SPECTRAL METHOD TO EVALUATE THE UNCERTAINTY OF DYNAMIC MEASUREMENTS

O.M. Vasilevskyi[1], M.Yu. Yakovlev[2], P.I. Kulakov[1]*
[1-]Vinnytsia National Technical University,
95, Khmelnytske Shose, Vinnytsia, 21021, Ukraine,
[2-]Hetman Petro Sahaidachny National Army Academy,
32, Heroes of Maidan street, Lviv, 79012, Ukraine,
e-mail: o.vasilevskyi@gmail.com

Article suggested spectral method of assessing the measurement devices dynamic uncertainty that allows to investigate measurement accuracy in dynamic operating conditions in frequency domain and to estimate amplitude values of dynamic uncertainty based on input signal frequency characteristic and spectral function. The results were approbated when evaluating the engines vibration acceleration dynamic measurements uncertainty. It was established that maximum value of vibration acceleration dynamic measurement uncertainty amounts to 0.137 m/s^2 for observation time of 600 sec and vibration acceleration signal nominal value of 0.35 m/s^2 at a frequency of 6 kHz.
References 10, table 1, figures 2.

Key words: dynamic uncertainty of measurement devices, quality assurance of dynamic measurements, spectral function, frequency characteristic, vibration acceleration.

Introduction. Conducting experiments using measuring devices (MD) under dynamic conditions is becoming increasingly widespread in many areas including scientific research, technology, manufacturing industry, commerce, and medicine. Dynamic measurements are related primarily to the study of the regularities in the passage of physical processes in the subjects under study. Therefore, the role of such measurements is particularly significant, firstly, in the fields of science related to the study of the structure of matter, the analysis and synthesis of new substances and materials, the study of objects under experimental conditions. Secondly, it is important in the fields of technology, manufacturing and medicine, which are characterized by the creation of new technological processes and the testing of new MDs, by taking the concept of representation of the quality of measurements into account.

When compiling a report on the results of dynamic measurements, it is necessary to demonstrate quantitative values of the quality of measurements so that their reliability can be correctly assessed [1, 4, 7, 9]. Without such values, the results of dynamic measurements cannot be compared, neither with each other nor with reference values. Therefore, it is necessary to propose methods for estimating the quality characteristics of dynamic measurements. In this case, it is necessary to take into account the fact that during dynamic measurements a transient mode of operation of the MD will also be present at some stage, during which the signal from the output of the measuring device changes significantly over time. These circumstances are due to the inertial properties of the MD, since they consist, as a rule, of a set of different masses and springs, capacitances and inductances, and other inertial elements that lead to the manifestation of dynamic uncertainty. The equation of the transformation of the MD, which displays its statics properties, is unacceptable in a dynamic mode. In this case, we must go to the differential equations that describe the dynamic relationship between the output $y(t)$ and the input $x(t)$ values of the measuring devices [5, 6].

In view of the above, there is a need to develop methods for estimating the uncertainty of dynamic measurements that would meet international requirements for estimating the characteristics of the quality of measurements, which is a topical scientific task in the field of metrology.

The purpose of this article is the development and mathematical description of a method for estimating the uncertainty of dynamic measurements, which would allow taking into account the inertial properties of the measuring device and the passage of the measurement signal through it.

To represent a differential equation in the frequency domain, the differentiation symbol for the time coordinate d/dt is replaced by $j\omega$ and thus the transfer function $H(j\omega)$ of the corresponding measuring transducer is obtained [5, 8].

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*ORCID ID : http://orcid.org/0000-0002-0167-2218

Convenient for practical use are the frequency characteristics of measuring instruments, which are given in Table [8].

It is also known that the concept of estimating and expressing measurement uncertainties existing in international practice [9], does not describe how it is possible to assess dynamic uncertainties when conducting metrological work.

Frequency characteristics of the MM	**Typical Units**
$H(j\omega) = K$, where K is the transmission coefficient	Non-inertial (ideal measurement transducer)
$H(j\omega) = K/(1+j\omega\tau)$, where τ is the time constant determined by the parameters of the MM	Aperiodic (temperature transducer)
$H(j\omega) = K/j\omega$	Integrated (integrated amplification)
$H(j\omega) = K(1+j\omega\tau)$	Forcing (differential amplification)
$H(j\omega) = \exp(-j\omega\tau)$	Delay (analog-to-digital converters)
$H(j\omega) = K\big/\left(1+j\omega\tau_1-\omega^2\tau_2^2\right) =$ $= K\big/\left(1+2j\omega\beta\tau-\omega^2\tau^2\right)$	Oscillating (electromechanical transducers)

