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MAGNETIC FIELD CALCULATION OF BRUSHLESS DIRECT CURRENT MOTOR WITH SMOOTH STATOR BY SECONDARY SOURCES METHOD

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A mathematical model of a brushless direct current motor (BLCM) with high-corrosion permanent magnets is developed. It is based on the secondary sources method, and allowed the problem of calculating the magnetic permeability distribution in the engine ferromagnetic elements to reduce to solving a system of integral equations of the simple layer density and the volume density magnetic charges. An iterative method of finding the magnetic permeability is proposed taking into account the nonlinear dependence of the magnetic permeability. References 10, figures 3.
Key words**:** ac converter-fed motor, magnetic permeability, nonlinearity, secondary sources method.

The BLCM with high-corrosion permanent magnets are increasingly used in various areas of modern technology due to their reliability, due to the lack of slippery electrical contacts and good regulatory characteristics [1 – 3, 6]. Compared with commutator and asynchronous motors, the BLCM have a number of advantages such as: increased mechanical moment and power per unit volume, the possibility of continuous operation without overheating at low angular velocity with high mechanical moment on the shaft, increased overload capacity, good adjusting properties and dynamic characteristics [5, 7].

Let's consider a BLCM with an nonsalient pole stator and an salient-pole rotor (Fig. 1). Stator *1* and rotor *4* of the BLCM are made of a laminated soft magnetic material. The stator is a pipe along which the grooves for the winding are cut. Winding *2* has nine coils. The cylindrical rotor is placed inside the stator coaxial axis of symmetry. Eight permanent magnets are glued on the surface of the rotor.

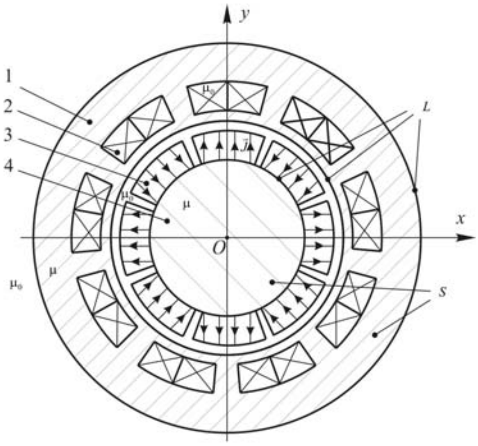

Рис. 1

Previously in work [10] was formulated a three-dimensional boundary value problem for calculating the characteristics of the magnetic field in the BLCM, taking into account the magnetic properties of the medium. To simplify the calculation, the magnetic field was taken as a plane-parallel, it was neglected hysteresis of ferromagnetic materials, and it was assumed that *B=B(H)*, where *B*, *H* is the induction and magnetic field strength. In the paper [8] on the basis of the secondary sources method, the boundary value problem of calculating the characteristics of a plane-parallel magnetic field in the BLCM, taking into account the nonlinearity of the magnetic characteristic of steel, was reduced to a system of integral equations for fictitious magnetic charges, which made it possible to narrow the search area of unknowns. In the kernel of the transformed integral equations there is a function $\nabla_Q \mu(Q)$, whose calculation is complicated by numerical solution of these equations. Therefore, in work [9] using the Green's identity and the properties of the potential of a simple layer of charge, the kernels of the integral equations are modified in the direction of reducing the components containing the function $\nabla_Q \mu(Q)$

$$\sigma(Q)-\frac{1}{\pi}\oint_L \sigma(M)K_1(M,Q)dL_M=\frac{1}{\pi}\int_S \rho(M)K_2(M,Q)dS_M+F^{\sigma}(Q)\,, \qquad (1)$$

$$\rho(Q)+\frac{1}{2\pi}\int_S \rho(M)K_3(M,Q)dS_M=-\frac{1}{2\pi}\oint_L \sigma(M)K_4(M,Q)dL_M-F^{\rho}(Q)\,, \qquad (2)$$

