parameters, when $\int i_s d(Li_r) = i_s L i_r / 2$ (magnetic energy and coenergy are equal) and with variable parameters, when the energy expression is written using integral dependencies.

With constant parameters, the expression $dW_{msi.s}/d\Theta$ is as follows:

$$\frac{dW_{msi.s}}{d\Theta}=\frac{d}{d\Theta}\left[i_{si}\sum_{\nu=\nu_1}^{\nu_N}\sum_{k=1}^{z_2}M_{ik\nu}\cos\left(\nu\Theta-\delta_{i\nu}+k\nu\delta_k\right)i_{k\nu}\right]/2\,. \tag{4}$$

Let's transform (3) taking into account (4) and ratios (1), take the derivative by the angle of rotor rotation and get

$$M_{esi}-\frac{dW_{msi.r}}{d\Theta}=\frac{d}{d\Theta}\left[\frac{i_{si}}{2}\sum_{\nu=\nu_1}^{\nu_N}\sum_{k=1}^{z_2}M_{ik\nu}\cos\left(\nu\Theta-\delta_{i\nu}+k\nu\delta_k\right)i_{k\nu}\right]=\frac{i_{si}}{2}\sum_{\nu=\nu_1}^{\nu_N}\nu M_{ik\nu}\left(i_{r\nu}^{R}\sin\delta_{i\nu}-i_{r\nu}^{I}\cos\delta_{i\nu}\right). \tag{5}$$

During transformations, it is taken into account that: $e^{-j\delta_{i\nu}}+e^{j\delta_{i\nu}}=2\cos\delta_{i\nu}$; $je^{-j\delta_{i\nu}}-je^{j\delta_{i\nu}}=2\sin\delta_{i\nu}$;

$$\cos\left(\nu\Theta-\delta_{i\nu}+k\nu\delta_k\right)=\left(e^{j\nu k\delta_k}\cdot e^{-j\delta_{i\nu}}\cdot e^{j\nu\Theta}+e^{-j\nu k\delta_k}\cdot e^{j\delta_{i\nu}}\cdot e^{-j\nu\Theta}\right)/2\,;$$

$$\sum_{k=1}^{z_2}M_{ik\nu}\cos\left(\nu\Theta-\delta_{i\nu}+k\nu\delta_k\right)i_{k\nu}=\sum_{k=1}^{z_2}M_{ik\nu}\left(\bar{i}_{k\nu}\cdot e^{-j\delta_{i\nu}}+\overset{*}{\bar{i}}_{k\nu}\cdot e^{j\delta_{i\nu}}\right)/2=M_{ik\nu}\left(\bar{i}_{r\nu}\cdot e^{-j\delta_{i\nu}}+\overset{*}{\bar{i}}_{r\nu}\cdot e^{j\delta_{i\nu}}\right)/2\,.$$

To determine $dW_{msi.r}/d\Theta$ we will perform similar transformations. At the same time, consider the equation of electrical balance for circuits z_2 of the rotor according to each harmonic of the stator MMF [12] (assuming that each harmonic of the stator induces its own system of currents in the rotor). The equation for the *k* rotor circuit

$$\begin{gathered}0=\frac{d}{dt}\left[2\left(m_n+m_л\right)i_{k\nu}-m_n\left(i_{(k-1)\nu}+i_{(k+1)\nu}\right)+\sum_{n=1}^{z_2}M_{kk\nu}\cos\left[(n-k)\nu\delta_k\right]i_{n\nu}\right]+\\+2\left(r_c+r_{ук}\right)i_{k\nu}-r_c\left(i_{(k-1)\nu}+i_{(k+1)\nu}\right)+\frac{d}{dt}\sum_{i=1}^{V}M_{ki\nu}\cos\left(\nu\Theta+k\nu\delta_k-\delta_{i\nu}\right)i_{si}.\end{gathered} \tag{6}$$

