

**STUDY OF CHANGED MAIN FLUX REACTANCE OF SQUIRREL-CAGE INDUCTION MOTORS USING FIELD ANALYSIS OF THEIR STARTING CHARACTERISTICS**

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*The regularities in the change of the main flux reactance of squirrel-cage induction motors as slip functions are studied by quasi-3D field analysis to determine the equivalent parameters of equivalent circuit. The comparative analysis of the design conditions and calculated starting characteristics of the motors is carried out. As shown, the use of electromagnetic parameters which are equivalent to the parameters obtained by field model gives more high accuracy of calculation. As grounded, the coefficient of change of equivalent air gap is available and expedient to be taken in account for investigation of the motors. References 8, table 1, figures 2.*

**Key words:** induction motors, parameters of the equivalent circuit, field model, main flux reactance, start.

The study of starting characteristics is an important stage in design of induction motors (IM). The accuracy of calculation of these characteristics depends on the accuracy of parameters of the equivalent circuit of IM. The mathematical models of IM, which allow the separation of a magnetic flux of motor into both the main flux of mutual inductance and the leakage flux, are the most widespread for studying and designing of IM. Under this assumption, the path of mutual inductance flux as well as: the air gap, teeth and yokes of stator and rotor are already defined. In this case, the nonlinear properties of the electromagnetic parameters of IM are due to a changing in the main flux reactance, which depends on the total magnetomotive force (MMF) of the motor. Also, the consideration of different rotor parameters and stator leakage inductance for starting regime and design condition is very important [1,2]. These approaches give a good result at the IM design. At the same time, the errors in their calculations do not exceed a few percent, due to the application of both the tested constructive variants [1] and the great empirical experience. The errors of calculations for starting characteristics have a significantly great value. They can reach tens of percent at determination of the starting currents and torque even taking into account the starting parameters from the reference book [1].

The accurate calculation of the electromagnetic parameters for IM is an important factor in increasing the reliability of the analysis results of their starting characteristics. It should be noted that the structure of the equations of electric equilibrium is determined by the assumption about the independence in the fluxes of mutual inductance and leakage. It provides a small time and stability of the calculation process using available computing facilities, but it, also, to reduce the analysis accuracy due to the simplified accounting of electromagnetic communications in the motor. Field models of IM are constructed with smaller number of limiting assumptions and to provide greater accuracy of analysis, in comparison with circuit models [3,4]. But, their application takes significant amounts of computer memory and counting time. In this case, the specification of constructing the effective algorithms for studying the operating conditions and designing of IM becomes very complicated.

Thus, the specification of a refined analysis for both the operating and starting characteristics of IM should be solved within the framework of existing structures of circuit mathematical models with the determination of the electromagnetic parameters using mathematical or physical models without the above-mentioned bounding assumptions. Such an approach can lead to the equivalence of the field model by the circuit. In the analysis of squirrel-cage induction motors, such an equivalence is doing using the nonlinear dependences of electromagnetic parameters as the function both currents and slip. The work [5] is devoted to the possibility of determination such dependences using the results of physical experiment. The development of the equivalent circuit model of IM with two-dimensional electromagnetic parameters was carried out in [3,4] using the quasi-3D field analysis. Such an equating raises the accuracy of analysis of operating conditions, but the character of changing for the main flux reactance as the slip function does not coincide with the regularities by results of studying the circuit models of IM [2]. This needs to be proved.

**The purpose of the paper** is to find and ground the regularities in the change of the main flux reactance of squirrel-cage induction motors using field analysis for their further adaptation to study of operating conditions and design of the induction motors.

