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PROBABILISTIC PROPERTIES OF ELECTRICAL CHARACTERISTICS OF CAPACITOR CHARGE CIRCUIT WITH STOCHASTIC ACTIVE RESISTANCE

A.A. Shcherba[1]*, N.I. Suprunovska[1], D.S. Ivashchenko[2]**
[1] Institute of Electrodynamics National Academy of Sciences of Ukraine,
pr. Peremohy, 56, Kyiv, 03057, Ukraine. E-mail: iednat1@gmail.com
[2] Oracle, 1501 4th Ave, Seattle, WA 98101, US. E-mail: ivas-90@mail.ru

The approach to the determination of the probability properties (probability density function, probability distribution function, mathematical expectation) of the electrical characteristics of the circuits of electric discharge installations whose active resistance can be changed at random is proposed. It is assumed that such a stochastic resistance is characterized by a continuous random variable whose probabilistic properties are known. As an example, probabilistic properties of the voltage on a capacitor in a first-order circuit with a stochastic active resistance having a uniform probability distribution were investigated. References 10, figures 3.
Keywords: transient processes, stochastic resistance, random process, continuous probability distribution.

Introduction. The electric load resistance of electric discharge installations (EDI) using linear [1–3] and nonlinear [4] reservoir capacitors, usually depends nonlinearly on the reservoir capacitor characteristics [1, 5], or it stochastically changes [2, 7]. This fact complicates significantly both the analysis of transient processes in EDI's circuits and the synthesis of their new structures [5, 7, 8]. If EDI load resistance varies randomly, all of the basic characteristics (in particular charge voltage of the reservoir capacitor and discharge currents in the load) also become random variables. At the same time, in order to optimize the operating modes of the EDI [2, 4, 6] and the synthesis of their new structures [5], it is very important to estimate the change ranges of the main characteristics of EDI if any parameter of their circuits (capacitance, inductance or active resistance) varies randomly. Therefore, the determination of the probabilistic properties of the electrical characteristics of the EDI's circuits with elements whose parameters can vary stochastically is an important scientific task.

The aim of the work was to determine the probabilistic properties of the electrical characteristics of the EDI with reservoir capacitor and stochastically changing active resistance.

To solve the problem in the paper, methods for determining the probability characteristics of functional transformations over random variables, presented in [7–9], were used.

In the research it is assumed that the active resistance R of the capacitor charging circuit of EDI is a stochastic parameter, and it is linear during the discharge time of the capacitor, but it can vary between its discharges according to the known law of probability distribution.

The assumption is also made that R is a continuous random variable (that is, it's possible values r constitute a continuous set), and the law of variation of continuous random variable R is known – i.e. it's probabilistic characteristics (the probability density function $f(r)$ and the probability distribution function $F(r)$ with all their parameters are known).

Using the probabilistic characteristics of functional transformations over random variables for determining the probabilistic properties of electrical characteristics of the circuit with a stochastically changing parameter. From a mathematical point of view, the electrical characteristics of the EDI's circuit are functional transformations of the circuit parameters. Thus, if one-valued functional transformation of a random variable X is given: $Y = \varphi(X)$, then Y will also be a random variable, and its possible values y will be completely determined by the possible values x of the random variable X.

According to our assumptions, we assume that for a continuous random variable X its probability density function $f_X(x)$ is known. Then if the functional transformation $Y = \varphi(X)$ is monotonic, then its probability density function $f_Y(y)$ is defined by the following expression [7]:

$$f_Y(y) = f_X(x(y)) \cdot \left| x'(y) \right| . \tag{1}$$

Here $x(y)$ is the inverse function with respect to function $y(x)$, and $\left| x'(y) \right|$ is the absolute value of the derivative of this inverse function.

To find the mathematical expectation of the unknown continuous random variable $Y = \varphi(X)$, the following formula can be used [7]:

$$M[Y] = \int_{-\infty}^{+\infty} \varphi(x) \cdot f_X(x) dx. \tag{2}$$

ORCID: * https://orcid.org/0000-0002-0200-369X ; ** https://orcid.org/0000-0001-7499-9142

Let's use the considered probabilistic characteristics of functional transformations over random variables to determine the probabilistic properties of electrical characteristics in EDI's circuit of the first order with the stochastically changing active resistance R.

As an example, we consider one of the most popular distributions of random variable – the uniform distribution.

