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IMPEDANCE BOUNDARY CONDITION OF NON-UNIFORM ELECTROMAGNETIC FIELD PENETRATION INTO CONDUCTING HALF-SPACE

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The influence of the non-uniformity of the external field to its distribution on the surface of the conducting half-space is investigated on the basis of exact analytical and approximate asymptotic calculation methods of an arbitrary three-dimensional electromagnetic field. The function that generalizes the impedance boundary condition for diffusion of non-uniform field into conducting body is proposed. On the basis of calculations results comparison by exact and approximate methods for concrete model of electromagnetic system the admissible value of the upper limit of the introduced small parameter is established. References 10, figures 5.
Key words: three-dimensional electromagnetic field, strong skin effect, impedance boundary condition.

Introduction. In the case of high-speed pulse or high-frequency processes there is a strong skin effect in the conducting elements of electromagnetic devices, in which the current and electromagnetic field are concentrated in a thin surface layer. In this paper, sinusoidal electromagnetic fields are considered in the quasi-stationary approximation, when wave processes can be neglected [1, 2]. Under such limitations, simplified approaches are widely used to solve specific problems and develop appropriate numerical calculation methods [3, 4]. Despite the long history of development, the study of the electromagnetic field with strong skin effect remains an urgent task.

Features of electromagnetic field penetration into conducting body, including its surface distribution, depend not only on electrical conductivity, relative magnetic permeability of the medium and field frequency, but also on geometry of boundary surfaces and features of field distribution of external sources near the surface.

For a body with ideal conductivity, the depth of field penetration $\delta = \sqrt{2/(\omega\mu\mu_0\gamma)}$ goes to zero $\delta \to 0$. Here μ is relative magnetic permeability, γ is specific electrical conductivity, ω is cyclic field frequency. In this case, it is sufficient to use a mathematical model in which the tangential component of the electric field intensity and the normal component of the magnetic field intensity are equal to zero on the surface of the conducting body $\boldsymbol{E}_\tau = 0$, $H_n = 0$ (Fig. 1) [5]. The finite, but significantly smaller penetration depth of field compared to the size of the body and the nonzero value of the electric field intensity are taken into account in an approximate mathematical model using the concept of impedance boundary condition formulated by M. Leontovich [6, 7]. It is assumed that the local electromagnetic field penetrates into the metal body in the same way as a uniform field penetrates to the conducting half-space (Fig. 2). Moreover, the magnitude of the magnetic field tangential to the body surface $H_\tau = H_{\|}(z=0)$ is determined from the independent solution of the external problem under the condition $H_n = H_z = 0$. The field vectors at the boundary are connected by the impedance boundary condition:

$$\dot{\boldsymbol{E}}_\tau = \varsigma\left[\boldsymbol{e}_z \times \dot{\boldsymbol{H}}_\tau\right], \qquad (1)$$

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where $\varsigma = p/\gamma = \sqrt{i\omega\mu\mu_0\gamma}/\gamma$ is the effective surface impedance. In this model representation, the tangent component to the surface of the half-space $\boldsymbol{H}_\tau = \boldsymbol{H}_{\|}(z=0)$ is equal to twice value of the tangent component of the magnetic field of external sources $\boldsymbol{H}_{\|}(z=0) = 2\boldsymbol{H}_{0\|}$ [8].

Detailed review of research on the use of the concept of surface impedance in the modeling of electrodynamics problems is presented, in particular, in [3, 9]. At the same time, in most of the cited papers, mathematical models of the diffusion of non-uniform electromagnetic field are limited of a small value of penetration depth or insignificant field non-uniformity at the body surface.