Similarly to the previous case, we transform (6), as well as the equations of other rotor contours, sum up their losses when the currents decrease, and taking into account all the branch of the stator winding, we get, taking into account that $M_{ik\nu}=M_{ki\nu}$, the full expression of the electromagnetic torque for an random number of MMF harmonics and for an random structure of the stator winding

$$M_e=\sum_{i=1}^{V}i_{si}\sum_{\nu=\nu_1}^{\nu_N}\nu M_{ik\nu}\left(i_{r\nu}^{R}\sin\delta_{i\nu}-i_{r\nu}^{I}\cos\delta_{i\nu}\right). \tag{7}$$

In the case of motor symmetry, the torque expression (7) is equivalent to the traditional expression [12]. It was obtained with the assumption of equality for magnetic energies and coenergies. If the above transformations are performed without this assumption, with the involvement of integral dependencies, then the expression will be as follows:

$$\begin{aligned}M_{ek}&=\sum_{i=1}^{V}\sum_{\nu=\nu_1}^{\nu_N}\left[\int_0^{i_{si}}\nu M_{ik\nu}\left(i_{r\nu}^{R}\sin\delta_{i\nu}-i_{r\nu}^{I}\cos\delta_{i\nu}\right)di_{si}+\int_0^{i_{r\nu}^{R}}i_{si}\nu M_{ik\nu}\sin\delta_{i\nu}di_{r\nu}^{R}-\int_0^{i_{r\nu}^{I}}i_{si}\nu M_{ik\nu}\cos\delta_{i\nu}di_{r\nu}^{I}\right]=\\&=\sum_{i=1}^{V}\sum_{\nu=\nu_1}^{\nu_N}\left[\int_0^{t_z}\nu M_{ik\nu}\left(i_{r\nu}^{R}\sin\delta_{i\nu}-i_{r\nu}^{I}\cos\delta_{i\nu}\right)\frac{di_{si}}{dt}dt+\int_0^{t_z}i_{si}\nu M_{ik\nu}\sin\delta_{i\nu}\frac{di_{r\nu}^{R}}{dt}dt-\int_0^{t_z}i_{si}\nu M_{ik\nu}\cos\delta_{i\nu}\frac{di_{r\nu}^{I}}{dt}dt\right].\end{aligned} \tag{8}$$

Comparison (7) and (8) makes it possible to investigate the amount of refinement by directly determining the integrals in the expression of the electromagnetic torque without the assumption of equality for magnetic energies and coenergies

$$k_M=M_{ek}/M_e\,. \tag{9}$$

The value of the correction factor (9) is equal to one in the unsaturated mode. With increasing saturation, the value increases. Information about the dependence of its change will allow to determine the areas of expedient application of this clarification. In turn, a refined analysis of operating conditions taking into account the torque correction factor k_M (9) requires, at each calculation point, the study of the transient process of current attenuation to determine the integrals (8).

The algorithm for calculating the operating condition with the determination of the torque specified value M_{ek} can be significantly improved if the dependence of the change k_M as the function of total MMF F_m. The invariance of this dependence was established from the analysis of different operation conditions for the investigated IM. Therefore, having formed it based on the study of one mode, it can be used to determine the adjusted moment for different operation conditions according to the transformed expression (9)

$$M_{ek} = k_M(F_m)M_e . \tag{10}$$

Mathematical model for determining the dependence of the torque correction factor k_M as the function of total MMF F_m, or in the function of the saturation coefficient k_μ is based on the mathematical model of the dynamic operating conditions of IM EMTS [12]. This model is intended for research and design of IM, allows to study the transient processes of currents attenuation and to determine the integrals in this process (8). The initial conditions of the calculation are the instantaneous values of the currents at the given calculation point, zero speed. For this calculation, the differential equations of electrical balance of the IM EMTS model [12], which were obtained by transforming (2), (6) into variables (1), were used

$$u_{si} = r_{si}i_{si} + \frac{d}{dt}\sum_{q=1}^{V}\left(m_{iq} + \sum_{\nu=\nu_1}^{\nu_N} M_{iq\nu}\cos\left(\delta_{i\nu} - \delta_{q\nu}\right)\right)i_{sq} + \frac{d}{dt}\sum_{\nu=\nu_1}^{\nu_N} M_{ik\nu}\left(i_{r\nu}^{R}\cos\delta_{i\nu} + i_{r\nu}^{I}\sin\delta_{i\nu}\right); \tag{11}$$