The mathematical model of IM that provides both the accuracy of field methods and speed of circular methods was developed in [6] using field analysis. The field analysis was carried by Comsol Multiphysics software package using the developed techniques and algorithms. It was obtained expressions for determining the parameters of equivalent circuit using quasi-3D analysis based on equations of balance for the powers in circuit of the phase windings of both stator and rotor [7]. The calculated value of the main flux reactance is obtained using field analysis in the cross plane of

the motor [7] (a model of quasi-statics with currents that are perpendicular to the plane of analysis) according to the following expression:

$$x_m = \frac{-P_{ec}}{3sI_s I_r^I}, \text{ here } P_{ec} = l_\delta \sum_{i=1}^{z_2} \left( \int_{sci} \frac{J_{mi}^2}{2\gamma_{c2}} ds \right); \quad I_r^R + jI_r^I = \sum_{i=1}^{z_2} \left[ \sqrt{2} e^{j(i-1)p\delta_k} \int_{sci} (J_{mi}^R + jJ_{mi}^I) ds \right] / (12W_s K_{o\delta}), \quad (1)$$

where  $P_{ec}$  is the power of electrical losses in all rotor bars;  $i=1, \dots, z_2$  is a serial number of the bar (losses in the bar  $i$  and determination by integrating over the area of the conductor  $sci$  according to represented expression);  $I_s$  is the effective value of time complex of current for the stator phase, that is specified at the field analysis with a zero initial phase;  $J_{mi}, J_{mi}^R, J_{mi}^I$  is the actual effective value of time complex of current density, its real and imaginary parts;  $I_r^R, I_r^I$  is real and imaginary components of complex for rotor current by the field of operating harmonic for MMF with the number of the poles pairs  $p$ , (MMF is reduced to parameters of stator);  $s$  is the slip;  $\gamma_{c2}$  is the specific conductivity of rotor winding;  $l_\delta$  is the length of the motor's magnetic circuit;  $K_{o\delta}, W_s$  is the winding coefficient and the number of stator phase turns;  $\delta_k = 2\pi / z_2$  is the angle between neighboring bars.

Estimation of adequacy of the developed mathematical model is carried out for rated and circuited conditions for motors 4A80A2U3, 4A80A4U3, 4A132M8U3. The result of analysis is shown in Table. Data from reference book [1] according to which an error  $\Delta$  is defined were accepted as a base for comparison.

Parameters of regimes		Notation, units of measurement	IM 4A80A2U3/4A80A4U3/4A132M8U3				
			Data from reference book	Calculation using parameters of equivalent circuit			
				Constant parameters (from reference book)	With losses in steel, mechanical and additional losses		
					Parameters obtained by circuit model $k_\mu, x_{1r}, x'_{2r}, r'_{2r}$	Parameters obtained by field model	
absolute value / $\Delta, \%$							
Rated (75°C/90°C/75°C)	Useful power	$P_{2n}, W$	1500/1100/5500	<u>1541/1113/5090</u> 2,73/1,18/7,45	<u>1503/1122/5467</u> 0,23/2,0/0,6	<u>1505/1094/5514</u> <b>0,33/0,55/0,25</b>	
	Stator current	$I_{1n}, A$	3,3/2,74/13,57	<u>3,012/2,6/11,96</u> 8,73/5,11/11,86	<u>3,222/2,769/13,36</u> 2,36/1,06/1,55	<u>3,326/2,766/13,5</u> <b>0,79/0,95/0,52</b>	
	Efficiency	$\eta_n$	0,81/0,75/0,83	<u>0,877/0,816/0,88</u> 8,27/8,8/6,02	<u>0,816/0,763/0,845</u> 0,74/1,73/1,81	<u>0,814/0,764/0,85</u> <b>0,49/1,87/2,41</b>	
	Mechanical power	$\cos \varphi_n$	0,85/0,81/0,74	<u>0,885/0,795/0,73</u> 4,12/1,85/1,35	<u>0,866/0,806/0,734</u> 1,88/0,49/0,81	<u>0,842/0,784/0,726</u> <b>0,94/3,2/1,89</b>	
	Slip	$s_n, \%$	0,042/0,054/0,041	<u>0,042/0,054/0,041</u> -/-/-	<u>0,042/0,054/0,041</u> -/-/-	<u>0,042/0,054/0,041</u> -/-/-	
	torque	$M_n, N \cdot m$	4,98/7,4/73,02	<u>5,121/7,49/67,58</u> 2,73/1,22/7,45	<u>4,99/7,55/72,58</u> 0,23/2,03/0,6	<u>5,001/7,36/73,21</u> <b>0,33/0,54/0,25</b>	
Starting	$v = v_p$	torque	$M_n, N \cdot m$	10,46/14,8/138,7	<u>12,45/14,52/113,4</u> 19,02/1,89/18,24	<u>11,47/15,22/134,6</u> 9,66/2,84/2,96	<u>10,4(50°C)/15(90°C)/11(50°C)</u> <b>0,19/1,49/19,97</b>
		current	$I_n, A$	21,45/13,7/74,64	<u>20,55/12,48/56,14</u> 4,2/8,91/24,79	<u>19,78/12,71/62,73</u> 7,79/7,23/15,96	<u>21,4(50°C)/13,8(90°C)/67,4(50°C)</u> <b>0,1/0,44/9,71</b>
	$v = v_p \times (1;5;7)$	torque	$M_n, N \cdot m$	-/-/-	-/-/-	<u>10,66/13,35/124,2</u> 1,91/8,45/10,45	<u>-/14,69(75°C)/132(50°C)</u> <b>-/0,74/4,83</b>
		current	$I_n, A$	-/-/-	-/-/-	<u>19,47/12,16/61,19</u> 9,23/11,24/18,02	<u>-/13,77(75°C)/71,7(50°C)</u> <b>-/0,51/3,86</b>

