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SIMULATION OF ELECTROMAGNETIC-ACOUSTIC CONVERSION PROCESS UNDER TORSION WAVES EXCITATION. PART 3

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Mathematical simulation and calculation of electromagnetic fields in the electromagnetic-acoustic transducer of rational design are performed under non-dispersive torsional waves excitation in tubular electrically conductive ferromagnetic hollow rods of small diameter, taking into account spatial, frequency, energy and material factors. The results of the research can be used to simulate and construct exciting EMATs for measuring, monitoring, and diagnostic equipment in the energy, nuclear, chemical and other industries in view of ultrasonic studies of ferromagnetic tubular products. References 10, figures 5.

Key words: mathematical simulation, ultrasonic transducer model, non-dispersive torsional waves, tubular product, skin layer, conversion loss.

Introduction. A significant part of products and equipment used in power engineering and other industries is manufactured using essentially different technologies [1−4]. Ultrasonic methods are used to diagnose their condition and measure the properties of materials. Traditional ultrasonic methods of diagnostics are not always applicable to such complex products, therefore development of new ultrasonic methods and means of diagnostics and measurements is required. To effectively evaluate the quality of products and their properties, the use of torsional ultrasonic waves is promising [5]. Previously performed theoretical [6−7], model [8] and experimental developments [9] did not fully develop concepts for creating excitation and reception facilities for torsional ultrasonic waves and their application in industry. So in works [6−7] theoretical and model studies on the excitation of torsional waves with the use of electromagnetic-acoustic conversion (EMA) were performed. In work [6], a differential equation for running non-dispersive torsional oscillations in a hollow electrically conductive ferromagnetic rod is formulated and its general solution is given for an object of investigation of infinite length. A solution of the differential equation [6] was found in work [7] in the part of determining the electromagnetic field intensity produced by the exciting high-frequency EMA coil of the transducer (EMAT) in the region between this coil and a tubular product of ferromagnetic material. The wave characteristic of the EMAT source of the variable magnetic field is determined.

For designing, calculating and manufacturing effective EMA transducers with predefined characteristics for measuring, controlling, diagnosing and investigating the physical and mechanical properties of tubular products, it is necessary to continue researching the solutions of the differential equation obtained, to determine the values of the interrelated electromagnetic fields in the areas that are elements of the EMAT and to formulate recommendations on the choice of EMAT design options.

The aim of the work is mathematical and computer research of the factors determining the rational design of the EMAT for ultrasonic diagnostics of hollow rods and small diameter tubes with non-dispersive torsional waves.

Contents and results of research. To achieve this goal, it is necessary to find the solutions of the differential equation [6] explicitly by finding the interrelated electromagnetic fields in different domains of the EMAT model step-by-step, taking into account all the factors affecting the design of the passing through transducer. In this case it is necessary to use the solutions obtained in work [7].

Calculation of the alternating magnetic field in a hollow ferromagnetic rod and the determination of the eddy current loss coefficient

Neglecting the displacement currents and taking into account the axial symmetry of the electromagnetic field, the Maxwell equations for the domain $\rho \le R$ are written in the following form:

$$\frac{\partial H_\rho^*}{\partial z} - \frac{\partial H_z^*}{\partial \rho} = r_2 E_\vartheta^* , \qquad -\frac{\partial E_\vartheta^*}{\partial z} = -i\omega\mu_1^\varepsilon H_\rho^* , \qquad \frac{1}{\rho} E_\vartheta^* + \frac{\partial E_\vartheta^*}{\partial \rho} = -i\omega\mu_3^\varepsilon H_z^* ,$$

where $\vec{H}^*$ and $\vec{E}^*$ – amplitudes of the intensity vectors of the alternating magnetic and electric fields in the volume of the ferromagnetic rod; r_2 and μ_m^ε ($m = 1;3$) – components of the tensors of specific electric conductivity and magnetic permeability of a ferromagnet polarized by a constant magnetic field. From the last relation it follows that

$$-\frac{\partial^2 H_\rho^*}{\partial z^2} + \frac{\partial^2 H_z^*}{\partial \rho \partial z} = -i\omega\mu_1^\varepsilon r_2 H_\rho^*, \tag{1}$$

$$\frac{1}{\rho}\frac{\partial H_\rho^*}{\partial z} - \frac{1}{\rho}\frac{\partial H_z^*}{\partial \rho} + \frac{\partial^2 H_\rho^*}{\partial \rho \partial z} - \frac{\partial^2 H_z^*}{\partial \rho^2} = -i\omega\mu_3^\varepsilon r_2 H_z^*. \tag{2}$$

Applying the integral Fourier transform [10] to the left and right sides of equations (1) and (2), we obtain a system of ordinary differential equations of the following form

$$k_s^2 H_\rho^*(\rho,\pm k_s) \mp ik_s \frac{\partial H_z^*(\rho,\pm k_s)}{\partial \rho} = -i\omega\mu_1^\varepsilon r_2 H_\rho^*(\rho,\pm k_s), \tag{3}$$

$$\mp\frac{ik_s}{\rho} H_\rho^*(\rho,\pm k_s) - \frac{1}{\rho}\frac{\partial H_z^*(\rho,\pm k_s)}{\partial \rho} \mp ik_s \frac{\partial H_\rho^*(\rho,\pm k_s)}{\partial \rho} - \frac{\partial^2 H_z^*(\rho,\pm k_s)}{\partial \rho^2} = -i\omega\mu_3^\varepsilon r_2 H_z^*(\rho,\pm k_s). \tag{4}$$

It follows from equation (3) that the integral images of the radial and axial components of the magnetic field vector of the coil are linearly related, thus,

$$H_\rho^*(\rho,\pm k_s) = \pm\frac{ik_s}{k_s^2 + i\omega\mu_1^\varepsilon r_2}\frac{\partial H_z^*(\rho,\pm k_s)}{\partial \rho}. \tag{5}$$