where

$$K_1(M,Q)=\lambda(Q)\frac{\vec{r}_{MQ}\vec{n}_Q}{r_{MQ}^2}-\frac{1}{L}\int_L \lambda(P)\frac{\vec{r}_{MP}\vec{n}_P}{r_{MP}^2}dL_P\,, \qquad K_2(M,Q)=\lambda(Q)\frac{\vec{r}_{MQ}\vec{n}_Q}{r_{MQ}^2}-\frac{1}{L}\int_L \lambda(P)\frac{\vec{r}_{MP}\vec{n}_P}{r_{MP}^2}dL_P-\frac{\pi}{L}\,,$$

$$F^{\sigma}(Q)=2\mu_0\left[\lambda(Q)\vec{H}^{(B)}(Q)\vec{n}_Q-\frac{1}{L}\int_L \lambda(P)\vec{H}^{(B)}(P)\vec{n}_P dL_P\right], \qquad K_3(M,Q)=\frac{\vec{r}_{MQ}\nabla_Q\mu(Q)}{\mu(Q)r_{MQ}^2}-\frac{1}{S}\int_L \ln\frac{\mu(P)}{\mu_0}\frac{\vec{r}_{MP}\vec{n}_P}{r_{MP}^2}dL_P+\frac{2\pi}{S}\ln\frac{\mu(M)}{\mu_0}\,,$$

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$$K_4(M,Q)=\frac{\vec{r}_{MQ}\nabla_Q\mu(Q)}{\mu(Q)r_{MQ}^2}-\frac{1}{S}\int_L \ln\frac{\mu(P)}{\mu_0}\frac{\vec{r}_{MP}\vec{n}_P}{r_{MP}^2}dL_P+\frac{\pi}{S}\ln\frac{\mu(M)}{\mu_0}+\frac{2\pi}{S}, \qquad F^{\rho}(Q)=\mu_0\left[\frac{\vec{H}^{(B)}(Q)\nabla_Q\mu(Q)}{\mu(Q)}-\frac{1}{S}\int_S\frac{\vec{H}^{(B)}(P)\nabla_P\mu(Q)}{\mu(P)}dS_P\right],$$

where $\sigma_M(Q)$ is the density of a simple layer of magnetic charges at the point Q of the boundary L of the ferromagnetic bodies; $\sigma_M(M)$ is similarly at the point M; $\lambda(Q)=[\mu(Q)-\mu_0]/[\mu(Q)+\mu_0]$, $\mu(Q)$ is the magnetic permeability at the point Q of the ferromagnetic medium, which is depended of the intensity of the magnetic field; μ_0 is magnetic permeability of the external medium to the ferromagnetic bodies, $\mu_0=4\pi\cdot10^{-7}$ Гн/м; $\vec{r}_{QM}$ is the position vector, which is directed from the integration point M to the observation point Q; $\vec{n}_Q$ is normal to the boundary L, which is directed from the ferromagnetic bodies to the outside; $\rho_M(Q)$ is the volume density of magnetic charges at the point Q of cross-section S of ferromagnetic bodies; $\vec{H}^{(B)}(Q)$ is the magnetic field intensity created as permanent magnets of the rotor, and the currents in the windings of the stator.

The kernel of the integral equation (2) includes the function $\nabla_Q\mu(Q)$. The advantage of the secondary sources method is that this function can be explicitly expressed through the density of the magnetic field sources.

Consider the method of calculation of the function $\nabla_Q\mu(Q)$. If the function $\mu(Q)=\mu(H(Q))$ is known, then

$$\nabla_Q\mu(Q)=\frac{\partial\mu}{\partial H}\left[\frac{\partial H(r_Q,\alpha_Q)}{\partial r_Q}\vec{e}_r(Q)+\frac{\partial H(r_Q,\alpha_Q)}{r_Q\partial\alpha_Q}\vec{e}_\alpha(Q)\right], \tag{3}$$

where the magnetic field intensity of the magnetic field sources is determined by the expression

$$\vec{H}(Q)=\frac{1}{2\pi\mu_0}\oint_L\sigma(M)\frac{\vec{r}_{MQ}}{r_{MQ}^2}dL_M+\frac{1}{2\pi\mu_0}\int_S\rho(M)\frac{\vec{r}_{MQ}}{r_{MQ}^2}dS_M+\frac{1}{2\pi}\oint_{L_M}\sigma_M(M)\frac{\vec{r}_{MQ}}{r_{MQ}^2}dL_M+\frac{1}{2\pi}\int_{S_W}\frac{\vec{\delta}_W(M)\times\vec{r}_{MQ}}{r_{MQ}^2}dS_M, \tag{4}$$

where $\sigma(M)$ is the simple layer density of magnetic charges; $\rho(M)$ is the volume density of magnetic charges; $\sigma_M(M)$ is simple layer of magnetic charges inputed on the boundary of permanent magnets for their replacement in the calculation model; $\vec{\delta}_W(M)$ – current density in windings of the stator coil.