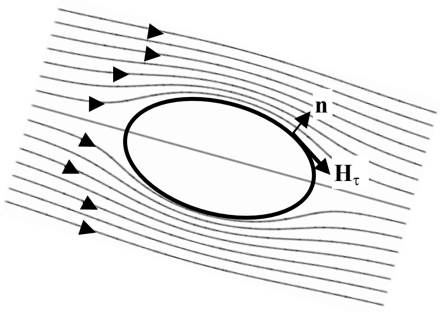

Fig. 1

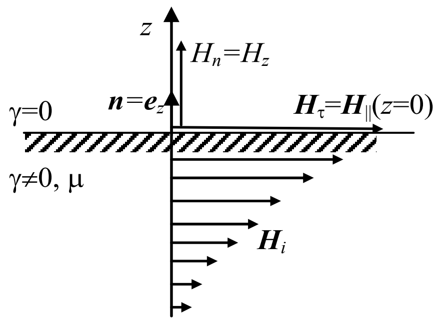

Fig. 2

The researches on the analytical solution of a three-dimensional problem for an electromagnetic field near conducting half-space, which does not contain restrictions on the configuration of the external field, electrophysical properties of media and field frequency are systematized in [8]. The results made it possible to generalize the concept of a strong skin effect to the case of diffusion of non-uniform electromagnetic field into conducting body. The skin effect can be considered as strong, when the penetration depth of the field δ is small compared to the characteristic dimensions not only of the conducting body but also of the entire electromagnetic system, including distances r from external field sources to the boundary surface of the body. Introduction the appropriate quantitative parameter $\varepsilon_1 = \mu\delta/\left(\sqrt{2}r_1\right)$ makes it possible to find an approximate solution of the problem and analyze the resulting errors. Here r_1 is the distance between the observation point in the dielectric region and the point of the external field source mirror reflected from the interface of the media.

The aim of the work is to generalize the concept of impedance boundary condition for strong skin effect in the case of diffusion of non-uniform electromagnetic field into conducting half-space and study the resulting error of the approximate asymptotic calculation method for a particular model of electromagnetic system.

Asymptotic solution of the problem for non-uniform electromagnetic field and generalized impedance boundary condition.

When the parameter ε_1 is small $\varepsilon_1 < 1$, the exact expressions for the three-dimensional electromagnetic field can be simplified and the solution can be represented in the form of asymptotic series expansion of potentials and field intensities [8]. At the interface of the media in the dielectric region $z = 0$, where the parameter $\varepsilon_1 = \varepsilon$ takes the largest value, the complex-value amplitudes of electric $\dot{\boldsymbol{E}} = \dot{\boldsymbol{E}}_{\|} + \dot{\boldsymbol{E}}_{\perp}$ and magnetic $\dot{\boldsymbol{H}} = \dot{\boldsymbol{H}}_{\|} + \dot{\boldsymbol{H}}_{\perp}$ field intensities are determined by the magnitude of the external magnetic field $\dot{\boldsymbol{H}}_0 = \dot{\boldsymbol{H}}_{0\|} + \dot{\boldsymbol{H}}_{0\perp}$ and its derivatives with respect to the coordinate z:

$$\dot{\boldsymbol{E}}_{\|}(z=0) = \sum_{n=0}^{N} \dot{\boldsymbol{E}}_{\| n} = \varsigma \sum_{n=0}^{N} 2a_n(\mu)\left(\frac{\varepsilon}{\sqrt{i}}\right)^n \left\{ r_0^n \frac{\partial^{(n)}}{\partial z^n} \boldsymbol{e}_z \times \dot{\boldsymbol{H}}_{0\|} \right\}\Bigg|_{z=0}, \quad (2)$$

$$\dot{\boldsymbol{H}}_{\|}(z=0) = \sum_{n=0}^{N+1} \dot{\boldsymbol{H}}_{\| n} = -\sum_{n=0}^{N+1} 2a_{n-1}(\mu)\left(\frac{\varepsilon}{\sqrt{i}}\right)^n \left\{ r_0^n \frac{\partial^{(n)} \dot{\boldsymbol{H}}_{0\|}}{\partial z^n} \right\}\Bigg|_{z=0}, \quad (3)$$

$$\dot{\boldsymbol{H}}_{\perp}(z=0)=\sum_{n=0}^{N}\dot{\boldsymbol{H}}_{\perp n}=-\sum_{n=0}^{N}2a_n(\mu)\left(\frac{\varepsilon}{\sqrt{i}}\right)^{n+1}\left\{r_0^{n+1}\frac{\partial^{(n+1)}\dot{\boldsymbol{H}}_{0\perp}}{\partial z^{n+1}}\right\}\Bigg|_{z=0}. \quad (4)$$