Determination of the probabilistic properties of the capacitor voltage in an electrical circuit with a stochastic active resistance having a uniform probability distribution. Fig. 1 shows the EDI's charging circuit of the first order (with one reactive element) with a stochastically varying active resistance R, through which the capacitor C is charged from the DC voltage generator (DCVG) of the voltage U_{DCVG}. The resistance R is a random variable that is characterized by a continuous uniform distribution over the interval $[r_{min}; r_{max}]$: $R \in [r_{min}; r_{max}]$. Within the limits of separate transient capacitor charging process initiated by the opening of the thyristor VT, the value R is a fixed value and does not change during this transient process. However, in each subsequent transient the value R can randomly take another value $R \in [r_{min}; r_{max}]$. In view of the fact that R is a random variable, the capacitor voltage $U_C(t)$ (which depends on the value R) will also be a random variable. Let's define the probabilistic properties of the random variable $U_C(t)$.

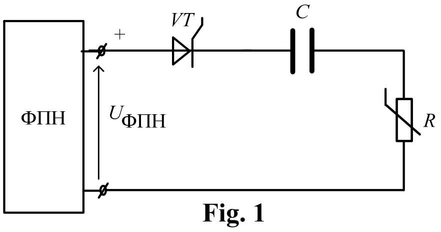

Fig. 1

The probability distribution function $F_R(r)$ of a uniformly distributed random variable R is determined by the expression [7]

$$F_R(r) = P(R < r) = \begin{cases} 0, & r < r_{min} \\ (r - r_{min})/(r_{max} - r_{min}), & r \in [r_{min}; r_{max}] \\ 1, & r > r_{max} \end{cases}. \quad (3)$$

Here $P(R < r)$ is the probability that a random variable R will take a value less than r.

The probability density function $f_R(r)$ (which by definition is a derivative of the probability distribution function: $f_R(r) = F'_R(r)$) is given by the formula [6, 7]

$$f_R(r) = \begin{cases} 0, & r \notin [r_{min}; r_{max}] \\ 1/(r_{max} - r_{min}), & r \in [r_{min}; r_{max}] \end{cases}. \quad (4)$$

Suppose that the capacitor was discharged initially: $U_C(0) = 0$. Then the capacitor voltage at time t is calculated as [10]

$$U_C(t) = U_{DCVG}\left(1 - e^{-t/rC}\right). \quad (5)$$

Further, for notational convenience, $U_C(t)$ will be written as U_C.

Let's determine the probability density function of the random variable U_C: $f_{U_C}(U_C)$. For this purpose, using (5), we express the resistance r through the capacitor voltage U_C (that is, we define the inverse function $r(U_C)$ with respect to the function $U_C(r)$)

$$r(U_C) = t/\left(C \cdot \ln(1 - U_C/U_{DCVG})\right). \quad (6)$$

Then, according to (1), the function $f_{U_C}(U_C)$ takes the form

$$f_{U_C}(U_C) = f_R(r(U_C)) \cdot |r'(U_C)|. \quad (7)$$

Let us find the derivative $r'(U_C)$.

$$r'(U_C) = t/\left[CU_{DCVG}(\ln(1 - U_C/U_{DCVG}))^2(1 - U_C/U_{DCVG})\right]. \quad (8)$$

Substituting (4) and (8) into (7), we obtain the final expression for the function $f_{U_C}(U_C)$:

$$f_{U_C}(U_C) = \begin{cases} 0, & U_C \notin [U_{C\,min}; U_{C\,max}] \\ (1/(r_{max} - r_{min})) \cdot t/\left[CU_{DCVG}(\ln(1 - U_C/U_{DCVG}))^2(1 - U_C/U_{DCVG})\right], & U_C \in [U_{C\,min}; U_{C\,max}] \end{cases}. \quad (9)$$

Here $U_{C\,min} = U_{DCVG}\left(1 - e^{-t/r_{max}C}\right)$, $U_{C\,max} = U_{DCVG}\left(1 - e^{-t/r_{min}C}\right)$.

It should be noted that the minimum capacitor voltage is reached at the maximum value of the active resistance and vice versa.