$$0 = r_{r\nu}i_{r\nu}^{R} + \frac{d}{dt}\left[L_{r\nu}i_{r\nu}^{R} + \frac{z_2}{2}\sum_{i=1}^{V} M_{ki\nu}\cos\delta_{i\nu}i_{si}\right] + \nu\omega_r\left[L_{r\nu}i_{r\nu}^{I} + \frac{z_2}{2}\sum_{i=1}^{V} M_{ki\nu}\sin\delta_{i\nu}i_{si}\right]; \tag{12}$$

$$0 = r_{r\nu}i_{r\nu}^{I} + \frac{d}{dt}\left[L_{r\nu}i_{r\nu}^{I} + \frac{z_2}{2}\sum_{i=1}^{V} M_{ki\nu}\sin\delta_{i\nu}i_{si}\right] - \nu\omega_r\left[L_{r\nu}i_{r\nu}^{R} + \frac{z_2}{2}\sum_{i=1}^{V} M_{ki\nu}\cos\delta_{i\nu}i_{si}\right], \tag{13}$$

where $L_{r\nu} = l_{r\nu} + \frac{z_2}{2}M_{kk\nu}$; $r_{r\nu} = 2r_{ук} + 2r_c\left(1 - \cos\nu\delta_k\right)$; $l_{r\nu} = 2m_л + 2m_n\left(1 - \cos\nu\delta_k\right)$; $\omega_r = \frac{d}{dt}\Theta$ is the rotor rotation frequency.

Taking into account the nonlinearity of the electromagnetic parameters, the solution of the system (11) – (13) requires the determination of the derivative of their change by time. The study was carried out taking into account the change of inductive parameters according to the main field as a function of the saturation coefficient k_μ, [12]. This coefficient changes according to the change in the total MMF of the motor

$$F_{m\nu} = \left|\sum_{i=1}^{V}\overline{f}_{вi\nu}i_{si} + f_{r\nu}\left(i_{r\nu}^{R} + ji_{r\nu}^{I}\right)\right|, \tag{14}$$

where $\overline{f}_{вi\nu}$ are the spatial complexes of single MMF branches of the stator winding; $f_{r\nu}$ are rotor contours [12].

The time derivative from the change of inductive parameters matrix with respect to the main field has the following expression:

$$\frac{d}{dt}\left(\left[M\ \right]\right) = \sum_{\nu=\nu_1}^{\nu_N}\left(\frac{\partial}{\partial k_{\mu\nu}}\left(\left[M_\nu\right]\right)\frac{\partial k_{\mu\nu}}{\partial F_{m\nu}}\frac{dF_{m\nu}}{dt}\right). \tag{15}$$

The solution of the system (11) – (13) taking into account (15) is possible in the presence of dependencies $k_{\mu\nu}\left(F_{m\nu}\right)$, $\frac{\partial k_{\mu\nu}}{\partial F_{m\nu}}\left(F_{m\nu}\right)$, which are previously defined. The value $\frac{dF_{m\nu}}{dt}$ is determined by solving the equations system as a derivative in time from expression (15) by the values of derivative currents.

Numerical experiment on the study of the torque correction factor k_M, (9) is performed on the example of a serial two-pole IM, with a power of 1500 W: 4A80A2U3. The mathematical model of IM EMTS [12] was used, taking into account the change in the value of electromagnetic parameters by the main field when the saturation factor of the magnetic circuit changes. The study was performed taking into account only the main MMF harmonic. The calculation results for the dependence of the saturation factor of the magnetic circuit k_μ are shown in Fig. 1. The dashed line corresponds to the dependence determined by the method of the magnetic circle sections [2], the solid line - using field analysis [5].

It can be seen that up to twice the nominal MMF value, both dependencies k_μ practically match up. Further discrepancies are caused by the algorithm for taking into account the field flattening in [2] and the sinusoidal conditions of the currents in the field analysis with the quasi-static model in [13].