Substituting relation (5) into equation (4), we obtain the standard Bessel equation for the integral image of the axial component of the intensity vector of the magnetic field of the coil

$$\frac{\partial^2 H_z^*(\rho,\pm k_s)}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial H_z^*(\rho,\pm k_s)}{\partial \rho} - \zeta^2 H_z^*(\rho,\pm k_s) = 0,$$

the solution of which has the form

$$H_z^*(\rho,\pm k_s) = C I_0(\zeta\rho) + D K_0(\zeta\rho), \tag{6}$$

where C and D – the constants to be determined; the symbol ζ in the differential equation and its solution designates a complex wave number that takes into account the existence of eddy currents (skin effect) and is defined as follows: $\zeta = \sqrt{\mu_3^\varepsilon\left(k_s^2 + i\omega\mu_1^\varepsilon r_2\right)/\mu_1^\varepsilon}$.

Substituting the general solution (6) into relation (5), we obtain an expression for calculating the radial component $H_\rho^*(\rho,\pm k_s)$:

$$H_\rho^*(\rho,\pm k_s) = \pm\frac{i\mu_3^\varepsilon k_s}{\mu_1^\varepsilon \zeta}\left[C I_1(\zeta\rho) - D K_1(\zeta\rho)\right]. \tag{7}$$

Calculating the divergence of the vector $\vec{H}^*(\rho,\pm k_s)$, which components are given by relations (6) and (7), we obtain

$$div\,\vec{H}^*(\rho,\pm k_s) = \pm ik_s\left(\frac{\mu_3^\varepsilon}{\mu_1^\varepsilon} = 1\right)\left[C I_0(\zeta\rho) + D K_0(\zeta\rho)\right]. \tag{8}$$

It is obvious that in the case of an isotropy of the magnetic permeability, that is, when $\mu_1^\varepsilon = \mu_3^\varepsilon$, the right side of expression (8) becomes 0. Otherwise $div\,\vec{H}^*(\rho,\pm k_s) \neq 0$.

When performing calculations, it is expedient to divide the infinite domain ($0 \le \rho < \infty$, $-\infty < z < \infty$) into subdomains according to [7, fig. 1]:

- domain No. 1 ($R \le \rho < \infty$, $0 \le \vartheta \le 2\pi$, $-\infty < z < \infty$) – air gap under coil loops;

- domain No. 2 ($r_1 \le \rho \le R$, $0 \le \vartheta \le 2\pi$, $-\infty < z < \infty$) – volume of an electrically conductive ferromagnetic tube;

- domain No. 3 ($R_0 \le \rho \le r_1$, $0 \le \vartheta \le 2\pi$, $-\infty < z < \infty$) – air gap between the inner surface of the tube and the lateral surface of the electric current conductor;

- domain No. 4 ($0 \le \rho \le R_0$, $0 \le \vartheta \le 2\pi$, $-\infty < z < \infty$) – volume of the electric current conductor.

The alternating magnetic field in the domains No. 1 and No. 2 was determined in work [7]. Next, consider an alternating magnetic field in the air gap in the domain No. 3. In this domain, an alternating magnetic field satisfies Maxwell's equations in vacuum, which for amplitude values varying in time according to the field law $e^{i\omega t}$ and are written as follows

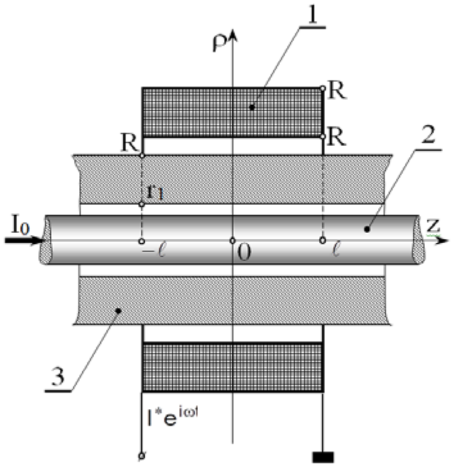

Fig. 1

$$rot\,\vec{H}^{(3)}(\rho,z) = i\omega\chi_0\vec{E}^{(3)}(\rho,z), \quad (9)$$

$$rot\,\vec{E}^{(3)}(\rho,z) = -i\omega\mu_0\vec{H}^{(3)}(\rho,z), \quad (10)$$

where $\vec{H}^{(3)}(\rho,z)$ and $\vec{E}^{(3)}(\rho,z)$ – amplitude values of the intensity vectors of the magnetic and electric field in the domain No. 3; $\chi_0 = 8{,}85\cdot 10^{-12}$ F/m – dielectric constant of vacuum or dielectric constant. Having determined the rotor from the left and right sides of equation (9) and substituting the right side of equation (10) into the obtained result in place of $rot\,\vec{E}^{(3)}(\rho,z)$, we obtain an equation for determining the components of the vector $\vec{H}^{(3)}(\rho,z)$

$$rot\,rot\,\vec{H}^{(3)}(\rho,z) - k_0^2\vec{H}^{(3)}(\rho,z) = 0, \quad (11)$$

where $k_0 = \omega/c$ – wave number of electromagnetic oscillations in vacuum; $c = 1/\sqrt{\chi_0\mu_0}$ – propagation speed of electromagnetic waves in vacuum.