The derivative $\partial\mu/\partial H$ is evaluated by dependence $\mu(H)$. For it we can use one of the mathematical models of the magnetization curve [4]. To calculate the partial derivatives of the relation (3), we will take into account that the magnetic field intensity module through its components is expressed as

$$H(r_Q,\alpha_Q)=\sqrt{H_r^2(r_Q,\alpha_Q)+H_\alpha^2(r_Q,\alpha_Q)}.$$

Thus we find

$$\frac{\partial H(r_Q,\alpha_Q)}{\partial r_Q}=\frac{1}{H(r_Q,\alpha_Q)}\left(H_r(r_Q,\alpha_Q)\frac{\partial H_r(r_Q,\alpha_Q)}{\partial r_Q}+H_\alpha(r_Q,\alpha_Q)\frac{\partial H_\alpha(r_Q,\alpha_Q)}{\partial r_Q}\right), \tag{5}$$

$$\frac{\partial H(r_Q,\alpha_Q)}{r_Q\partial\alpha_Q}=\frac{1}{r_QH(r_Q,\alpha_Q)}\left(H_r(r_Q,\alpha_Q)\frac{\partial H_r(r_Q,\alpha_Q)}{\partial\alpha_Q}+H_\alpha(r_Q,\alpha_Q)\frac{\partial H_\alpha(r_Q,\alpha_Q)}{\partial\alpha_Q}\right). \tag{6}$$

We find partial derivatives in (5), (6), using explicit expressions for the magnetic field intensity (4). We consider first the case when the intensity of the magnetic field is calculated from a simple layer of magnetic charges (componentally in a cylindrical coordinate system)

$$H_r^\sigma(Q)=\frac{1}{2\pi\mu_0}\oint_L\sigma(M)\frac{r_Q-r_M\cos(\alpha_Q-\alpha_M)}{r_{MQ}^2}dL_M, \quad H_r^\sigma(Q)=\frac{1}{2\pi\mu_0}\oint_L\sigma(M)\frac{r_M\sin(\alpha_Q-\alpha_M)}{r_{MQ}^2}dL_M. \tag{7}$$

Then, by performing simple transformations, we find

$$\frac{\partial H_r^\sigma(r_Q,\alpha_Q)}{\partial r_Q}=\frac{1}{2\pi\mu_0}\oint_L\sigma(M)P_{rr}(Q,M)dL_M, \qquad \frac{\partial H_\alpha^\sigma(r_Q,\alpha_Q)}{\partial r_Q}=\frac{1}{2\pi\mu_0}\oint_L\sigma(M)P_{\alpha r}(Q,M)dL_M,$$

$$\frac{\partial H_r^\sigma(r_Q,\alpha_Q)}{\partial\alpha_Q}=\frac{1}{2\pi\mu_0}\oint_L\sigma(M)P_{r\alpha}(Q,M)dL_M, \qquad \frac{\partial H_\alpha^\sigma(r_Q,\alpha_Q)}{\partial\alpha_Q}=\frac{1}{2\pi\mu_0}\oint_L\sigma(M)P_{\alpha\alpha}(Q,M)dL_M,$$

where $P_{rr}(Q,M)=-\frac{r_Q^2-2r_Mr_Q\cos(\alpha_Q-\alpha_M)+r_M^2\cos(2(\alpha_Q-\alpha_M))}{r_{MQ}^4}$, $P_{\alpha r}(Q,M)=-\frac{2r_M\sin(\alpha_Q-\alpha_M)[r_Q-r_M cos(\alpha_Q-\alpha_M)]}{r_{MQ}^4}$,

$$P_{r\alpha}(Q,M)=\frac{r_M(r_M^2-r_Q^2)\sin(\alpha_Q-\alpha_M)}{r_{MQ}^4}, \qquad P_{\alpha\alpha}(Q,M)=-\frac{r_M(2r_Mr_Q-(r_M^2+r_Q^2)\cos(\alpha_Q-\alpha_M))}{r_{MQ}^2}.$$

We have analogous expressions for corresponding partial derivatives of the components of the magnetic field strength, which is created by permanent magnets and volume magnetic charges.