From [1, 9], it is only known that there are methods in existence of evaluation of type A and type B, as well as forms of representation of uncertainties: standard, combined and extended. The definitions of these uncertainties are presented in [9]. Such distinguished works as [6, 7] in which dynamic uncertainty is calculated as a standard uncertainty of type B, which is determined by the dynamic error divided by the square root of three (assuming a uniform distribution law). The use of classical theories of determining the dynamic error in the expression of dynamic uncertainty is unacceptable, since the concept of measurement uncertainty, which is set out in an international standard [9], moves from the notion of measurement error, as such, which is not known and is not subject to being determined, as opposed to measurement uncertainty which can be evaluated. For a given measurement such a result does not have a single value, but has an infinite number of values scattered around the result. Therefore, there is a need to develop a new method for estimating dynamic uncertainty, which could be determined without using the dynamic error used in the theory of errors.

Method for estimating the uncertainty of dynamic measurements. If the equation of the transducer under measurement can be represented in the form

$$Y = K_C X, \tag{1}$$

where X is the measured value of the physical quantity (input signal); K_C is the coefficient of the conversion of the measuring device and Y is the measurement result (output signal), then the mathematical expectation for the input signal will be equal to $M[X]$, and the mathematical expectation of the output signal will be equal to

$$M[Y] = K_C M[X], \tag{2}$$

where $M[Y]$ and $M[X]$ are the corresponding mathematical expectations of the output and input signals of the measuring device, respectively.

The spectral density of the input signal $X(t)$ has the form [4, 8]

$$H_X(\omega) = \lim \left(2\mathrm{T}\right)^{-1} \left|X(j\omega)\right|^2, \quad \text{when } T \to \infty, \tag{3}$$

where $X(j\omega)$ is the Fourier image obtained by replacing the value in the operand of the image $X(s)$ by the values of s for $j\omega$; T is the time of observation; $\omega=2\pi f$.

The expression for the spectral density of the output signal can be represented by the expression

$$H_Y(\omega) = \lim \left(2\mathrm{T}\right)^{-1} \left|\mathrm{Y}(j\omega)\right|^2, \quad \text{when } T \to \infty. \tag{4}$$

The relationship between the images of the output and input values gives us an expression for the transfer function of the measuring device [4, 8]

$$\mathrm{K}_C(s) = \frac{Y(s)}{X(s)} = \sum_{k=0}^{m} B_k s^k \Big/ \sum_{q=0}^{n} A_q s^q , \tag{5}$$

where $Y(s)$, $X(s)$ are the operator images of $Y(t)$ output and $X(t)$ input signals, respectively; k, q are the order of the derivatives of Y and X, respectively; A_q, B_k are the coefficients of the differential equation.

Therefore, we can write that [7, 8]

$$H_Y(\omega) = \left|K_C(j\omega)\right|^2 H_X(\omega), \qquad (6)$$

where $K_C(j\omega)$ is the frequency characteristic of the measuring transducer.

The uncertainty of the output signal for dynamic measurements can be defined as the square root of the integral of the spectral density of the output signal over all frequencies [8]

$$\mathrm{u_D}(\omega) = \pi^{-1/2}\left(\int\limits_0^{\infty}\left|K_C(j\omega)\right|^2 H_X(\omega)\,d\omega\right)^{1/2} = \pi^{-1/2}\left(\mathrm{T}^{-1}\int\limits_0^{\infty}\left|K_C(j\omega)\right|^2\left|X(j\omega)\right|^2 d\omega\right)^{1/2}, \qquad (7)$$

where $\left|K_C(j\omega)\right|$ is the frequency response module of the measuring device, used for dynamic measurements.

The frequency response module of the measuring device is determined by the formula

$$\left|K_C(j\omega)\right| = \left(a^2(\omega) + b^2(\omega)\right)^{1/2}, \qquad (8)$$

where $a(\omega)$, $b(\omega)$ are respectively, the real and imaginary parts of the frequency response MD $K_C(j\omega)$ [8].

The spectral function of the incoming signal $X(j\omega)$ is related to its time function $X(t)$ by the Laplace expression

$$X(j\omega) = \int\limits_0^{\infty} X(t)e^{-j\omega_0 t}\,dt, \qquad (9)$$

where ω_0 is the cyclic frequency of the input signal [4 - 8].

For a finite time interval, the infinity sign may be replace by the total observation time T.

To represent uncertainty of dynamic measurement (7) as a function dependent on time $u_D(t)$, the Fourier expression for inverse transformation may be executed

$$u_D(t) = \pi^{-1/2}\int\limits_0^{\infty} u_D(\omega)e^{j\omega t}\,d\omega = \pi^{-1/2}\left[\int\limits_0^{\infty} u_D(\omega)\cos(\omega t)\,d\omega + j\int\limits_0^{\infty} u_D(\omega)\sin(\omega t)\,d\omega\right]. \qquad (10)$$

Thus, the uncertainty that is introduced due to the limited properties of the measuring device used for the dynamic measurements can be estimated in the time domain, based on the model equation of the spectral function of the input signal and the frequency response of the measuring instrument used by formula (10).