In the case of an axisymmetric field, the vector equation (11) splits into two scalar equations of the following form:

$$-\frac{\partial^2 H_\rho^{(3)}(\rho,z)}{\partial z^2} + \frac{\partial^2 H_z^{(3)}(\rho,z)}{\partial\rho\,\partial z} - k_0^2 H_\rho^{(3)}(\rho,z) = 0, \quad (12)$$

$$\frac{1}{\rho}\left[\frac{\partial H_\rho^{(3)}(\rho,z)}{\partial z} - \frac{\partial H_z^{(3)}(\rho,z)}{\partial\rho}\right] + \frac{\partial}{\partial\rho}\left[\frac{\partial H_\rho^{(3)}(\rho,z)}{\partial z} - \frac{\partial H_z^{(3)}(\rho,z)}{\partial\rho}\right] - k_0^2 H_z^{(3)}(\rho,z) = 0. \quad (13)$$

From the limiting conditions [7, expression 3] it follows that

$$\lim_{L_3\to\infty}\left\{H_\beta^{(3)}(\rho,z), \frac{\partial H_\beta^{(3)}(\rho,z)}{\partial\beta}\right\} = 0, \quad \beta = (\rho,z). \quad (14)$$

where $L_3 = \sqrt{\rho^2 + z^2}$ – distance from the field source of the domain No. 3. The conditions (14) allow us to apply the integral transformation over the coordinate z for solving the system of equations (12), (13). We introduce the notation:

$$H_\beta^{(3)}(\rho,\pm k_s) = \int_{-\infty}^{\infty} H_\beta^{(3)}(\rho,z)e^{\pm ik_s z}dz, \quad \beta = (\rho,z), \quad (15)$$

The values $H_\beta^{(3)}(\rho,\pm k_s)$ will be called integral images of the alternating magnetic field intensity vector components amplitude values in the air gap, domain No. 3.

By applying the transformation (15) to equations (12), (13), we obtain

$$\left(k_s^2 - k_0^2\right)H_\rho^{(3)}(\rho,\pm k_s) \mp ik_s\frac{\partial H_z^{(3)}(\rho,\pm k_s)}{\partial\rho} = 0, \quad (16)$$

$$\frac{1}{\rho}\left[\mp ik_s H_\rho^{(3)}(\rho,\pm k_s) - \frac{\partial H_z^{(3)}(\rho,\pm k_s)}{\partial\rho}\right] +$$

$$+\frac{\partial}{\partial\rho}\left[\mp ik_s H_\rho^{(3)}(\rho,\pm k_s) - \frac{\partial H_z^{(3)}(\rho,\pm k_s)}{\partial\rho}\right] - k_0^2 H_z^{(3)}(\rho,\pm k_s) = 0. \quad (17)$$

From the equation (16) it follows that

$$H_\rho^{(3)}(\rho,\pm k_s) = \pm \frac{ik_s}{k_s^2 - k_0^2}\frac{\partial H_z^{(3)}(\rho,\pm k_s)}{\partial \rho}. \tag{18}$$

Substituting relation (18) into equation (17), we transform it to the form

$$\frac{\partial^2 H_z^{(3)}(\rho,\pm k_s)}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial H_z^{(3)}(\rho,\pm k_s)}{\partial \rho} - (k_s^2 - k_0^2) H_z^{(3)}(\rho,\pm k_s) = 0, \tag{19}$$

Since $k_s^2 >> k_0^2$, the general solution of equation (19) is written as follows

$$H_z^{(3)}(\rho,\pm k_s) = E\,I_0(k_s\rho) + F\,K_0(k_s\rho), \tag{20}$$

where E and F – constants to be determined.

Substituting expression (20) into relation (18), and taking into account that $k_s^2 >> k_0^2$, we obtain

$$H_\rho^{(3)}(\rho,\pm k_s) = \pm i\left[E\,I_1(k_s\rho) - F\,K_1(k_s\rho)\right]. \tag{21}$$

The alternating magnetic field in the volume of the electric current conductor, domain No. 4, is calculated by the method of determining the alternating magnetic field [7, domain No. 2]. The final result of the calculations is written in the following form

$$H_z^{(4)}(\rho,\pm k_s) = H\,I_0(\zeta_0\rho),\; H_\rho^{(4)}(\rho,\pm k_s) = \pm i\frac{k_s}{\zeta_0}H\,I_1(\zeta_0\rho), \tag{22}$$

where H – constant; $\zeta_0 = \sqrt{k_s^2 + i\omega r_0\mu_0}$; r_0 – Specific electric conductivity of the material of the central conductor of electric current.

The constants B, C, D, E, F and H are determined from the conditions of conjugation of fields on the boundaries of media sections with different electrical and magnetic properties. These conditions are written as follows

$$H_z^{(1)}(R,\pm k_s) - H_z^{*}(R,\pm k_s) = 0, \qquad \mu_0 H_\rho^{(1)}(R,\pm k_s) - \mu_1^\varepsilon H_\rho^{*}(R,\pm k_s) = 0,$$
$$H_z^{*}(r_1,\pm k_s) - H_z^{(3)}(r_1,\pm k_s) = 0, \qquad \mu_1^\varepsilon H_\rho^{*}(r_1,\pm k_s) - \mu_0 H_\rho^{(3)}(r_1,\pm k_s) = 0,$$
$$H_z^{(3)}(R_0,\pm k_s) - H_z^{(4)}(R,\pm k_s) = 0, \qquad H_\rho^{(3)}(R_0,\pm k_s) - H_\rho^{(4)}(R_0,\pm k_s) = 0 \tag{23}$$

Conditions (23) for the conjugation of fields form a non-homogeneous system of six algebraic equations, in which there are six unknown constants B, C, D, E, F and H. It is obvious that such a system of equations is solved in a unique way with respect to the unknown constants.