Let's consider the case when the magnetic field intensity is calculated from the currents of the coils (componentally in the cylindrical coordinate system)

$$H_r^{\delta}(Q)=\frac{1}{2\pi}\int_{S_W}\delta_W(M)\frac{-r_M\sin(\alpha_Q-\alpha_M)}{r_{MQ}^2}dS_M\ , \qquad H_{\alpha}^{\delta}(Q)=\frac{1}{2\pi}\int_{S_W}\delta_W(M)\frac{r_Q-r_M\cos(\alpha_Q-\alpha_M)}{r_{MQ}^2}dS_M\ .$$

After simple transformations we arrive to the expression

$$\frac{\partial H_r^{\delta}(r_Q,\alpha_Q)}{\partial r_Q}=-\frac{1}{2\pi}\oint_{S_W}\delta_W(M)P_{\alpha r}(Q,M)dS_M\ , \qquad \frac{\partial H_{\alpha}^{\sigma}(r_Q,\alpha_Q)}{\partial r_Q}=\frac{1}{2\pi}\oint_{S_W}\delta_W(M)P_{rr}(Q,M)dS_M\ ,$$

$$\frac{\partial H_r^{\sigma}(r_Q,\alpha_Q)}{\partial \alpha_Q}=-\frac{1}{2\pi}\oint_{S_W}\delta_W(M)P_{\alpha\alpha}(Q,M)dS_M\ , \qquad \frac{\partial H_{\alpha}^{\sigma}(r_Q,\alpha_Q)}{\partial \alpha_Q}=\frac{1}{2\pi}\oint_{S_W}\delta_W(M)P_{r\alpha}(Q,M)dS_M\ .$$

If we know the distribution of the simple layer density $\sigma(Q)$ of magnetic charges on the boundary of ferromagnetic bodies, the volume density $\rho(Q)$ of magnetic charges in the cross section of massive conductors, the distribution of the current density $\delta_W(Q)$ in the coils of the winding, the distribution of the magnetic charges density $\sigma_M(Q)$ on the boundary of the permanent magnets, then in accordance with the above relations can be calculated partial derivatives (5), (6) and, accordingly, the function $\nabla_Q\mu(Q)$ by the formula (3). Taking into account that the latter function is explicitly expressed in terms of the densities of the magnetic field sources, this, on the one hand, greatly simplifies the formation of the kernel of the integral equation (2), and, on the other hand, raises the accuracy of its calculation in the transition to the finite-dimensional analogue for a numerical solution of the system of equations (1), (2).

Based on the developed mathematical model, an iterative method for finding the magnetic permeability in the cross section of ferromagnetic bodies taking into account the nonlinear dependence characteristic was developed.

Calculation example. Consider the calculation of the permanent magnets' field in the electric motor (Fig. 2) on the basis of the developed mathematical model: *1* – stator winding; *2* – stator; *3* – homogeneously magnetized permanent magnets; *4* – rotor shaft. The magnetic properties of the ferromagnetic medium (shown in Fig. 2 by numbers *2* and *4*) are represented by the dependence $B(H)$. In Fig. 3 is shown the result of the calculation of the *r*-components magnetic induction in the gap between the permanent magnets and the interior of the stator, taking into account (solid line) and without taking into account (dotted line) the nonlinear properties of the ferromagnetic materials.