The comparative analysis is carried out according to the parameters of three mathematical models (using the simulation model [8]), which to realize: 1) the calculation of the equivalent circuit using reference parameters for rated and circuited conditions without taking into account losses in steel, mechanical and additional losses; 2) the calculation with parameters according to [2], taking into account the saturation of the main magnetic flux paths, the changes both the rotor active impedance and leakage parameters in the starting regime according to reference book [1]; 3) the calculation of parameters with nonlinear dependences based on quasi-3D field analysis [6]. The analysis of the IM regimes for the second and third models is made taking into account losses in steel, mechanical and additional losses. The value of the active impedances is determined using the operating temperature indicated in the table. The application of a mathematical model with parameters by the field analysis decreases the errors in modeling for operating conditions: to a level of 1% in the whole range of the slip variation for IM 4A80A2U3, 4A80A4U3; up to a level of 1% in design conditions and up to 4 ... 5% - in circuited conditions for IM 4A132M8U3 with increased proportion of spatial harmonics of MMF.

The use of variable parameters based on the field analysis decreases the error in determining the values both of the starting current and the torque by several times in comparison with the changing in the saturation factor as a func-

tion of the total MMF. This is take place due to taking into account regimes with large slips of changing of the path for the mutual inductance flux which crosses an air gap and passes partially through the rotor slot area with winding. In this case, the equivalent non-magnetic gap increases in comparison with the constant value of the Carter coefficient [2]. This coefficient of change of the equivalent non-magnetic gap will be indicated  $k'_\delta$ . We take it into account when the coefficient of changing for the main flux reactance from expressions  $x_m$ , (1) and  $x_{m0}$  (at constant parameters) was determined:

$$k_\mu k'_\delta = \frac{x_{m0}}{x_m} = \frac{3}{2} \omega_0 \frac{\mu_0 l_\delta \pi R}{x_m \delta k_\delta} f_{sp}^2 = -\frac{9s}{2} \omega_0 \frac{\mu_0 l_\delta \pi R}{\delta k_\delta} f_{sp}^2 \frac{I_s I_r^I}{P_{ec}}, \quad (2)$$

where  $\delta, R$  is the values of the air gap and the radius of the stator boring;  $f_{sp} = 2K_{o\delta} W_s / p\pi$  is the unit MMF of stator phase by the operating harmonic;  $\omega_0$  is the angular frequency of network;  $\mu_0$  is the magnetic constant;  $k_\mu$  is the saturation factor of the magnetic circuit according to [2], with the flow path of the mutual inductance.

The dependence of the change of the coefficient for the equivalent non-magnetic gap  $k'_\delta$  is determined by slip from (2) for IM 4A80A2U3. In this case, we use the results of calculation the starting characteristics by the simulation model of the IM of electromechatron systems under parameters using quasi-3D field analysis [8,6], and also use the dependences  $k_\mu$  from the total MMF of motor as in [2]. Fig. 1 shows the dependencies as the function of frequency of rotor rotation with the stationary current: 1 are values  $k_\mu k'_\delta$ , determined according to (2); 2 is  $k_\mu$  given in [2]; 3 is  $k'_\delta = k_\mu k'_\delta / k_\mu$  calculated from the model with two-dimensional parameters (as the function of slip and currents) using the field analysis, the sixth column of the table.

