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MATHEMATICAL MODEL OF A FLEXIBLE MICRO GRID INTEGRATED INTO THE COUNTRY GRID

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The functioning of local electric networks ('Micro Grid') is described, which due to the development of Smart Network technologies, distributed generation and electricity storage systems received an incentive for development. The structural decomposition of the market model is suggested for the 'Top-Level Market' and 'Micro Grid' local systems to reflect the interaction of the 'Micro Grid' with the wholesale electricity market. Load regulation instruments are described, which ensure the flexibility of the Micro Grid. A mathematical model of the 'Micro Grid' system is proposed, which formulated as a Unit commitment problem. The computational experiment shows the adequacy of the results of modelling of 'Micro Grid' systems. References 9, figures 4, tables 2.

Keywords: Micro Grid, power system flexibility, market decomposition, mathematical model, Unit commitment problem.

Introduction. The emergence of efficient technologies for distributed generation, and primarily renewable energy systems (RESs), as well as network equipment, intelligent measuring instruments, data transmission systems and control methods, which are collectively described by the concept of Smart Grids, has led to a change in the structure of the power system and the interaction of agents in the energy market.

The high efficiency of distributed generation sources stimulates large consumers, as well as consumer associations, to create their own generation sources. The emergence of such balanced energy complexes of consumers-producers forms a new kind of participants in the energy system, which, depending on the level of own consumption and the level of market prices, can act as consumers, and as producers. The term 'Micro Grid' is used below to describe them.

The scale of the market participant plays a decisive role in its market behavior. Large producers or consumers may have market power, leading to imperfect competition [1]. At the same time, the structure of Micro Grids and their binding to individual consumers or groups of consumers implies the operation of effective pricing mechanisms in them.

Research has focused on the impact of Smart Grids on the level of competition in the energy market. The [2] examines the methodological basis for determining the effectiveness of Smart Grids implementation and its impact on the level of competition. The strategic behavior of players using accumulative technologies is investigated in the work [3]. In [4] authors explore the behavior and interaction of Micro Grids operators using game theory techniques. At the same time, a simplified view of Micro Grids is considered as a complex of consumer and unmanaged generation.

The open question remains the modeling of the interaction of the Micro Grid's operator, using a modern set of distributed generation technologies, with the wholesale electricity market, operating in conditions of imperfect competition. In [5], the decomposition of the power system is performed with the allocation of the wholesale market as the Top-level and the set of Micro Grids as the bottom level.

Consider the work of Micro Grid, which includes:

- consumer;
- maneuverable fossil fuel generator (FFG);
- electricity storage system (ESS);
- uncontrolled RES generator.

Each of the listed members may represent a number of producers and/or consumers of a similar type.

The following instruments are used to control the Micro Grid load:

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- the FFG participation in load regulation;
- an efficient lithium-ion batteries;
- the centralized management of RES;
- involvement of consumers in regulating electricity demand.

The Micro Grid is also connected to the Top-level market.

When covering the demand for electricity, the minimum cost criterion is used. RES, which have a minimum cost of production, are loaded first, followed by maneuverable FFG, ESS and external sources, depending on the ratio of their production costs and demand. The possibility of offloading RES and consumers is assumed, if the costs of offloading them are lower than the costs of increasing production.

In essence, the task of modeling such a system is similar to the task of optimal load of power equipment [6-9]. Such models are used effectively for short-term power system forecasting and allow for maneuverability of equipment.

The purpose of the study is to develop a mathematical model of the optimal load of the generating capacities of the Micro-grid and the power transmission lines connecting it to the Country Grid.