Since by definition the probability density function $f_{U_C}(U_C)$ is a derivative of the probability distribution function $F(U_C)$, then to find $F(U_C)$, we integrate (9):

$$F_{U_C}(U_C)=\int_0^{U_C} f_{U_C}(U_C)dU_C=\begin{cases}0, & U_C<U_{C\,min}\\ \left(1/(r_{max}-r_{min})\right)\cdot t\cdot \ln\left((U_{DCVG}-U_C)/(U_{DCVG}-U_{C\,min})\right)/C, & U_C\in\left[U_{C\,min};U_{C\,max}\right)\\ 1, & U_C\ge U_{C\,max}\end{cases}. \quad (10)$$

It is obvious that the distribution of the random variable U_C **is not uniform**, since a uniform distribution would be characterized by the expression [7]

$$F_{U_C}(U_C)=\begin{cases}0, & U_C<U_{C\,min}\\ (U_C-U_{C\,min})/(U_{C\,max}-U_{C\,min}), & U_C\in\left[U_{C\,min};U_{C\,max}\right]\\ 1, & U_C>U_{C\,max}\end{cases}.$$

Let's calculate, as an example, the probabilistic properties of a random variable $U_C(t)$ for the following circuit parameters: $U_{DCVG}=100$ V, $C=0.1$ F, $r_{min}=0.1$ Ohm, $r_{max}=1$ Ohm. The time moment under consideration we take $t=10^{-4}$ s.

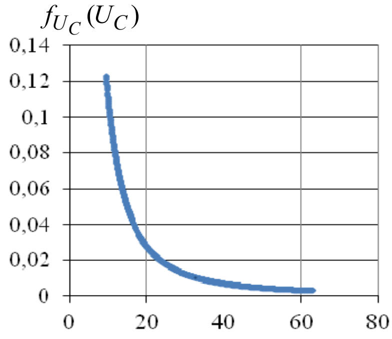

Fig. 2

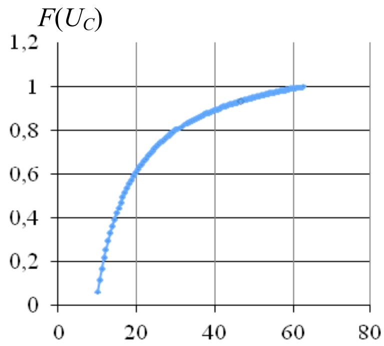

Fig. 3

The values of the maximum and minimum possible capacitor voltages at these parameters are defined as

$$U_{C\min}=U_{DCVG}\left(1-e^{-t/r_{\max}C}\right)=9.5\text{ V} \quad (11)$$

$$U_{C\max}=U_{DCVG}\left(1-e^{-t/r_{\min}C}\right)=63.2\text{ V} \quad (12)$$

The graphs of the probability density function $f_{U_C}(U_C)$ and the probability distribution function $F(U_C)$ of the random capacitor voltage $U_C(t)$ at time $t=10^{-4}$ s are presented in Fig. 2 and 3.

Let's define the mathematical expectation $M[U_C]$ of the random variable U_C, using (2), (4), (5).

$$M[U_C]=\int_{r_{min}}^{r_{max}} U_C(r)\cdot f_R(r)dr=\int_{r_{min}}^{r_{max}}\left(U_{DCVG}\left(1-e^{-t/rC}\right)/(r_{max}-r_{min})\right)dr. \quad (13)$$

Integrating (13) and performing the transformations, we obtain the final expression for $M[U_C]$:

$$M[U_C]=U_{\Phi\Pi H}\left[r_{max}\left(1-e^{-t/(r_{max}C)}\right)-r_{min}\left(1-e^{-t/(r_{min}C)}\right)-t\left(Ei\left(-\frac{t}{r_{max}C}\right)-Ei\left(\frac{-t}{r_{min}C}\right)\right)\Big/C\right]\Big/(r_{max}-r_{min}). \quad (14)$$

Here $Ei(x)$ is a special function, which is known in mathematics as an integral exponential function $Ei(x)=\int_{-\infty}^{x}\frac{e^t}{t}dt$.

It should be noted that the value of the mathematical expectation of the capacitor voltage $M[U_C]$ **is not equal** to the capacitor voltage when the resistance of the circuit is equal to its mathematical expectation $M[R]$:

$$M[U_C]\neq U_C\big|_{r=M[R]}. \quad (15)$$

For example, for considered circuit parameters, the mathematical expectation of the capacitor voltage at time $t=10^{-4}$ s, according to (14), is $M[U_C]=21.7$V.