The solutions for the constants C and D, which determine the amplitude values of the components of the intensity vector of the alternating magnetic field in the volume of the ferromagnetic tube, have the form

$$C = \frac{\mu_0}{\mu_3^\varepsilon} I^* N W_\kappa(\ell,R,k_s)\frac{C_{11}}{(k_sR)\Delta_0}, \quad D = -\frac{\mu_0}{\mu_3^\varepsilon} I^* N W_\kappa(\ell,R,k_s)\frac{D_{11}}{(k_sR)\Delta_0}, \tag{24}$$

where Δ_0, C_{11} and D_{11} – determinants of the following matrices

$$\Delta_0 = \begin{Vmatrix} -K_0(k_sR) & -I_0(\zeta R) & -K_0(\zeta R) & 0 & 0 & 0 \\ \frac{\mu_0}{\mu_3^\varepsilon}K_1(k_sR) & -\frac{k_s}{\zeta}I_1(\zeta R) & \frac{k_s}{\zeta}K_1(\zeta R) & 0 & 0 & 0 \\ 0 & I_0(\zeta r_1) & K_0(\zeta r_1) & -I_0(k_s r_1) & -K_0(k_s r_1) & 0 \\ 0 & \frac{k_s}{\zeta}I_1(\zeta r_1) & -\frac{k_s}{\zeta}K_1(\zeta r_1) & -\frac{\mu_0}{\mu_3^\varepsilon}I_1(k_s r_1) & \frac{\mu_0}{\mu_3^\varepsilon}K_1(k_s r_1) & 0 \\ 0 & 0 & 0 & I_0(k_sR_0) & K_0(k_sR_0) & -I_0(\zeta_0R_0) \\ 0 & 0 & 0 & I_1(k_sR_0) & -K_1(k_sR_0) & -\frac{k_s}{\zeta_0}I_1(\zeta_0R_0) \end{Vmatrix};$$

$$C_{11}=\begin{Vmatrix} K_0(\zeta r_1) & -I_0(k_s r_1) & -K_0(k_s r_1) & 0 \\ -\frac{k_s}{\zeta}K_1(\zeta r_1) & -\frac{\mu_0}{\mu_3^{\varepsilon}}I_1(k_s r_1) & \frac{\mu_0}{\mu_3^{\varepsilon}}K_1(k_s r_1) & 0 \\ 0 & I_0(k_s R_0) & K_0(k_s R_0) & -I_0(\zeta_0 R_0) \\ 0 & I_1(k_s R_0) & -K_1(k_s R_0) & -\frac{k_s}{\zeta_0}I_1(\zeta_0 R_0) \end{Vmatrix};$$

$$D_{11}=\begin{Vmatrix} I_0(\zeta r_1) & -I_0(k_s r_1) & -K_0(k_s r_1) & 0 \\ \frac{k_s}{\zeta}I_1(\zeta r_1) & -\frac{\mu_0}{\mu_3^{\varepsilon}}I_1(k_s r_1) & \frac{\mu_0}{\mu_3^{\varepsilon}}K_1(k_s r_1) & 0 \\ 0 & I_0(k_s R_0) & K_0(k_s R_0) & -I_0(\zeta_0 R_0) \\ 0 & I_1(k_s R_0) & -K_1(k_s R_0) & -\frac{k_s}{\zeta_0}I_1(\zeta_0 R_0) \end{Vmatrix}.$$

Substituting relations (24) into formula (7) for calculating the component $H_\rho^*(\rho,\pm k_s)$, and the result obtained – in the formula [7, expression 4] we obtain

$$\mu^*(\pm k_s)=\frac{(m_1-m_2)}{2}I_0\left\{RH_\rho^*(R,\pm k_s)\ -r_1H_\rho^*(r_1,\pm k_s)+\right.$$

$$\left.+\int_{r_1}^{R}\rho\left[\frac{1}{\rho}H_\rho^*(\rho,\pm k_s)+\frac{\partial H_\rho^*(\rho,\pm k_s)}{\partial\rho}\mp i\,k_sH_z^*(\rho,\pm k_s)\right]d\rho\right\},$$

to determine the integral image $\mu^*(\pm k_s)$ of the linear density of external torque, we obtain an expression for calculating the amplitudes $\Phi^{(\pm)}$ of the angles of rotation of the cross sections in the front of a non-dispersive torsional wave

$$\Phi^{(\pm)}=\Phi_0W_\kappa(\ell,R,k_s)W_{вт}(k_s,\Pi),\qquad(25)$$

where Φ_0 – absolute sensitivity of the EMAT model in the excitation mode of non-dispersive torsional waves; $W_\kappa(\ell,R,k_s)$ – wave characteristic of the source of an alternating magnetic field or the coefficient of interference losses; $W_{вт}(k_s,\Pi)$ – coefficient of loss of excitation efficiency of torsional waves, caused by eddy currents (skin effect); The symbol Π in the list of arguments denotes a set of geometric and physicomechanical parameters of the material of the hollow ferromagnetic rod and the central conductor of the electric current.

Absolute sensitivity Φ_0 is determined by the following expression

$$\Phi_0=\frac{(m_1-m_2)\mu_0}{4\mu_1^{\varepsilon}\left[GJ_p\right]^B}I_0I^*R^2N\ .$$

In the case when $m_1=0,5\ H/m$; $m_2=-0,25\ H/m$; $\mu_1^{\varepsilon}=30\,\mu_0$; $I_0=10\ A$; $I^*=1\ A$; $R=2\cdot10^{-3}\ m$; $r_1=10^{-3}\ m$; $G=70\ GPa$ and $N=10$, absolute sensitivity $\Phi_0=6,063\cdot10^{-6}\ rad$, which is $1,25$ arcsec.