The methodology of research for the torque correction factor k_M corresponds to the following steps:

1) calculation of the steady-state IM mode taking into account the influence of saturation on the electromagnetic parameters when the saturation factor changes [5], definition M_e (7); 2) determination of ininstantaneous system currents (11) – (13) at the calculation point for their use as initial conditions for calculating current attenuation; 3) calculation of the damping mode currents with zero voltage at the input IM terminals and zero rotor speed (ensures constancy of the rotor angle rotation and determination of the reserve of only magnetic energy without the influence of mechanical energy) and determination of integrals of the refined electromagnetic torque M_{ek} expression (8); 4) determination of the factor k_M (9).

Calculations showed that in order to obtain a stable dependence of the change of the factor (9), it is necessary to ensure a strict correspondence of the dependences k_μ, $\partial k_\mu / \partial F_m$. For this purpose, the calculated dependence according to the field analysis k_μ (Fig. 1) is approximated by a polynomial in the curve region dependence, which was used to determine the derivative $\partial k_\mu / \partial F_m$, Fig. 1. This area, for IM 4A80A2U3, is limited by points 1 and 2 with coordinates: $k_1 = 100$; $f_1 = 1{,}15$; $k_2 = 200$; $f_2 = 1{,}31$. Outside the area, the dependence has a linear character, for which point 3 is used: $k_3 = 500$; $f_3 = 2{,}08$. Conditions for forming the polynomial of the bending region: the polynomial values at the boundaries region correspond to the specified points coordinates, the derivative of the polynomial value at the boundaries region correspond to the direction of the dependence linear sections.

In accordance with the following specified conditions, for three sections of the approximation dependence, the limits of the sections of the change in the MMF value *F*, the corresponding expressions of the dependence k_μ and its derivative were formed

$$F \in \left[F < f_1; f_1 \le F \le f_2; F > f_2\right];$$

$$k_\mu = \left[k_1; k_1 + b(F - f_1)^c; k_2 + e(F - f_2)\right];$$

$$\partial k_\mu / \partial F_m = \left[0; bc(F - f_1)^{c-1}; e\right],$$

where $e = (k_3 - k_2)/(f_3 - f_2)$;

$$c = e(f_2 - f_1)/(k_2 - k_1); \quad b = (k_2 - k_1)/(f_2 - f_1)^c.$$

The graphs of the approximation function (coincides with the given curve) and its derivative are shown in Fig. 1.

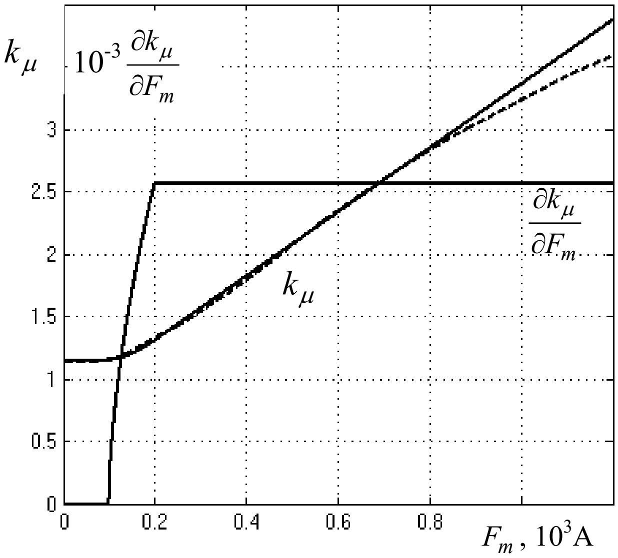

Fig. 1

The obtained approximate dependences of the saturation factor function and its derivative, the matrix of the derivative from the inductive parameters change (15) provide the possibility of solving the system (11) – (13). Thus, it is possible to investigate the change in the torque correction factor k_M according to the outlined methodology. The study k_M was carried out on the example of calculations of IM 4A80A2U3 operating conditions with three specified values of the angular speed of rotor rotation and with variations in the voltage value.

The results of the study are summarized in Table 1, where, according to the specified speed and the effective value of the phase voltage, the following values are given (calculated taking into account the nonlinearity of the electromagnetic parameters): current, saturation factor, MMF, torques, k_M. The line of the nominal mode is marked in bold.