Here $a_n(\mu)$ are the Taylor series coefficients of the function $\left[\chi/\sqrt{i}+\sqrt{1+\left(\chi/\left(\mu\sqrt{i}\right)\right)^2}\right]^{-1}=\sum_{n=0}^{\infty}a_n(\mu)\left(\chi/\sqrt{i}\right)^n$, i is imaginary unit. The first several expansion coefficients in the power series of the function have the values $a_0=1, a_1=-1, a_2=1-1/\left(2\mu^2\right),\ldots$, it is accepted $a_{-1}=-1$; r_0 is distance r_1 for points on the surface.

From the series (2) and (3) it is seen that the approximate Leontovich's impedance boundary condition (1) is valid only for the first two terms. The deviation takes place starting with $n=2$ i.e. with term of the series proportional ε^2 to which $\dot{\boldsymbol{E}}_{\parallel 2}=\left[1-1/\left(2\mu^2\right)\right]\varsigma\boldsymbol{e}_z\times\dot{\boldsymbol{H}}_{\parallel 2}$. Hence, the generalized impedance boundary condition, which connects the field intensities on the interface $z=0$ and is not limited to the first two terms of the series, can be represented as follows:

$$\dot{\boldsymbol{T}}=\dot{\boldsymbol{E}}_{\parallel}-\varsigma\dot{\boldsymbol{H}}_{\parallel}=\varsigma\sum_{n=2}^{N}2\left(a_n+a_{n-1}\right)\left(\frac{\varepsilon}{\sqrt{i}}\right)^{n}\left\{r_0^{n}\frac{\partial^{(n)}}{\partial z^{n}}\boldsymbol{e}_z\times\dot{\boldsymbol{H}}_{0\parallel}\right\}\Bigg|_{z=0}, \quad (5)$$

where the summation begins from $n=2$. As follows from (5), the first nonzero term of the series of functions $\dot{\boldsymbol{T}}$ is proportional to ε^2, and the argument of complex-value amplitude $\dot{\boldsymbol{T}}$ is equal to $-\pi/4$:

$$\dot{\boldsymbol{T}}\approx 2|\varsigma|\left(1-\frac{1}{2\mu^2}\right)\frac{\varepsilon^2}{\sqrt{i}}\left\{r_0^{2}\frac{\partial^{(2)}}{\partial z^{2}}\boldsymbol{e}_z\times\dot{\boldsymbol{H}}_{0\parallel}\right\}\Bigg|_{z=0}. \quad (6)$$

From series (4) it is seen that the fulfillment of another condition in the model of the ideal skin effect about the equality of zero normal to the surface component of the magnetic field strength is more rigid. It is performed only for the zero term of the asymptotic series and is violated, starting with the term proportional to the small parameter in the first degree ε^1.