At the same time, the capacitor voltage that would be observed in the case if the circuit resistance r was equal to the resistance mathematical expectation $M[R]$ (which for uniform distribution is $M[R]=(r_{min}+r_{max})/2=0.55$ Ohm) at $t=10^{-4}$ s will reach the value (accordingly (5)) $U_C\big|_{r=M[R]}=16.6$V.

This difference in values is explained by the fact that the dependence of the electrical characteristics of the circuit on its resistance is not linear, and, as a consequence, the form of the probability density function (and, consequently, its expectation depending on this function) for the circuit resistance differs from the corresponding functions for the circuit electrical characteristics.

Conclusion. The probabilistic properties (probability density function, probability distribution function, mathematical expectation) of the electrical characteristic (capacitor voltage) of electric discharge installation circuit

with stochastic parameter (active resistance) that changes according to the known probability distribution law (in particular, uniform probability distribution) are determined in the paper.

It is shown that the analysis of processes in circuits with a stochastic parameter, based on the intuitive use of its mathematical expectation, gives an inaccurate result at calculation of circuit electrical characteristics. Therefore it is necessary to use the mathematical expectation of the considered circuit electrical characteristic (mathematical expectation of capacitor voltage), which generally differs from the corresponding electrical characteristic, calculated at the deterministic parameter value (that equals to its mathematical expectation).

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ІМОВІРНІСНІ ВЛАСТИВОСТІ ЕЛЕКТРИЧНИХ ХАРАКТЕРИСТИК ЗАРЯДНОГО КОЛА КОНДЕНСАТОРА ІЗ СТОХАСТИЧНИМ АКТИВНИМ ОПОРОМ

А.А. Щерба[1], чл.-кор. НАН України; **Н.І. Супруновська**[1], докт.техн.наук, **Д.С. Іващенко**[2]
[1] - Інститут електродинаміки НАН України,
пр. Перемоги, 56, Київ, 03057, Україна, **e-mail: iednat1@gmail.com**
[2] – Oracle, 1501 4th Ave, Seattle, WA 98101, US, **e-mail: ivas-90@mail.ru**

Запропоновано підхід до визначення імовірнісних властивостей (функції густини розподілу ймовірностей, функції розподілу ймовірностей, математичного сподівання) електричних характеристик кіл електророзрядних установок, активний опір яких може змінюватися випадковим чином. Передбачається, що такий стохастичний опір характеризується безперервною випадковою величиною, імовірнісні властивості якої відомі. Як приклад були досліджені імовірнісні властивості напруги на конденсаторі в колі першого порядку зі стохастичним активним опором, що має рівномірний розподіл ймовірностей. Бібл. 10, рис. 3.

Ключові слова: перехідні процеси, стохастичний опір, випадковий процес, безперервний розподіл ймовірностей.

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ВЕРОЯТНОСТНЫЕ СВОЙСТВА ЭЛЕКТРИЧЕСКИХ ХАРАКТЕРИСТИК ЗАРЯДНОЙ ЦЕПИ КОНДЕНСАТОРА СО СТОХАСТИЧЕСКИМ АКТИВНЫМ СОПРОТИВЛЕНИЕМ

А.А. Щерба[1], чл.-корр. НАН Украины; **Н.И. Супруновская**[1], докт.техн.наук, **Д.С. Иващенко**[2]
[1] - Институт электродинамики НАН Украины,
пр. Победы, 56, Киев, 03057, Украина, **e-mail: iednat1@gmail.com**
[2] – Oracle, 1501 4th Ave, Seattle, WA 98101, US, **e-mail: ivas-90@mail.ru**

Предложен подход к определению вероятностных свойств (функции плотности распределения вероятностей, функции распределения вероятностей, математического ожидания) электрических характеристик цепей электроразрядных установок, активное сопротивление которых может изменяться случайным образом. Предполагается, что такое стохастическое сопротивление характеризуется непрерывной случайной величиной, вероятностные свойства которой известны. В качестве примера были исследованы вероятностные свойства напряжения на конденсаторе в цепи первого порядка со стохастическим активным сопротивлением, имеющим равномерное распределение вероятностей. Библ. 10, рис. 3.

Ключевые слова: переходные процессы, стохастическое сопротивление, случайный процесс, непрерывное распределение вероятностей.

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