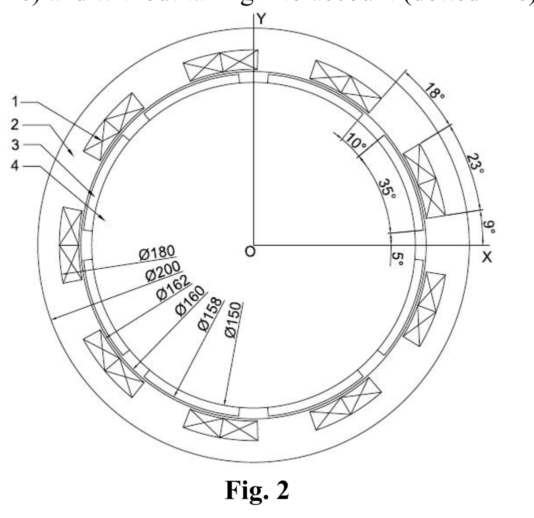

Fig. 2

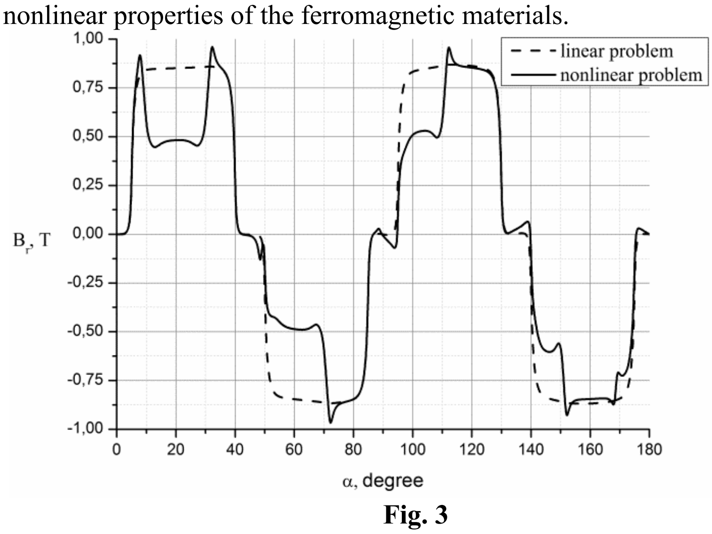

Fig. 3

As we see from the submitted results, in zones corresponding to the location of jumper placement we have to take into account the nonlinear properties of ferromagnetic materials (in particular, the jumper ones themselves). In the same way, a comparative analysis of the calculations performed with the same task by means of the software product COMSOL Multiphysics was made, the average deviation was 3.5%**.**

Thus, an iterative method for finding the magnetic permeability with a nonlinear characteristic is developed, which is based on the reduction of the problem of determining the characteristics of the magnetic field to the solution of the system of nonlinear integral equations for the density of a simple layer and the density of volume magnetic charges.

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РАСЧЕТ МАГНИТНОГО ПОЛЯ МАГНИТОЭЛЕКТРИЧЕСКОЙ МАШИНЫ С ГЛАДКИМ СТАТОРОМ МЕТОДОМ ВТОРИЧНЫХ ИСТОЧНИКОВ

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Разработана математическая модель вентильного двигателя с высококоэрцитивными постоянными магнитами, в основе которой лежит метод вторичных источников, который позволил задачу определения распределения магнитной проницаемости в ферромагнитных элементах двигателя свести к решению системы интегральных уравнений для плотности простого слоя и плотности объемных магнитных зарядов. Предложен итерационный метод нахождения магнитной проницаемости с учетом её нелинейной зависимости. Библ. 10, рис. 3.

Ключевые слова: вентильный двигатель, магнитная проницаемость, метод вторичных источников, нелинейность

РОЗРАХУНОК МАГНІТНОГО ПОЛЯ МАГНІТОЕЛЕКТРИЧНОЇ МАШИНИ ІЗ ГЛАДКИМ СТАТОРОМ МЕТОДОМ ВТОРИННИХ ДЖЕРЕЛ

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Розроблено математичну модель вентильного двигуна з висококоерцитивними постійними магнітами, в основі якої лежить метод вторинних джерел, що дав змогу завдання визначення розподілу магнітної проникності у феромагнітних елементах двигуна звести до розв'язання системи інтегральних рівнянь для густини простого шару й густини об'ємних магнітних зарядів. Запропоновано ітераційний метод знаходження магнітної проникності з урахуванням її нелінійної залежності. Бібл. 10, рис. 3.

Ключові слова: вентильний двигун, магнітна проникність, нелінійність, метод вторинних джерел.

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