1. Mathematical description of 'micro grid' model.

1.1. Model parameters:

t – the number of the time interval of the weekly forecast period
T – set of numbers t
T_0 – the number of the extreme right time interval of the weekly period
$\overline{p}^{RnW}$ – potentially achievable volume of electricity production from RES
$\overline{\Delta p}^{RnW}$ – the maximum allowable amount of unloading of electricity production from RES
$\overline{l}$ – potentially achievable amount of electricity consumption
$\overline{\Delta l}$ – the maximum allowable amount of unloading of electricity consumers
c^{SU} – the cost of starting the FFG
$\overline{C}$ – operating costs at minimal FFG load
$\tilde{c}$ – the coefficient of elasticity of operating costs to the load of the FFG
$\underline{P}$ – the minimum load of the FFG
$\overline{P}$ – maximum load of the FFG
c^{SD} – the cost of stopping the FFG
P^{SU} – the lower allowable load limit of the FFG during its start-up
P^{SD} – the upper permissible load limit of the FFG before stopping it
ΔP^{up} – the value of the maximum increase in load on the FFG
ΔP^{down} – the value of the maximum reduction of the load on the FFG
η^{P} – efficiency of ESS equipment, operating in charging mode
η^{G} – efficiency of ESS equipment, operating in discharge mode
c^{P} – specific operating costs for ESS operation in charging mode
c^{G} – specific operating costs of ESS operation in discharge mode
$\overline{q}$ – the maximum amount of energy that ESS can store
$\underline{q}$ – the minimum amount of energy that ESS can store
$\overline{p}^{P}$ – maximum ESS load when operating in charging mode
$\overline{p}^{G}$ – maximum ESS load when operating in discharge mode
c^{RnW} – specific operating costs for the production of electricity from RES
$c^{\Delta l}$ – specific operating costs for unloading electricity consumers
$c^{\Delta RnW}$ – specific operating costs for unloading installations for the production of electricity from RES
Pr^{TL} – the prices in the 'Top-Level Market'
Pr^{TSO} – the transmission price
H – maximum allowable load of transmission lines that connect the Micro Grid with the Country Grid

1.2. Model variables:

$Cost$ – operating costs for electricity production by the FFG
$Cost^{SU}$ – costs for starting the FFG
$Cost^{SD}$ – costs to stop the FFG
$Cost^{RnW}$ – operating costs for electricity generation from RES
$Cost^{S}$ – operating costs for ESS, which operates in discharge mode
$Cost^{\Delta l}$ – costs for reduction of consumption load
p – the load of the unit in discharge mode
p^{RnW} – the load of RES
Δp^{RnW} – the volume of unloading of RES
l – the load of consumption
Δl – the volume of unloading of consumers
$Cost^{Ext}$ – the cost of electricity purchase in the 'Top-Level Market'
$Income$ – the income from electricity sale to the 'Top-Level Market'
p^{Ext} – the load of purchase in the 'Top-Level Market'
p^{Int} – the load of sale to the 'Top-Level Market'
l – the load of consumption
$\tilde{p}$ – variable load of the FFG
u – a binary function that characterizes the state of the FFG and takes a value of 1 if the FFG is operating in load mode, and a value of 0 if the unit is not operating
x – a binary function that characterizes the state of the FFG and takes a value of 1 if the FFG is running in stop mode, and a value of 0 in all other cases
y – binary function that characterizes the state of the FFG: the function takes the value 1 if the FFG is running in startup mode, and the value 0 in all other cases
p^{P} – the load of ESS, operating in charging mode
p^{G} – the load of ESS, operating in discharge mode
q – the amount of energy stored at the ESS
u^{P} – binary function that characterizes the state of ESS: the function takes the value 1 if the system is operating in charging mode, and the value 0 if the system is not operating in this mode
x^{P} – binary function that characterizes the state of ESS: the function takes a value of 1 if the system is in a state of stop charging mode, and a value of 0 in all other cases
y^{P} – binary function that characterizes the state of ESS: the function takes the value 1 if the system is in the start state of the charging mode, and the value 0 in all other cases
u^{G} – binary function that characterizes the state of ESS: the function takes the value 1 if the system is operating in discharge mode, and the value 0 if the system is not operating in this mode
x^{G} – binary function that characterizes the state of ESS: the function takes the value 1 if the system is in the state of stopping the discharge mode, and the value 0 in all other cases
y^{G} – binary function that characterizes the state of ESS: the function takes the value 1 if the system is in the start state of the discharge mode, and the value 0 in all other cases
n^{EI} – binary function that takes the value 1 if the Micro Grid export electricity, and the value 0 in case of its import.