Evaluation of the uncertainty of the dynamic measurement of vibration acceleration. In accordance with [2], in a linear approximation, the mechanical oscillating system can be represented by one or a combination of the links of the first and second order, i.e. oscillatory link. Therefore, to analyze a mechanical oscillation system, we can use the well-known application of the theory of linear systems. In this case, the vibrations that are registered at the point of positioning of the accelerometer on the node of the bearings of the electric motor represent its response to the impact of the vector generating process [2]. The differential equation describing the dynamic relationship of the input and output values of the vibration acceleration measuring transducer has the form [3, 4]

$$\frac{d^2X_s(t)}{dt^2} + 2h\frac{dX_s(t)}{dt} + h_\mathrm{k}^2 X_s(t) = \frac{F_0}{m}\cos(\omega_0 t), \qquad (11)$$

where $F(t) = F_0\cos(\omega_0 t)$ is the harmonic forced power of the oscillation of the surface of the object (input value); F_0 is the force amplitude; ω_0 is the angular frequency of forced power; $X_s(t)$ represents the the mechanical vibrations of the inertial mass; m is the mass of the accelerometer; c is the damping variable; k is the equivalent rigidity of the piezoelements, $h = c/2m$ is the damping coefficient; $h_k = \sqrt{k/m}$ is the critical value damping coefficient [3, 4].

The accelerometer is located on the bearings of the electric motor and performs spot metering of the vibration in the frequency range from 6 kHz to 10 kHz, which is created by the operation of the roller bearings. Vibrations generated by other excitatory forces should be considered as interference [2].

The transfer function of the measuring device will take the form of

$$H(s)=\frac{K_{MM}}{s^2+2hs+h_k^2}, \tag{12}$$

where K_{MM} is the coefficient of proportionality of the measuring channel of vibration acceleration.

Turning to the domains of frequency and separating the real and imaginary parts, we obtain an expression for the module of the frequency characteristics of the measuring device for vibration acceleration

$$\left|K_C(j\omega)\right|=\left|\frac{K_{MM}}{(j\omega)^2+2h(j\omega)+h_k^2}\right|=\left[\frac{K_{MM}}{\omega^4-2\omega^2h_k^2+4\omega^2h^2+h_k^4}\right]^{1/2}. \tag{13}$$

The input signal $F_0m^{-1}\cos(\omega_0t)$ of vibration acceleration has the form of

$$X(j\omega)=j\omega F_0\left(\omega_0^2+(j\omega)^2\right)^{-1}m^{-1}, \tag{14}$$

where ω_0 is the cyclic frequency input vibration acceleration, which ranges from 6 to 10 kHz that is, with a minimum value of 18,849.5 and the maximum value is 31,415.9 radians/second.

The module image of the input vibration acceleration is written as

$$\left|X(j\omega)\right|=\omega F_0\left(\omega_0^2-\omega^2\right)^{-1}m^{-1}. \tag{15}$$

From source literature [3, 4], it is known that the amplitude of forced harmonic power F_0 is $3\cdot10^{-4}$ m. The mass of the accelerometer is $m=4\cdot10^{-2}$ kg. The damping variable for the piezoelectric accelerometers is equal to 0,5, equivalent rigidity of the piezoelements is k=2, and the minimum observation time T=300 s. The proportionality factor or gain K_{MM} of the measuring channel of the vibration acceleration is 10^5.

Substituting the resulting values of the module of the frequency characteristics (13) and the image of the input signal (15) in equation (7), we obtain an expression for the evaluation of the uncertainty of dynamic measurement of vibration acceleration in the spectral area

$$u_D(\omega)=\pi^{-1/2}\left(T^{-1}\int_0^\infty\frac{K_{MM}\omega^2F_0^2\left(\omega_0^2-\omega^2\right)^{-2}m^{-2}}{\omega^4-2\omega^2h_k^2+4\omega^2h^2+h_k^4}d\omega\right)^{1/2}. \tag{16}$$

To represent the characteristicsof the changes in the uncertainty in the dynamic measurement vibration acceleration in the time domain, which is caused by the inertial properties of the measuring transducer in its dynamic mode we must express a Fourier expression for inverse transformation in the form of (10).