The coefficient of eddy current losses $W_{вт}(k_s,\Pi)$ is calculated by the formula

$$W_{вт}(k_s,\Pi)=\frac{1}{(k_sR)\Delta_0}\left\{\frac{1}{\zeta R}\left[C_{11}I_1(\zeta R)+D_{11}K_1(\zeta R)\right]-\right.$$

$$\left.-\frac{(r_1/R)^2}{\zeta r_1}\left[C_{11}I_1(\zeta r_1)+D_{11}K_1(\zeta r_1)\right]+\frac{1}{R}\int_{r_1}^{R}\rho\,div\vec{H}^*(\rho,\pm k_s)\,d\rho\right\}.\qquad(26)$$

Fig. 2 shows the results of calculations using the formula (26) of the frequency-dependent change of the coefficient $W_{вт}(k_s,\Pi)$ of the excitation efficiency losses of torsional waves due to the skin effect. On the abscissa axes in Fig. 2, *a* and Fig. 2, *b* the values of the dimensionless wave number k_sR are set apart.

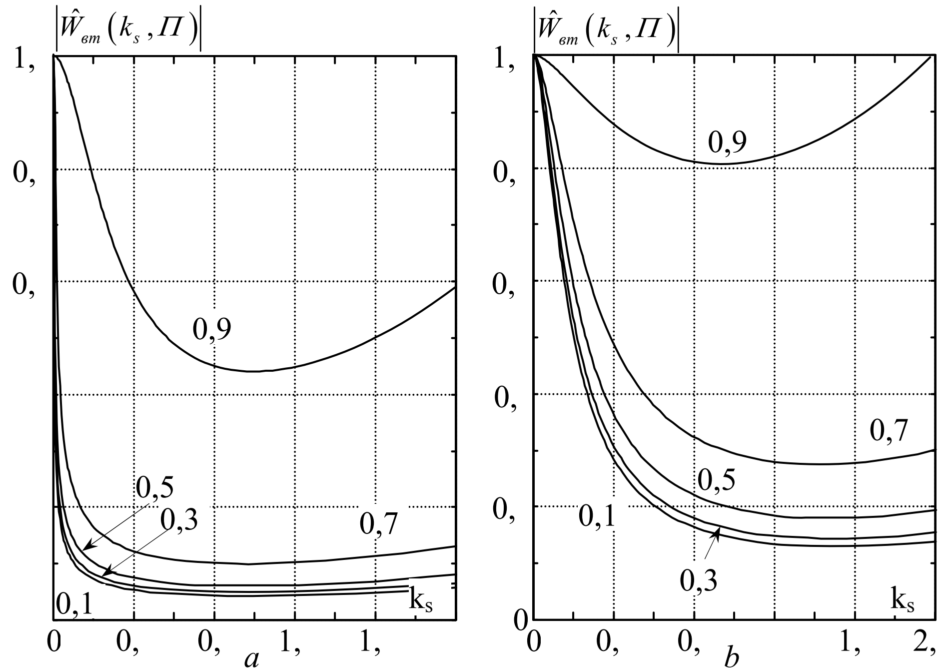

Fig. 2

At the speed of the shear waves $v_s = 3500\ m/s$ and the radius $R = 2 \cdot 10^{-3}\ m$ ((these values were taken when performing the calculations, the results of which are shown in Figure 2), the value $k_s R = 1$ corresponds to a cyclic frequency $f = 278{,}5\ kHz$. In order to avoid division by 0, calculations begin with a value $k_s R = 0{,}0001$, that is, from three hertz. On the ordinate axes in Fig. 2 the modules of the normalized value of the coefficient $W_{вт}(k_s, \Pi)$ are set apart. At the same time $\left|\hat{W}_{вт}(k_s, \Pi)\right| = \left|W_{вт}(k_s, \Pi)/W_0\right|$, where $W_0 = W_{вт}(k_s, \Pi)$ at the value $k_s R = 0{,}0001$. The calculations were made for the following set of ameters: the specific electrical conductivity of the central conductor of the electric current $r_0 = 60\ MS/м$; radius of the conductor $R_0 = 0{,}05R$; magnetic permeability of a ferromagnetic rod $\mu_1^{\varepsilon} = \mu_3^{\varepsilon} = 30\mu_0$ ($\mu_0 = 4\pi \cdot 10^{-7}\ H/m$); thus $div\,\vec{H}^*(\rho, \pm k_s) = 0$ the integral in formula (26) is also equal to 0. The specific electric conductivity r_2 of the rod was assumed to be equal to $14{,}3\ MS/m$, which approximately corresponds to the electrical conductivity of nickel. The graphs constructed with this value of conductivity are shown in Fig. 2, *a*. The data in Fig. 2, *b* were calculated at $r_2 = 1\ S/m$, which roughly corresponds to the electrical conductivity of magnetostrictive ferrites. The variable parameter of the family of curves shown in Fig. 2 is the relative radius of the cavity of the ferromagnetic rod, that is, the value r_1/R. The numerical values of the ratio r_1/R are indicated in the field of the figures near the corresponding curves. The values of the normalizing factors W_0 for the same values of the parameters are shown in Table 1. For magnetostrictive ferrites, i.e., in the case of conductivity $r_2 = 1\ S/m$, the real parts of the normalizing factor W_0 in the range of values $r_1/R \le 0{,}55$ are insignificant (in the third decimal place) differ from the values shown in Table 1. In the range $r_1/R > 0{,}60$ the real parts of the normalizing factor W_0 for the ferrite rod completely coincide with the values shown in Table 1. The imaginary parts of the normalizing factor are eight orders of magnitude smaller.

It follows from the foregoing analysis that in hollow ferromagnetic rods with thick walls substantial eddy currents are induced which already at comparatively low frequencies substantially reduce the amplitude values of the radial component $H_\rho^*(\rho, z)$ of the alternating magnetic field in the volume and on the surface of the hollow ferromagnetic rod. A decrease in the values $H_\rho^*(\rho, z)$ is accompanied by an increase in the domain where noticeable values of the radial components of the alternating magnetic field can be observed.