1.3. The objective function of the UC optimization problem

Operating costs of electricity supply in the Micro-Grid are minimized:

$$\sum_{\forall t \in T} \left(\begin{array}{l} Cost_t + Cost_t^{SU} + Cost_t^{SD} + \\ Cost_t^{S} + Cost_t^{RnW} + Cost_t^{\Delta l} + \\ Cost_t^{Ext} - Income_t \end{array} \right) \rightarrow min \, . \qquad (1)$$

1.4. Balance of electricity production and consumption

Balance of electricity production and consumption is represented by equations:

$$p_t^{RnW} - \Delta p_t^{RnW} + p_t + p_t^G + p_t^{Ext} = \\ l_t - \Delta l_t + p_t^P + p_t^{Int}, \quad \forall t \in T. \tag{2}$$

1.5. Operating costs for FFG

Operating costs for FFG:

$$Cost_t = \overline{C} u_t + \tilde{c}\, \tilde{p}_{t,}, \quad \forall t \in T\,; \tag{3}$$

$$\tilde{p}_t \le \left(\overline{P} - \underline{P}\right) u_t, \quad \forall t \in T\,; \tag{4}$$

$$p_t = \underline{P} u_t + \tilde{p}_t, \quad \forall t \in T. \tag{5}$$

1.6. The mode of FFG operation

The mode of operation of the FFG is described by the system of relations between the binary functions of its state, which has the form:

$$y_t - x_t = u_t - u_{t-1}, \quad \forall t \in T, \quad t \ne 1, \tag{6}$$

$$y_1 - x_1 = u_1 - u_{T_0}, \tag{7}$$

$$y_t + x_t \le 1, \quad \forall t \in T. \tag{8}$$

1.7. The cost of starting the FFG

The cost of starting the FFG is calculated by the formula:

$$Cost_t^{SU} = c^{SU}\, y_t, \quad \forall t \in T. \tag{9}$$

1.8. The cost of stopping the FFG

The cost of stopping the FFG is calculated by the formula:

$$Cost_t^{SD} = c^{SD}\, x_t, \quad \forall t \in T. \tag{10}$$

1.9. The current maximum attainable load of the FFG

The current maximum attainable load of the FFG is limited by the maximum installed capacity or its allowable load before stopping:

$$p_t \le p_{t-1} + \Delta P^{up} u_{t-1} + P^{SU} y_t, \quad \forall t \in T, \quad t \ne 1\,; \tag{11}$$

$$p_t \le p_{T_0} + \Delta P^{up} u_{T_0} + P^{SU} y_t, \quad t = 1. \tag{12}$$

The load of the unit is limited from the top by its current maximum achievable load:

$$p_t \le \overline{P} u_t, \quad \forall t \in T. \tag{13}$$

1.10. The current minimum achievable load of the FFG

The load of the FFG is limited from the bottom by its current minimum achievable load:

$$p_t \ge \underline{P} u_t, \quad \forall t \in T\,. \tag{14}$$

The current minimum achievable load of the FFG is limited by the technical capabilities to reduce its load during the shutdown:

$$p_t \ge p_{t-1} - \Delta P^{down} u_t - P^{SD} x_t, \quad \forall t \in T, \quad t \ne 1\,; \tag{15}$$

$$p_t \ge p_{T_0} - \Delta P^{down} u_t - P^{SD} x_t, \quad t = 1\,. \tag{16}$$

1.11. Description of ESS

Operating costs of ESS are determined by the formula:

$$Cost_t^S = c^P \left(1 - \eta^P\right) p_t^P - c^G \left(1 - \eta^G\right) p_t^G, \quad t \in T\,, \tag{17}$$