Generalized impedance boundary condition for a specific model of the electromagnetic system. Earlier in [10] calculations of three-dimensional electromagnetic field in the dielectric region were performed in the case when the external electromagnetic field is created by a circular contour with current $\dot{I}_0$ whose plane is perpendicular to the boundary surface of the conducting body. In this case, the components of the external magnetic field are $\dot{\boldsymbol{H}}_0=\frac{\dot{I}_0}{4\pi}\oint_l\frac{\boldsymbol{t}\times\boldsymbol{r}}{r^3}dl$, where $\boldsymbol{r}$ is the vector going from the contour point to the observation point, $\boldsymbol{t}$ is the vector tangent to the contour. The orientation of the Cartesian coordinate system is shown in Fig. 2 works [10] - the center of the coordinate system is on the interface, the vertical axis Oz, directed normally to the surface, passes through the center of the circle, the axis Ox coincides with the line of intersection of the plane of the circle with the interface. The geometric dimensions are as follows: radius of the contour $R=0.05$ m, distance from the center of the contour to the surface $H=0.06$ m, respectively, the minimum distance from the contour to the surface $h_0=0.01$ m. The electrophysical properties of the medium correspond to those of aluminum $\gamma=3{,}7\cdot 10^7\ 1/(\Omega\cdot\text{m})$, $\mu=1$. Frequency is variable. Despite the fact that the calculations were performed for specific geometric dimensions and properties of the material, the results due to the linearity of the problem in relative terms are valid for other parameters of the electromagnetic system.

The presence of exact and approximate solutions of the problem allows not only to determine the field intensities, and also to investigate the features of the generalized impedance boundary condition: calculate the function $\dot{\boldsymbol{T}}$, compare the results of exact and approximate approaches and analyze the influence of the parameter ε to the value of the function $\dot{\boldsymbol{T}}$ for different components of the electromagnetic field.

When integrating along circular contour, the distance r for different contour points is changed and the value of the parameter ε is changed accordingly for these points. Therefore, the results of calculations for the function $\dot{\boldsymbol{T}}$ and field intensities are presented depending on the maximum value of the parameter $\varepsilon=\delta/\left(\sqrt{2}r_{\min}\right)$, which corresponds to the minimum distance $r_{\min}$ between the contour and the observation point on the surface of the conductive body. Components of complex-value amplitudes of a function $\dot{T}_k^* = \left|\dot{T}_k\right| \exp(i\varphi_k)$, where $k = x, y$, were investigated for modules and arguments of the function. The results for the modules of complex-value amplitudes are presented in the form of relative values $T_{kE}^* = \left|\dot{T}_k\right| / \left|\dot{E}_k\right|$. This normalization allows, depending on the parameter ε to determine the relative effect of the non-uniformity of the electromagnetic field on the value of the function $\dot{\boldsymbol{T}}$, which for the model of penetration of the uniform field is equal to zero.

Fig. 3 – 5 show the values of different components of the function $\dot{\boldsymbol{T}}$ and the intensities of the electromagnetic field at different points on the surface, depending on parameter ε. In all figures, the solid curves correspond to the calculation by exact analytical expressions, the dotted curves correspond to the calculation by the approximate asymptotic method.

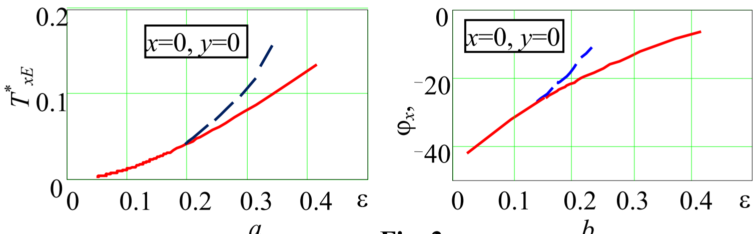

Fig. 3

At the point $x = 0, y = 0$ where the distance $r_{1\,\min} = h_0$ from the contour to the surface is the smallest, the electric field intensity $\dot{\boldsymbol{E}} = \dot{E}_x \boldsymbol{e}_x$ and, accordingly, the function $\dot{\boldsymbol{T}} = \dot{T}_x \boldsymbol{e}_x$ are directed along the axis x. At this point, the intensities of electric $\dot{E}_x$ and magnetic fields $\dot{H}_y$ on the surface are largest. The values of T_{xE}^* in dependence of the small parameter ε at this point is shown in Fig. 3 *a*. A characteristic feature is the approximately quadratic dependence on ε, which confirms the conclusion for the generalized impedance boundary condition with strong skin effect. The influence of the older terms of the series to the value of the argument of the function $\dot{T}_x$ is more significant. This can be seen from Fig. 3 *b* for the same point on the surface.