All this leads to a sharp decrease in the linear density of the external torques as the frequency of the change in the sign of the alternating magnetic field increases. The rate of decrease in the linear density of external torque and, as a consequence, the angles of rotation of the cross sections of the hollow ferromagnetic rod in the front of the torsional wave decreases in the frequency range, which corresponds to the values $k_s R > 1$. For ferromagnetic bars with thin walls in this frequency range, there is even a slight increase in the numerical values of the coefficient $W_{вт}(k_s, П)$.

Fig. 3 shows the graphs of the modulus of the function $W_{эап}(k_s, П) = W_к(\ell, R, k_s) W_{вт}(k_s, П)$, which takes into account all the losses of the excitation efficiency of torsional waves by a transducer, the design scheme of which is shown in Fig. 1. The function $W_{эап}(k_s, П)$ will be called the transfer (frequency) characteristic of the EMAT in the excitation mode of non-dispersive torsional waves.

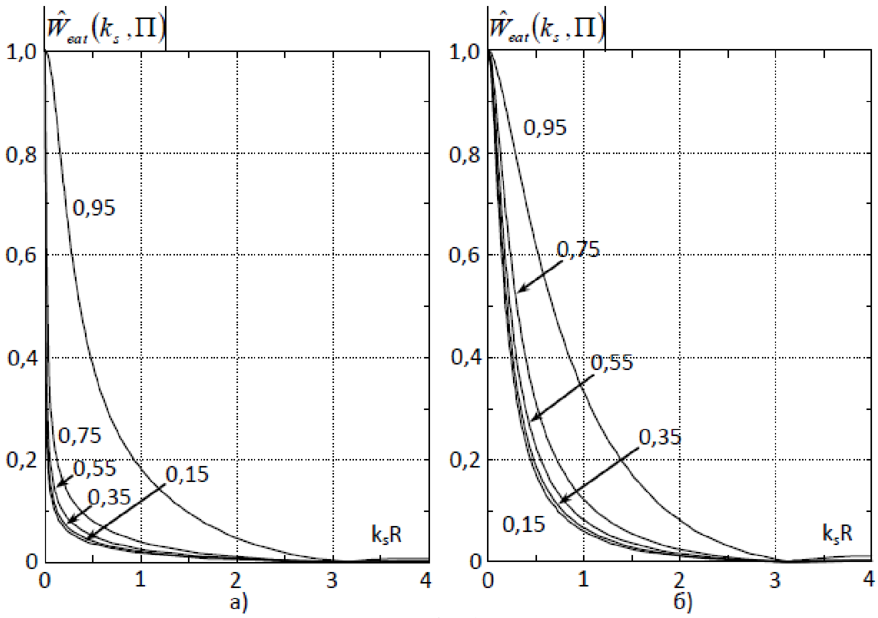

Fig. 3

The ordinate axes in Fig. 3 shows the normalized values of the transfer characteristic modulus, i.e., the values $\left|\hat{W}_{эап}(k_s, П)\right| = \left|W_{эап}(k_s, П)/W_0\right|$, where W_0 – the normalizing factor. The calculations were performed for the values of the physicomechanical parameters of the ferromagnetic rod and the central conductor of the electric current, the radius of the cross section of which is $R_0 = 0{,}1R$ ($R = 2 \cdot 10^{-3}$ m).The geometrical parameters of the coil – the source of the alternating magnetic field have the following values: $\ell = R$; $R_1 = 1{,}05R$ and $R_2 = 1{,}55R$. The variable parameter of the family of curves in Fig. 3 is the relative radius r_1/R of the hollow ferromagnetic rod, the numerical values of which are indicated in the field of the figures near the corresponding curves. Fig. 3, *a* shows graphs $\left|\hat{W}_{эап}(k_s, П)\right|$ for a hollow rod of nickel $r_2 = 14{,}3$ MS/m. Fig. 3, *b* shows the results of similar calculations for a hollow rod made of magnetostrictive ferrite ($r_2 = 1$ S/m).

From the analysis of the data shown in Fig. 3, it follows that the modulus of the transfer characteristic of a pass-through EMAT of a simple design reaches a maximum value at $\omega \to 0$. An increase in the values of the frequency of the change in the sign of the alternating magnetic field is accompanied by a sharp decrease in the absolute values of the function $W_{эап}(k_s, П)$, provided that $k_s \to \infty$ ($\omega \to \infty$) the numerical values

Table 1

Numerical values of the normalizing factor W_0 for different values of the ratio r_1/R ($r_2 = 14{,}3$ MS/m)

r_1/R	Re W_0	ImW_0
0,10	14,809	-0,677
0,15	14,625	-0,645
0,20	14,367	-0,603
0,25	14,034	-0,553
0,30	13,627	-0,497
0,35	13,144	-0,432
0,40	12,586	-0,375
0,45	11,952	-0,314
0,50	11,243	-0,255
0,55	10,458	-0,201
0,60	9,597	-0,151
0,65	8,661	-0,109
0,70	7,649	-0,073
0,75	6,562	-0,045
0,80	5,400	-0,025
0,85	4,163	-0,011
0,90	2,850	-0,003
0,95	1,463	-0,001

are $\left|W_{эап}\left(k_s, \Pi\right)\right| \to 0$.

Analysis of simulation results shown in Fig. 2 and Fig. 3, indicate that a simplified model of a passing through transducer, Fig. 1 will not be matched with the EMAT, which carries out the registration of ultrasonic waves. This is due to the fact that the potential difference at the receiver's electrical output is directly proportional to the rate of change of the magnetic flux through the electrical circuit of the receiver. The level of the electrical signal at the output of the receiver is directly proportional ω, that is, the circular frequency of the change in the sign of the magnetic flux. There is a contradiction – on the one hand, in the low-frequency region the electromagnetic-type receiver has a low sensitivity, which tends to be 0 when $\omega \to 0$, on the other hand, in the same frequency range the transfer characteristic of the radiating passing through transducer has maximum values. Obviously, the end-to-end transmission ratio from the electrical input to the electrical output of the ultrasonic device will have low values over the entire frequency range. First of all, it concerns the high-frequency range, which is most often used for practical measurements and diagnostics.