The analysis of dependencies in Table 1 shows that at all speeds, the patterns of changes in the torque correction factor k_M as the function of the total MMF F_m, or as the function of the saturation factor k_μ practically almost completely. This substantiates the possibility of increasing the analysis models adequacy of the IM saturated operation conditions, due to the precise determination of the torque value without the assumption of the equality of magnetic energies and coenergies, with the help of a pre-calculated dependence of the change in the torque correction factor k_M as a function of the total MMF F_m (10).

Table 1

ω_r, c^{-1}	U_f, V	I_f, A	k_μ	F_m, A	M_e, Nm	M_{ek}, Nm	k_M
300,21	55	0,74	1,15	70,61	0,3273	0,3273	1
	110	1,49	1,198	146,9	1,307	1,314	1,005
	165	2,3	1,503	275,2	2,914	2,983	1,024
	220	**3,26**	**2,099**	**507,3**	**5,08**	**5,339(5.18)**	**1,051**
	275	4,71	3,314	980,9(2050)	7,62(7.0)	8,431(8.08)	1,106
	385	13,75	11,21	4059(9500)	11,4(8.5)	15,83(15.85)	1,389
	440	21,47	17,63	6557	12,05	18,33 (20.07)	1,52
307	55	0,444	1,15	73,3	0,182	0,182	1
	110	0,9	1,21	154	0,7264	0,7307	1,006
	165	1,47	1,554	294,9	1,615	1,656	1,025
	220	2,321	2,224	556,1	2,801	2,959	1,056
	275	3,993	3,648	1111	4,156	4,658	1,121
	385	14,48	12,34	4496	5,953	8,423	1,415
	440	22,37	18,77	7001	6,239	9,603	1,539
310,5	165	1,134	1,581	305,5	0,8563	0,879	1,027
	220	2,008	2,293	582,9	1,482	1,57	1,059
	275	3,877	3,834	1184	2,184	2,466	1,129

For the IM studied, such a calculated dependence is shown in Fig. 2. From the data in Table 1 and Fig. 2, it can be seen that the refinement of the determination of the torque value (due to the increase in the adequacy of the formula for its calculation) for saturation to the nominal level is relatively low - does not exceed 5%. It is at the level of ordinary errors of calculation methods and has little effect on the research result. As the saturation increases, neglecting the refinement torque factor k_M becomes unacceptable - the error increases significantly. When the voltage increases by 25%, the error increases to 10%, by 75% - to 40%.

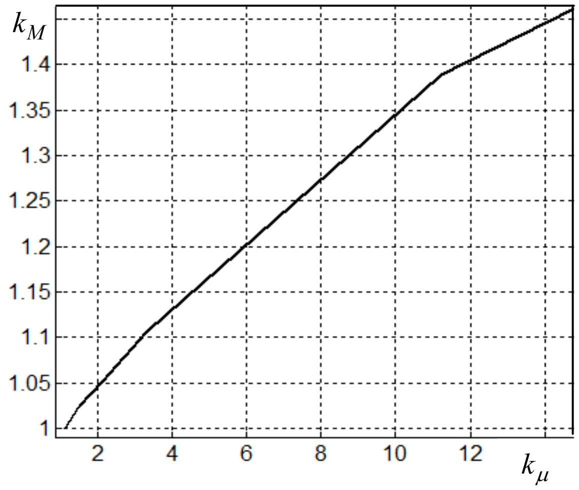

Fig. 2

An estimate of the reliability of the mathematical modeling results without the assumption of the equality of magnetic energy and coenergy can be performed by comparing the results of physical and mathematical experiments under the conditions of a change in the degree of saturation the motor magnetic core. As can be seen from the data in Fig. 2, the influence of this factor is significant at high saturations, when the precision in determining the electromagnetic torque value reaches tens of percent. Long-term operation of serial IMs in such conditions is impossible due to thermal overloads. This makes it difficult to obtain reliable data of a physical experiment.