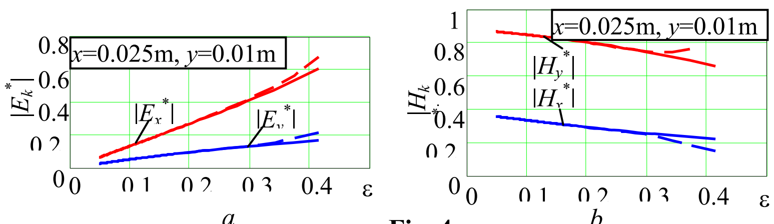

Fig. 4

If the observation points move away from the point closest to the contour on the surface, the electromagnetic field intensity decreases. In addition, at points on the surface that do not coincide with the axis x, all components of the electromagnetic field intensity are nonzero. The values of the field components can differ significantly from each other. This is illustrated in Fig. 4, which shows the values of the tangent components of the electric and magnetic field intensities at the point. In the figure, the magnitude of the intensities are normalized as follows $\dot{\boldsymbol{E}} = \dfrac{\mu_0 \left|\dot{I}_0\right| \omega}{4\pi} \dot{\boldsymbol{E}}^*$, $\dot{\boldsymbol{H}} = \dfrac{\left|\dot{I}_0\right|}{4\pi h_0} \dot{\boldsymbol{H}}^*$.

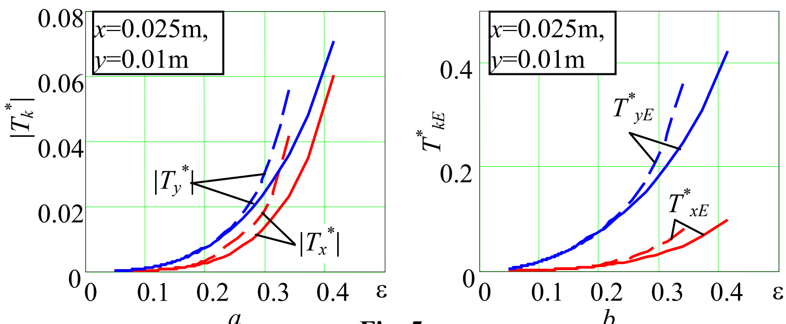

Fig. 5

Despite the significant difference in the values of the components of the field, the components of the function $\dot{\boldsymbol{T}}$ that characterizes the generalized impedance condition for non-uniform field differ much less. As can be seen from Fig. 5 *a*, the absolute values of the various tangent components of the function $\dot{\boldsymbol{T}}$ are values of the same order. The figure shows the normalized values, which are determined similarly to the electric field intensity $\dot{\boldsymbol{T}} = \frac{\mu_0 |\dot{I}_0| \omega}{4\pi} \dot{\boldsymbol{T}}^*$.

In this case of the specific model of the electromagnetic system, at the point of the surface we have relations $|\dot{E}_x| > |\dot{E}_y|, |\dot{H}_y| > |\dot{H}_x|$ for the components of the field intensities. However, for the components of the function $\dot{\boldsymbol{T}}$ we have the opposite inequality $|\dot{T}_x| < |\dot{T}_y|$. This feature is appropriately reflected in the relative value of the components of the function $\dot{\boldsymbol{T}}$ (Fig. 5 *b*) – larger values of the components T^*_{kE} occur for smaller value components of the electric field intensity $|\dot{E}^*_k|$. In all cases, the possibility of applying for non-uniform electromagnetic field the simplified model of penetration into conducting half-space of uniform electromagnetic field is performed only for fields that are characterized by very small value parameter ε. This implies the need to take into account the non-uniformity of the field in the development of methods for calculating electromagnetic fields using the impedance boundary condition.