Rational model of EMAT for control of tubular metal rod.

The emerging contradiction between radiation and the reception of torsional vibrations at high frequencies is due to the fact that the torsional oscillations of the adjacent sections of the product are realized in two opposite directions. This leads, when using the EMAT model considered earlier, to the total information signal tending to 0.

It follows from the foregoing that it is possible to realize the maximum transmission characteristic of the radiator at a frequency $\omega \neq 0$ by using two coils of the transducer, which are opposite in the magnetic field. Let us consider the model of the passing through EMAT, Fig. 4, which consists of two identical coils (positions 1 and 2). Катушки включены встречно по магнитному полю The coils are turned on opposite by the magnetic field. Coil 1 generates rotational torque with linear density $-\mu^*\left(z_1\right)e^{i\omega t}$, and coil 2 at the same time generates rotational torque $\mu^*\left(z_2\right)e^{i\omega t}$. The symbols z_1 and z_2 denotes axial axes of local coordinate systems, whose origins are at the points O_1 and O_2 and sagittal planes of the coils.

The angle of rotation $\Phi^{(\pm)}$ of the cross section of a hollow, polarized in the circumferential direction of the ferromagnetic rod will, with allowance for the previously derived relationships, be determined by the following expression

$$\Phi^{(\pm)} = -\frac{i}{2k_s\left[GJ_p\right]^B}\int_{-\infty}^{\infty}\left[-\mu^*\left(z_1\right) + \mu^*\left(z_2\right)\right]e^{\pm ik_s z}dz, \qquad (27)$$

where z – axial axis of a cylindrical coordinate system, whose origin is at the point O of the symmetry plane of the computational model (Fig. 4).

The symbol d denotes half the distance between the coils, then $z_1 = z + \left(\ell + d\right)$ and $z_2 = z - \left(\ell + d\right)$. Substituting into the formula (27) the values of the coordinate z, determined through the local coordinates z_1 and z_2, we obtain

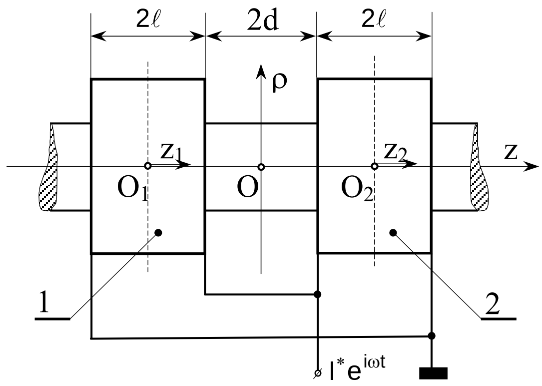

Fig. 4

$$\Phi^{(\pm)} = -\frac{i}{2k_s\left[GJ_p\right]^B}\left\{-e^{\mp ik_s(\ell+d)}\int_{-\infty}^{\infty}\mu^*\left(z_1\right)e^{\pm ik_s z_1}dz_1 + \right.$$

$$\left. + e^{\pm ik_s(\ell+d)}\int_{-\infty}^{\infty}\mu^*\left(z_2\right)e^{\pm ik_s z_2}dz_2\right\}, \qquad (28)$$

Since

$$-\frac{i}{2k_s\left[GJ_p\right]^B}\int_{-\infty}^{\infty}\mu^*\left(z_1\right)e^{\pm ik_s z_1}dz_1 = -\frac{i}{2k_s\left[GJ_p\right]^B}\int_{-\infty}^{\infty}\mu^*\left(z_2\right)e^{\pm ik_s z_2}dz_2 = \pm\Phi_0 W_{эап}\left(k_s, \Pi\right).$$

Then the expression (28) can be transformed by simple transformations to the form

$$\Phi^{(\pm)} = \pm \Phi_0 W^{(2)}_{эап}\left(k_s, \Pi\right) e^{\pm i\pi/2}, \qquad (29)$$

where $W^{(2)}_{эап}\left(k_s, \Pi\right) = 2\sin\left[k_s\left(\ell + d\right)\right] W_{эап}\left(k_s, \Pi\right)$ – transfer characteristic of two oppositely connected coil of passing through transducers of electromagnetic type in a mode of excitation of non-dispersive torsional waves.

The limiting transition shows that when $k_s = 0$ ($\omega = 0$) the transfer characteristic of the radiator is $W^{(2)}_{эап}\left(0, \Pi\right) = 0$. This allows us to state that the one shown in Fig. 4 variant of the passing through EMAT is self-consistent for cases of radiation and reception of torsional vibrations, since the effect of electromagnetic-acoustic transformation is reversible. For the measurements and diagnostics, two transducers are not required.

Fig. 5 shows the transmission characteristics modules $W^{(2)}_{эап}\left(k_s, \Pi\right)$ for the nickel (Fig. 5, *a*) and ferrite (Fig. 5, *b*) hollow ferromagnetic rod. While doing calculations it was assumed $d = 0$, i.e. coils are located without a backlash along the axis z. All other parameters have the same values that were used in building the curves in Fig. 3. The variable parameter of the family of curves is the dimensionless radius r_1/R of the hollow of the ferromagnetic rod. The curves in Fig. 5, *a* correspond to the values $r_1/R = 0{,}15$; $0{,}35$; $0{,}55$ and $0{,}75$ merge into one curve, i.e., they are indistinguishable within the resolution of the graphical representation of the results of computer simulation.