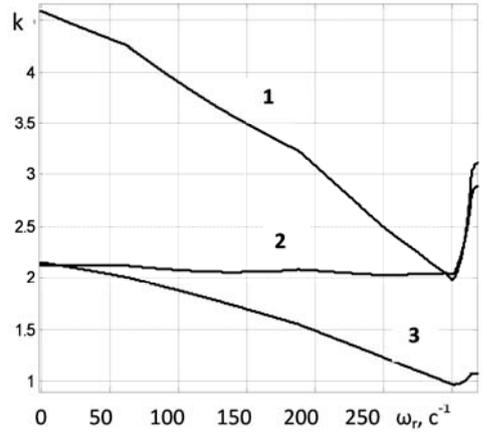


Fig. 1

The calculated results show that for the IM 4A80A2U3 coefficient changes from about 2.2 to 1, at slip changing from one to the nominal value, but its change is minimal at the further increase in speed.

The area of existence of reliable values of the product  $k_{\mu\delta} = k_\mu k'_\delta$  is investigated with expression (2). The analysis of this area lets substantiating the coefficient of equivalent non-magnetic gap  $k'_\delta$  at unit slip. The study was done using two-dimensional field analysis at plane of core laminations for short-circuit condition of motor 4A80A2U3. The results of the field analysis according to (2) are:  $I_s = 19,8 \text{ A}$ ;  $I_r^R + jI_r^I = (-19,48 - j0,367) \text{ A}$ ;  $P_{ec} = 2338 \text{ W}$ . The value  $k_{\mu\delta}$  depends on the imaginary component of the rotor current  $I_r^I$ , according to (2). This allows to make the necessary study with the given value of the real component of the rotor current  $I_r^R$ . The field of possible values of the rotor current will be reduced by the conditions:  $I_r < I_s$ ;  $x_{1n} > 0$ ;  $x'_{2n} > 0$ . A complex of lines is constructed in coordinates  $(I_r^R; k_{\mu\delta})$  for these conditions (fig. 2) that correspond to the constant values of the rotor current module. These isolines are indicated  $kI_s$ , where  $k=0.995\dots 0.97$ . Based on the field analysis of short-circuit conditions, the value  $x_m$  is less than in the operating condition. The value of coefficient  $k_{\mu\delta} = 2.48$  (a calculated point is indicated as CP according to data of the field analysis in fig. 2 testifies to it. The reducing  $k_{\mu\delta}$

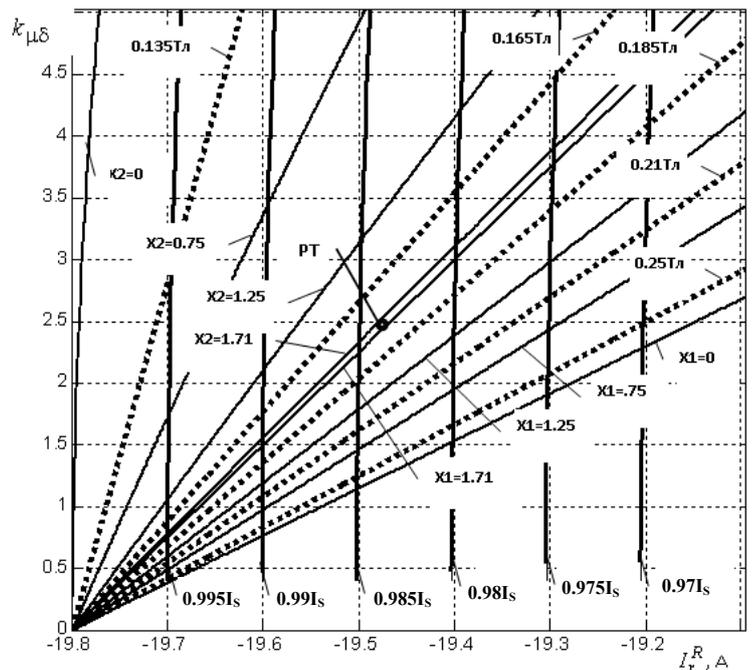


Fig. 2

to the nominal level is possible while increasing the rotor current at short-circuit condition. This does not correspond to the real relationship based on the field analysis. The regularities (fig. 2) were obtained by neglecting processes in the