In this work, the estimation of errors value with the assumption of equality of magnetic energies and coenergies was carried out during a comparative study of the nominal operating conditions of the same type series IMs with two and six poles. At the same time, we assume that the motors have the same temperature mode in nominal operating conditions, the same type of materials and manufacturing technology. As a result, the algorithms for calculating the electromagnetic parameters value must be the same.An increase in the number of serial IMs poles is accompanied by a decrease in the saturation degree, which is caused by a change in the ratio of the lengths of the magnetic flux paths through non-magnetic and magnetic media. The change in saturation under an unchanged calculation algorithm should be reflected in the calculated electromagnetic torque value and the accuracy of the mathematical analysis results of the nominal operating conditions. Let's check it in the next study.

The comparison was made based on the calculation of the nominal operating conditions of IMs 4A80A2U3 and 4A80A6U3. Reference data [14] about the parameters of their nominal operating conditions are given in Table 2. The initial information for the calculated determination of the operating condition parameters is the power supply voltage, the substitute circuit parameters, and the rotor angular speed. The speed is determined by solving the differential equation of the IM mechanical balance by the given inertia torque of the rotating parts and the resistance torque on the IM shaft (taken at the level of the nominal IM torque). The analysis is carried out according to the calculated values of the IM electromagnetic torque, taking into account the nonlinearity of the electromagnetic parameters, and torques of IM mechanical losses, in accordance with the adopted calculation algorithms. During the electromagnetic calculation of IM, the substitute circuit parameters are determined in accordance with [2], the electromagnetic torque (7) is increased taking into account expressions (8) - (10) and the dependence $k_M(F_m)$ from Table 1, losses in steel and additional ones are taken into account by connecting the appropriate resistances in parallel to the IM.

Table 2

Source of information			IM standard size	P_2	I_f		ω_r	Efficiency		$\cos\varphi$		k_μ	k_M
				W	A	δ,%	c^{-1}		δ,%		δ,%		
directory			4A80A2У3	1500	3,3	-	301	0,81	-	0,85	-	-	-
			4A80A6У3	750	2,22	-	95,9	0,69	-	0,74	-	-	-
calculation	Conditions for calculating parameters 1	$W_м \neq W_к$	4A80A2У3	1500	3,36	1,8	300,6	0,812	0,2	0,839	1,3	2,07	1,05
			4A80A6У3	750	2,2	0,9	96	0,707	2,5	0,731	1,2	1,32	1,01
		$W_м = W_к$	4A80A2У3	1500	3,46	**4,8**	299,8	0,771	**4,8**	0,849	0,1	2,05	1
			4A80A6У3	750	2,22	0,0	95,9	0,7	1,45	0,734	0,8	1,31	1
	Conditions for calculating parameters 2	$W_м = W_к$	4A80A2У3	1500	3,351	1,5	300,8	0,812	0,25	0,835	0,18	2,08	1
			4A80A6У3	750	2,174	**2,1**	96,5	0,736	**6,7**	0,715	**3,4**	1,34	1

The first four terms of the calculated results in Table 2 were obtained under the following parameters calculation conditions: the conductivity of the stator copper winding is 41 MSm/m; conductivity of aluminum rotor winding is 22.56 MSm/m; additional losses are taken at the level of 1% of the consumed active power [15]; mechanical losses of IM are determined according to [16], taking into account the influence of the elastic coupling [17]; losses in steel are determined based on the calculation of the main losses in steel [2] with the coefficient of technological factors influence 3.

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The results of refined calculations without the assumption of the equality of magnetic energy and co-energy (the first two lines of calculation data in Table 2) show that the error value (relative to the reference data) of the calculations results of currents and energy coefficients δ is approximately the same for both motors. The same level of errors, independly of the difference in the saturation factors values of the magnetic circuit and the increase of the torque (given in Table 2), confirms the feasibility of applying the studied refinement.