Energy balances of ESS are represented by equations:

$$q_t - q_{t-1} = \eta^P p_t^P - \frac{p_t^G}{\eta^G}, \quad \forall t \in T, \quad t \ne 1\,. \tag{18}$$

Complemented by the conditions of weekly cyclicity of accumulated energy:

$$q_0 = q_{T_0}\,. \tag{19}$$

The amount of stored energy is limited by the energy consumption of its storage systems:

$$\underline{q} \le q_t \le \overline{q}, \quad \forall t \in T\,. \tag{20}$$

The operation of ESS in the discharge mode is described by the following relations between the binary functions of the state of the systems:

$$y_t^G - x_t^G = u_t^G - u_{t-1}^G, \quad \forall t \in T, \quad t \neq 1, \tag{21}$$

$$y_1^G - x_1^G = u_1^G - u_{T_0}^G, \tag{22}$$

$$y_t^G + x_t^G \leq 1, \quad \forall t \in T. \tag{23}$$

The operation of ESS in the charging mode is described by the following relationships between binary functions of the state of the systems:

$$y_t^P - x_t^P = u_t^P - u_{t-1}^P, \quad \forall t \in T, \quad t \neq 1, \tag{24}$$

$$y_1^P - x_1^P = u_1^P - u_{T_0}^P, \tag{25}$$

$$y_t^P + x_t^P \leq 1, \quad \forall t \in T. \tag{26}$$

The inability to operate ESS simultaneously in discharge and charging modes is reflected in the limitations:

$$u_t^G + u_t^P \leq 1, \quad \forall t \in T. \tag{27}$$

Equipment of ESS, working in the discharge mode, has limited load, i.e.:

$$p_t^G \leq u_t^G \, \overline{p}^G, \quad p_t^G \geq 0, \quad \forall t \in T. \tag{28}$$

Load limitations of ESS are also taken into account during their operation in the charging mode:

$$p_t^P \leq u_t^P \, \overline{p}^P, \quad p_t^P \geq 0, \quad \forall t \in T. \tag{29}$$

1.12. Description of RES power generation systems

Operating costs for the production of electricity from RES are calculated by the formula:

$$Cost_t^{RnW} = c^{RnW} p_t^{RnW} + c^{\Delta RnW} \Delta p^{RnW}, \quad t \in T. \tag{30}$$

The projected maximum volumes of electricity production from RES are determined on the basis of retrospective models of the respective power plants.

Balancing of the Micro-Grid can be carried out by its operator by unloading power plants using RES. The possibilities of such balancing are limited, it means, we have balances of marginal and accepted volumes of production and volumes of unloading of power plants:

$$p_t^{RnW} + \Delta p_t^{RnW} = \overline{p}_t^{RnW}, \quad t \in T. \tag{31}$$

as well as restrictions on the volume of their unloading:

$$0 \leq \Delta p_t^{RnW} \leq \overline{\Delta p}_t^{RnW}, \quad t \in T. \tag{32}$$

1.13. Description of costs for unloading of consumers

Costs for unloading electricity consumers are determined by the formula:

$$Cost_t^{\Delta l} = c^{\Delta l} \Delta l_t, \quad t \in T. \tag{33}$$

Possibilities of such balancing are limited, it means, we have balances of demand of consumers load and the accepted volumes of its consumption and volumes of unloading of consumers:

$$l_t + \Delta l_t = \bar{l}_t, \quad t \in T, \tag{35}$$

as well as restrictions on the volume of their unloading:

$$0 \leq \Delta l_t \leq \overline{\Delta l_t}, \quad t \in T. \tag{36}$$

1.14. Purchasing of electricity in the 'Top-Level Market' system

The cost of electricity purchase in the 'Top-Level Market' system

$$Cost_t^{Ext} = \left(Pr_t^{TL} + Pr_t^{TSO}\right) p_t^{Ext}. \tag{37}$$

1.15. The sale of surplus electricity to the 'Top-Level Market' system

The income from electricity sale to the 'Top-Level Market' system

$$Income_t = Pr_t^{TL} \; p_t^{Int}. \tag{38}$$

2. Peculiarities of Computer Simulation of 'Micro Grid' System and Results of Computer Experiment

To perform computational experiments, the proposed mathematical model of the 'Micro Grid' system as a mixed integer programming problem (1)–(38) was implemented in IBM ILOG CPLEX Optimization Studio Version 20.1 using the OPL optimization programming language. The following computer experiments were performed using the obtained computer model.

Simulation is carried out for a week period of time with hourly process detail under the following conditions.

The pattern of electricity consumption is typical for an average city varies in the range from 7.8 to 18.6 MW (Fig. 1).

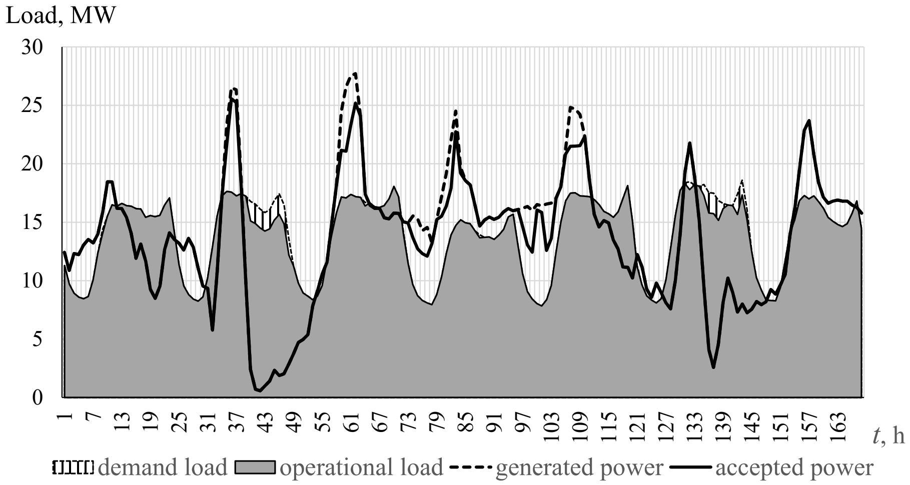

Fig. 1

The price of electricity in the 'Top-Level' market changes according to the formula:

$$\mathrm{Pr}_t^{TL} = 7.5 l_t - 47.5, \quad \forall t \in T.$$

The load of the RES varies depending on the weather conditions in the range from 0.6 to 27.7 MW (Fig. 1), its cost of electricity generation is 5 \$/MWh.

Characteristics of the FFG and the ESS induced in tables 1 and 2 respectively.

Table 1

c^{SU}	34.8 \$/StartUp
$\overline{C}$	30 \$/h
$\tilde{c}$	2 \$/MWh
$\underline{P}$	3.5 MW
$\overline{P}$	12.0 MW
c^{SD}	30.0 \$/ShurtDown
P^{SU}	3.5 MW/h
P^{SD}	12.0 MW/h
ΔP^{up}	8.5 MW/h
ΔP^{down}	1.5 MW/h

Table 2

η^{P}	0.85
η^{G}	0.85
c^{P}	40.0 \$/MWh
c^{G}	40.0 \$/MWh
$\overline{q}$	7.5 MWh
$\underline{q}$	0.0 MWh
$\overline{p}^{P}$	4.0 MW
$\overline{p}^{G}$	3.0 MW

The simulation results of the 'Micro Grid' system are shown below in Fig. 2, Fig. 3 and Fig. 4.