Comparison of the results obtained by analytical exact and approximate methods allows us to make conclusion about the allowable values of the parameters at which the use of simpler asymptotic approximation is acceptable. The error of the results of the electromagnetic field calculation by the approximate method depends on the complex parameter $\varepsilon = \mu\delta / \left(\sqrt{2} r_{\min}\right) = \sqrt{\mu / \omega\mu_0\gamma r^2_{\min}}$ that combines the electrophysical properties of the medium, field frequency and the minimum distance from field sources to the surface of the conductive body. As can be seen from Fig. 4, calculations of field intensities can be performed when the small parameter does not exceed the value $\varepsilon = 0.3$. In Fig. 3 and 5 the results of calculations of the function $\dot{\boldsymbol{T}}$ of the generalized impedance condition are also presented depending on the same complex parameter ε. It can be seen that sufficient accuracy is provided here for a slightly smaller range of values of the small parameter $\varepsilon \le 0.2$. This is due to the fact that the approximate expressions of the function do not take into account the first two terms of the asymptotic series, whose contribution to the value of field intensities is the largest.

Conclusion.

1. In the case of a strong skin effect the expansion of exact analytical expressions for a three-dimensional electromagnetic field into asymptotic series on the introduced small parameter allows to express the electromagnetic field intensities on the surface of the conductive half-space through the value of external magnetic field and its derivatives. This makes it possible to analyze the effect of external field non-uniformity at the surface of the conducting half-space and determine the effect of the introduced small parameter on the field value, as well as determine the validity of the approximate Leontovich's impedance boundary condition depending on the small parameter. The established deviation is the basis for the generalization of the impedance boundary condition and its presentation in the form that allows to obtain a quantitative estimate of this deviation.

2. The analysis of the function introduced as a generalization of the impedance boundary condition to the case of non-uniform electromagnetic fields confirmed the conclusion that the approximate Leontovich's impedance condition is valid only for the first two terms of the asymptotic series expansion. Comparison of the results obtained by exact and approximate methods allowed us to find the value of small parameter in which the use of simpler asymptotic approximation is acceptable and to conclude that its allowable limit value for generalizing the impedance boundary condition is smaller than the same value for fields on the surface of the conducting half-space.

3. Introduction of the generalized impedance boundary condition in the form of the function which, unlike the simplified approach, is not equal to zero, allows to formulate boundary value problems of finding the electromagnetic field using the impedance boundary condition in the case of diffusion of non-uniform field. The analysis of the emerging errors shows the permissible limits of the use of simplified and generalized approaches.

4. Since the conclusions regarding the quantitative indicators in this paper are based on the calculations of specific model of the electromagnetic system, it is advisable in the future to conduct research for standard sources of external fields, which may be elements of more complex systems.

Роботу виконано за бюджетною темою «Розробити нові математичні моделі та методи дослідження електрофізичних процесів і полів в електротехнічному обладнанні для вирішення задач його надійної експлуатації та діагностування» (шифр «КОМПЛЕКС-5»), КПКВК 6541030.

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ІМПЕДАНСНА ГРАНИЧНА УМОВА ПРОНИКНЕННЯ НЕОДНОРІДНОГО ЕЛЕКТРОМАГНІТНОГО ПОЛЯ В ЕЛЕКТРОПРОВІДНИЙ ПІВПРОСТІР

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На основі точного аналітичного і наближеного асимптотичного методів розрахунку довільного тривимірного електромагнітного поля досліджено вплив неоднорідності зовнішнього поля на його розподіл на поверхні електропровідного півпростору. Запропоновано використання функції, яка узагальнює імпедансну граничну умову на випадок дифузії неоднорідного поля в електропровідне тіло. На основі порівняння результатів розрахунків за точним і наближеним методами для конкретної моделі електромагнітної системи встановлено припустиме значення верхньої межі введеного малого параметру, за яким здійснюється розкладання в асимптотичний ряд. Бібл. 10, рис. 5.

Ключові слова: тривимірне електромагнітне поле, сильний скін-ефект, імпедансна гранична умова.

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