If the difference in magnetic energy and coenergy is not taken into account (Wm=Wk, the influence of the torque correction factor k_M is not taken into account, it is equal to one), as can be seen from the third and fourth terms of the calculation results of Table 2, the error of the IM calculation results of different polarity is significantly different. Without an accurate determination of the torque taking into account the value k_M it is impossible to simultaneously ensure a minimum error for IMs of different polarities. Thus, for a six-pole IM (fourth row of calculations in Table 2), the error in determining energy coefficients does not exceed 1.5%, and for a two-pole it increases by three times (third row). If the calculation conditions correspond to the minimum error for a two-pole IM (fifth row), then for a six-pole IM it increases significantly, according to the data of the sixth row of Table 2 – up to 6.7%. For this case (rows 5, 6), the following parameters calculation conditions are applied: the conductivity of the copper of the stator winding is 43 MSm/m; the conductivity of aluminum of the rotor winding is 23.2 MSm/m; additional losses are accepted at the level of 0.5% of the consumed active power, according to ДСТУ; IM mechanical losses are determined according to [16]; losses in steel are determined based on the calculation of the main losses in steel [2] with the coefficient of technological factors influence of 2.5.

Conclusions.

In order to carry out studies of IM operation conditions based on the refined expression of the electromagnetic torque, without the assumption of equality of magnetic energies and coenergies, mathematical models for determining the electromagnetic torque value based on the calculation of the currents attenuation process in the IM electric circuits were developed and implemented in software. The conducted studies showed: the amount of refinement in nominal operating conditions does not exceed 5%; with an increase in the saturation of the magnetic circuit, the error in determining the torque reaches tens of percent, which justifies the need to use the developed refined tools. The torque correction factor is determined as a ratio of the calculation results for two options: 1) Calculation of currents for a nonlinear formulation of the problem with a change in electromagnetic parameters in accordance with a change in the saturation factor and determination of the electromagnetic torque according to the formula with the assumption of equality of magnetic energy and coenergy; 2) Calculation of currents for a nonlinear formulation of the problem and determination of the electromagnetic torque according to the refined formula without the assumption of equality of magnetic energy and co-energy. According to the study results of the dependence of the change of the torque correction factor in the total MMF function, it is proposed to use it in practice for a simpler and easier numerical implementation of the first version calculation with the simultaneous use of the correction factor according to the previously found dependence of its change.

At applying the same algorithm for calculating the operating conditions parameters of the same type IMs of different polarity, it is impossible to simultaneously minimize the error of their calculations, unless a refined definition of the IM torque is applied without the assumption of the equality of magnetic energies and coenergies. Such clarification reduces the calculation error of operating condition parameters up to 3...4 times. A comparative study of the operating conditions parameters of the same type IMs of different polarities allows us to substantiate the feasibility of a refined definition the electromagnetic torque value without assuming the equality of magnetic and coenergies.

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УТОЧНЕННЯ ВИЗНАЧЕННЯ МОМЕНТУ АСИНХРОННОГО ДВИГУНА ЗА РЕЖИМІВ МАГНІТОПРОВОДУ У НАСИЧЕНІЙ ОБЛАСТІ

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За режимів роботи магнітопроводу асинхронного двигуна у насиченій області досліджено величину похибки формули розрахунку електромагнітного моменту за припущення рівності магнітних енергії і коенергії. Для серійних двигунів за номінальних режимів похибка не перевищує 5%, із збільшенням напруги на 75% похибка зростає до 40%. Уточнену величину електромагнітного моменту визначено за розрахунку величини інтегральних залежностей магнітної коенергії за кутом повороту ротора та у процесі розрахунку динамічного режиму затухання струмів у обмотках. Задля підвищення ефективності методики уточненого розрахунку електромагнітного моменту запропоновано до застосування залежність поправочного коефіцієнту у функції МРС машини. Застосування цієї, заздалегідь визначеної, залежності суттєво прискорює уточнені розрахунки. Обгрунтування достовірності результатів уточненого визначення електромагнітного моменту без припущення рівності магнітних енергії і коенергії здійснено за порівняння результатів розрахунку номінальних режимів роботи серійних двигунів із двома та шістьома полюсами, які мають суттєву розбіжність у ступені насичення. Бібл. 17, табл. 2, рис. 2.

Ключові слова: асинхронний двигун, насичений режим, електромагнітний момент, коенергія.

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