Maximum allowable load of transmission lines that connect the Micro-Grid with the Country Grid: $H = 4.0$ MW. It means that:

$$p_t^{Ext} \le n_t^{EI} H, \quad \forall t \in T, \tag{39}$$

$$p_t^{Int} \le \left(1 - n_t^{EI}\right) H, \quad \forall t \in T. \tag{40}$$

In Fig. 2. the changes in the load of the FFG, the power flow from the external energy system and to the external energy system are presented.

The value of the charge of the ESS is shown in Fig. 3.

In Fig. 4. shows the cost of electricity generation in 'Micro Grid' system and revenue from the sale of electricity to the 'Top-level' system.

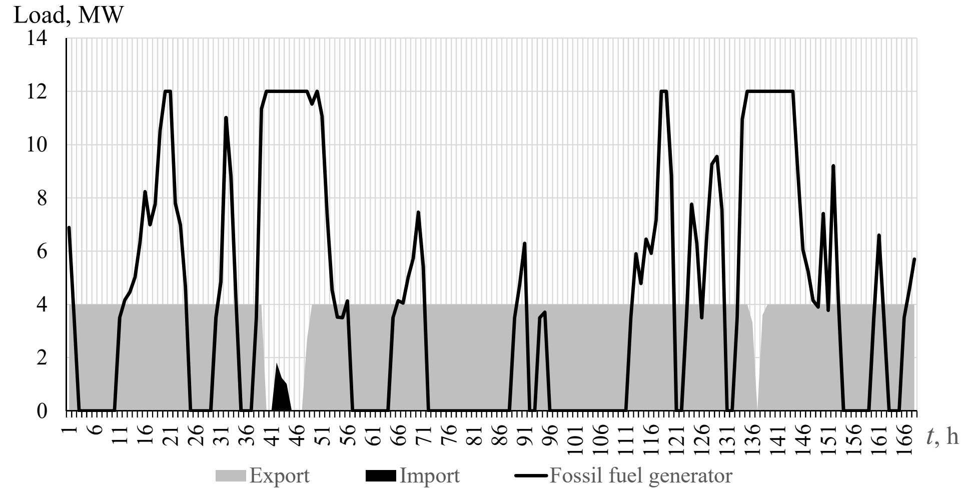

Fig. 2

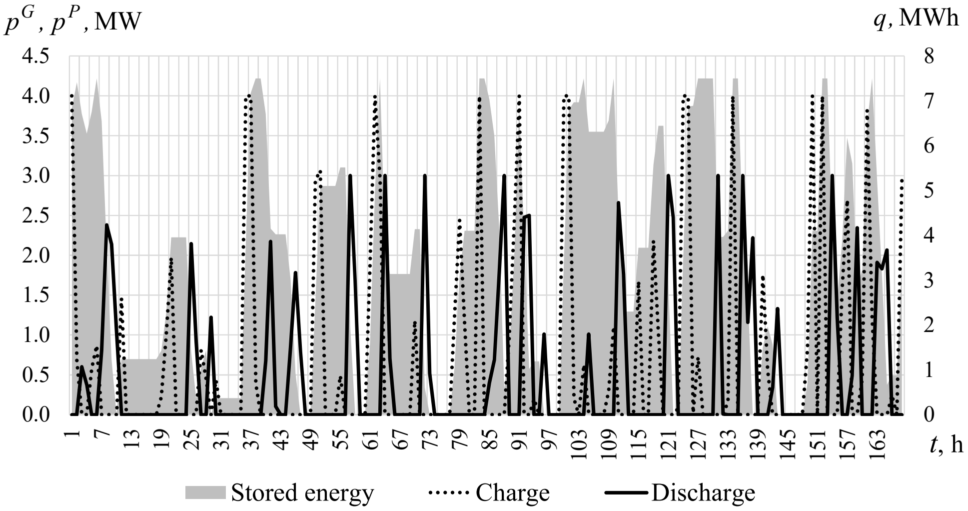

Fig. 3

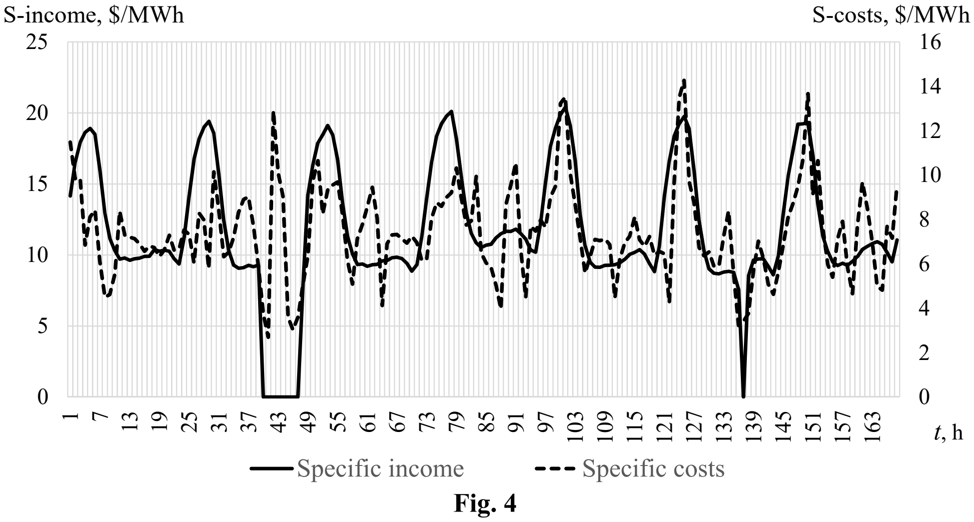

Fig. 4

Conclusions.

The development of distributed generation technologies, energy storage systems and Smart Grid technology, as well as modern trends in the consumer community's formation, encourage the study of "Micro-grids", which, depending on the price situation, can act as both consumers and producers of electricity.

Taking into account the fact that competition in wholesale markets is often imperfect, and "Micro-grids" have effective pricing mechanisms, the decomposition of the energy system is used, with the selection of the wholesale market as the Top-level and the set of "Micro-grids" as the lower level.

The presented model reflects the functioning of the 'Micro Grid', including a detailed description of traditional and renewable electricity generation technologies, as well as storage technologies. It also displays the main modern 'Micro Grids' mode management instruments. The peculiarities of the interaction of 'Micro Grid' with the Top-level market are taken into account.