frontal parts of the rotor. The parameters of the frontal parts significantly reduces the value  $x_m$ . Fig. 2 also shows the isolines of impedances of the stator and rotor leakage, the values of air gap induction. They are obtained from the data of the field analysis of the short-circuit condition using the expressions for parameters determination [6]. It is seen that at the same the impedance of the stator and rotor leakage, the isoline of the rotor current with the value from the field analysis ( $I_r = 19.484 = 0.984 I_s$ ) will cross isolines of the leakage impedances at the calculating point. The study showed the impossibility of substitution of the field model with a circuit in the case of traditional ratio of the electromagnetic parameters for IM, with minimum saturation factor in the starting regime and constant Carter coefficient.

**Conclusion.** As proved by field analysis of the starting characteristics of squirrel-cage induction motors, the value  $x_m$  decreases when the slip increases in distinction from the results for circuit models of the motors [2]. The use of equivalent (depending on slip and currents) nonlinear electromagnetic parameters of the motors provides high accuracy of simulation of starting characteristics by circuit methods. In this case, the field model is equivalent to the circuit model with the coefficient which characterizes the varied equivalent air gap. That promotes the effective improved algorithms to study the operating conditions and to design the induction motors using (depending on only MMF) electromagnetic parameters.

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УДК 621.313

#### ЗАКОНОМЕРНОСТИ ИЗМЕНЕНИЯ ГЛАВНОГО ИНДУКТИВНОГО СОПРОТИВЛЕНИЯ АСИНХРОННЫХ ДВИГАТЕЛЕЙ С КОРОТКОЗАМКНУТЫМ РОТОРОМ ПО РЕЗУЛЬТАТАМ ПОЛЕВОГО АНАЛИЗА ИХ ПУСКОВЫХ ХАРАКТЕРИСТИК

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По результатам квазитрехмерного полевого анализа с определением эквивалентных параметров схемы замещения исследованы закономерности изменения главного индуктивного сопротивления асинхронного двигателя с короткозамкнутым ротором в функции скольжения. Выполнен сравнительный анализ результатов расчетов пусковых характеристик, номинальных режимов асинхронных двигателей, показано повышение точности анализа при применении параметров, эквивалентизирующих полевую модель. Обосновано существование коэффициента изменения эквивалентного воздушного промежутка и целесообразность его учета при исследованиях. Библ. 8, табл. 1, рис. 2.

**Ключевые слова:** асинхронный двигатель, параметры схемы замещения, полевая модель, пуск.

УДК 621.313

#### ЗАКОНОМІРНОСТІ ЗМІНИ ГОЛОВНОГО ІНДУКТИВНОГО ОПОРУ АСИНХРОННИХ ДВИГУНІВ З КОРОТКОЗАМКНЕНИМ РОТОРОМ ЗА РЕЗУЛЬТАТАМИ ПОЛЬОВОГО АНАЛІЗУ ЇХНІХ ПУСКОВИХ ХАРАКТЕРИСТИК

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За результатами квазітривимірного польового аналізу з визначенням еквівалентних параметрів схеми заміщення досліджені закономірності зміни головного індуктивного опору асинхронного двигуна з короткозамкненим ротором у функції ковзання. Виконано порівняльний аналіз результатів розрахунків пускових характеристик, номінальних режимів асинхронних двигунів, показано зростання точності аналізу під час застосування параметрів, що еквівалентують польову модель. Обґрунтовано існування коефіцієнта зміни еквівалентного повітряного проміжка та доцільність його врахування при дослідженнях. Бібл. 8, табл. 1, рис. 2.

**Ключові слова:** асинхронний двигун, параметр схеми заміщення, польова модель, пуск.

Надійшла 05.03.2018  
Остаточний варіант 10.05.2018