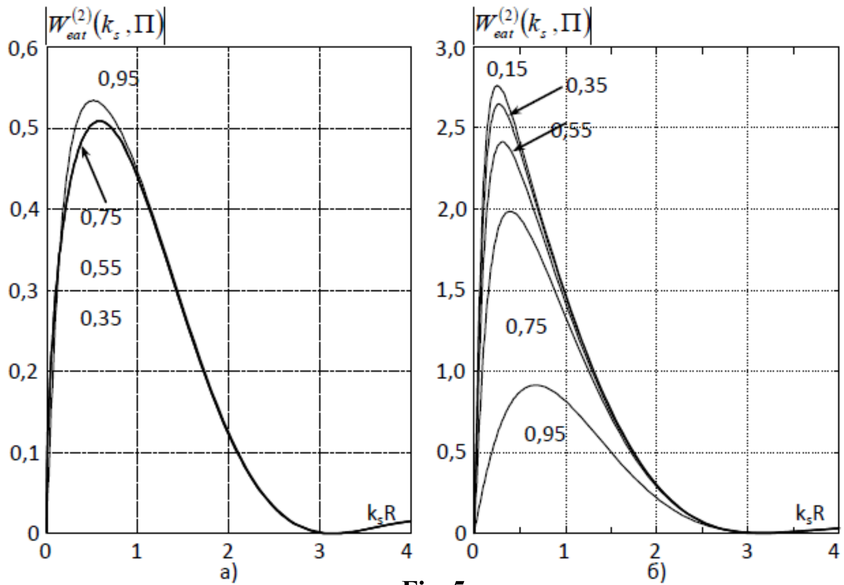

Fig. 5

From the analysis of results presented in Fig. 5, it follows that by manipulating the geometric parameters of a model of two oppositely connected identical coils, it is possible to control the position of the maximum value of the transfer characteristic $W^{(2)}_{эап}\left(k_s, \Pi\right)$ on the frequency axis (dimensionless wave numbers $k_s R$).

It is also obvious that an increase in the amount of pairs of coils in the EMAT (Fig. 4) included in the received variable magnetic field (information signal) can further increase its sensitivity.

Conclusions

1. The solution of the differential equation of forced torsional oscillations in a hollow electrically conductive ferromagnetic rod (tube), previously magnetized in the circumferential direction, is found in the form of an expression for the linear density of external torques. An expression is obtained for calculating the amplitudes of the angles of the rotations of the cross sections in the front of a traveling non-dispersive torsional wave through absolute sensitivity, the coefficient of interference losses, and the coefficient of loss of the excitation efficiency of torsional waves, caused by eddy currents (skin effect). The expression takes into account the complete set of geometric and physicomechanical parameters of the material of the hollow ferromagnetic rod, coils and the central conductor of the electric current, which allows designing electromechanical transducers taking into account the characteristics of the controlled tubular metal ware.

2. A rational physico-mathematical model of an electromagnetically-acoustic transducer is developed for excitation (reception) of torsional non-dispersive elastic oscillations. The basis of the model consists of two coils connected in a magnetic field and a magnetic field source in the form of a conductor with a current. The developed model takes into account the influence of the geometric dimensions of the coils of the converter and the product, their mutual arrangement, and also the physical and mechanical characteristics of the material of the metal product being studied. Converters of this type are intended for quality control, diagnostics, and measurement of physical and mechanical characteristics of tubular metal products.

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МОДЕЛЮВАННЯ ПРОЦЕСУ ЕЛЕКТРОМАГНІТНО-АКУСТИЧНОГО ПЕРЕТВОРЕННЯ ПРИ ЗБУДЖЕННІ КРУТИЛЬНИХ ХВИЛЬ. ЧАСТИНА 3

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Виконано математичне моделювання та розрахунок електромагнітних полів в електромагнітно-акустичному перетворювачі раціо-нальної конструкції при збудженні недиспергуючих крутильних хвиль у трубчастих електропровідних феромагнітних порожнистих стрижнях малого діаметра з урахуванням просторових, частотних, енергетичних і матеріальних чинників. Результати досліджень можуть бути використані для моделювання та конструювання збуджуючих ЕМАП для засобів вимірювань, контролю, діагностики в енергетичній, атомній, хімічній та інших галузях промисловості при ультразвукових дослідженнях феромагнітних виробів труб-частого типу. Бібл. 10, рис. 5.

Ключові слова: математичне моделювання, модель ультразвукового перетворювача, недиспергуючі крутильні хвилі, трубчастий виріб, скін-шар, втрати на перетворення.

УДК 620.179.16: 620.179.17

МОДЕЛИРОВАНИЕ ПРОЦЕССА ЭЛЕКТРОМАГНИТНО-АКУСТИЧЕСКОГО ПРЕОБРАЗОВАНИЯ ПРИ ВОЗБУЖДЕНИИ КРУТИЛЬНЫХ ВОЛН.ЧАСТЬ 3

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Выполнено математическое моделирование и расчет электромагнитных полей в электромагнитно-акустическом преобразователе рациональной конструкции при возбуждении недиспергирующих крутильных волн в трубчатых электропроводных ферромагнитных полых стержнях малого диаметра с учетом пространственных, частотных, энергетических и материальных факторов. Результаты исследований могут быть использованы для моделирования и конструирования возбуждающих ЭМАП для средств измерений, контроля, диагностики в энергетической, атомной, химической и других областях промышленности при ультразвуковых исследованиях ферромагнитных изделий трубчатого типа. Библ. 10, рис. 5.

Ключевые слова: математическое моделирование, модель ультразвукового преобразователя, недиспергирующие крутильные волны, трубчатое изделие, скин-слой, потери на преобразование.

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