The model is formulated as a Unit commitment problem, which allows to adequately reflect the loading modes of traditional generators and RES, as well as the peculiarities of ESS.

The use of this model in conjunction with known models of imperfect competition can adequately describe the functioning and development of energy systems in modern conditions.

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МАТЕМАТИЧНА МОДЕЛЬ ГНУЧКОЇ МІКРОМЕРЕЖІ, ІНТЕГРОВАНОЇ ДО ЗАГАЛЬНОДЕРЖАВНОЇ МЕРЕЖІ

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Описано функціонування локальних електричних мереж («Мікро-мереж»), які завдяки розвитку технологій Розумних Мереж, систем розподіленої генерації та накопичення електроенергії отримали стимул для розвитку. Пропонується структурна декомпозиція ринкової моделі для локальних систем «Ринок верхнього рівня» та «Мікро-мережа», для відображення взаємодії «Мікро-мереж» з оптовим ринком електроенергії. Описано інструменти регулювання навантаження, які забезпечують гнучкість Мікро-мережі. Запропоновано математичну модель «Мікро-мережі», яка сформульована як задача завантаження устаткування. Обчислювальний експеримент показує адекватність результатів моделювання «Мікро-мереж». Бібл. 9, рис. 4, табл. 2.

Ключові слова: Мікро-мережа, гнучкість енергосистеми, декомпозиція ринку, математична модель, проблема завантаження устаткування.

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