Since expression (10) consists of real and imaginary parts, and in assessing the uncertainty we are interested in the amplitude value of dynamic uncertainty, expression (10) may now be written as

$$\left|u_D(t)\right|=\left[\left(\int_0^\infty\pi^{-1/2}u_D(\omega)\cos(\omega t)d\omega\right)^2+\left(\int_0^\infty\pi^{-1/2}u_D(\omega)\sin(\omega t)d\omega\right)^2\right]^{1/2}. \tag{17}$$

For the solution of equation (17) in the light of equation (16) we used the Maple 9 mathematical package [10]. Substituting into expressions (16) and (17) the values for the impact coeffici-ents given above, we obtained the amplitude value of the dynamic uncertainty of the measure-ment of vibration acceleration, which equals 0,095 m/s^2 when the frequency of the input signal of the vibration acceleration is 10 kHz, and time of observation of the vibration acceleration T=300 s. If the time of observation is increased to 600 s at the same frequency of the input signal of the vibration acceleration, the value of dynamic uncertainty decreases to 0,065 m/s^2.

At the minimum frequency of the input signal of the vibration acceleration of 6 kHz, and with an observation time of 300 s, the value of dynamic uncertainty is 0,194 m/s^2. If the observation period increased to 600 s at a frequency of input signal of the vibration acceleration of 6 kHz, the value of dynamic uncertainty is reduced to 0,137 m/s^2 . The nominal value of the signal for vibration acceleration of the bearings of the electric motor of the motor is 0,35 m/s^2 [3, 4]. Characteristics of the change of dynamic uncertainty of the measurement of vibration acceleration depending on the time variable which were obtained using the Maple 9 mathematical package [10] are presented in Figures 1, with the minimum value of the frequency of the input signal of the vibration acceleration of 6 kHz with observation times of 300 s and 600 s, are respectively.

Characteristics of the change of dynamic uncertainty of the measurement depending on the time variable which were obtained using the Maple 9 mathematical package [10] are presented in Figures 2, with the maxi-mum value of the frequency of the input signal of the vibration accelera-tion of 10 kHz with observation times of 300 s and 600 s, are respectively.

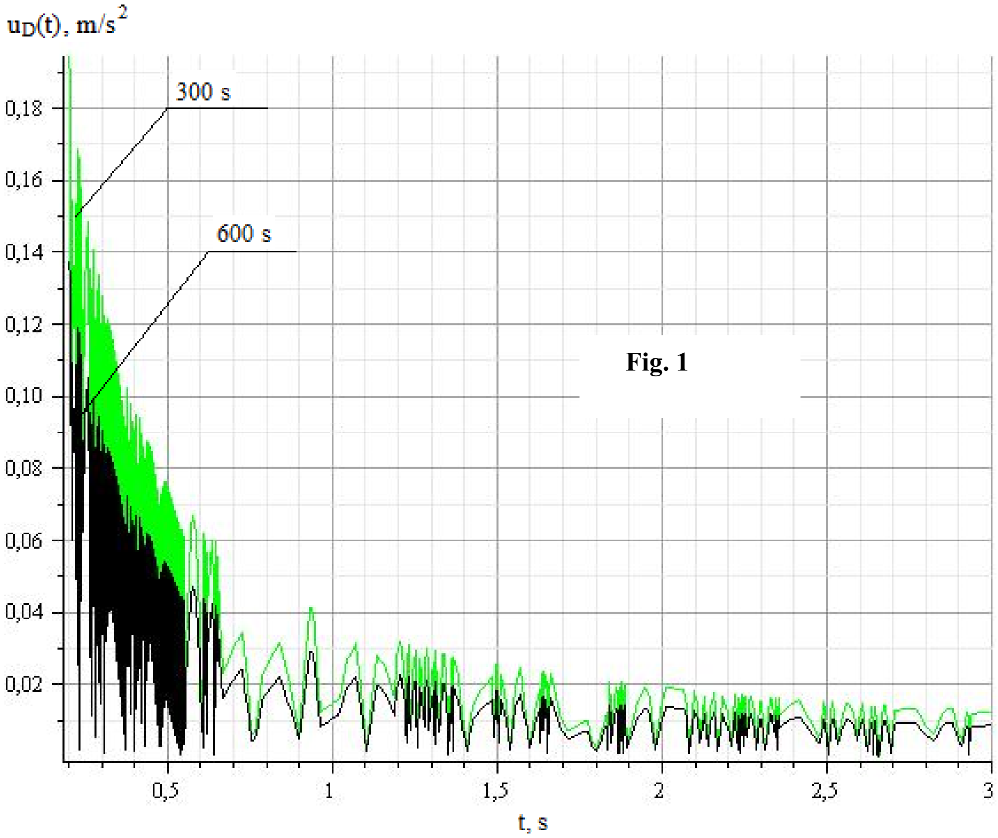

Fig. 1

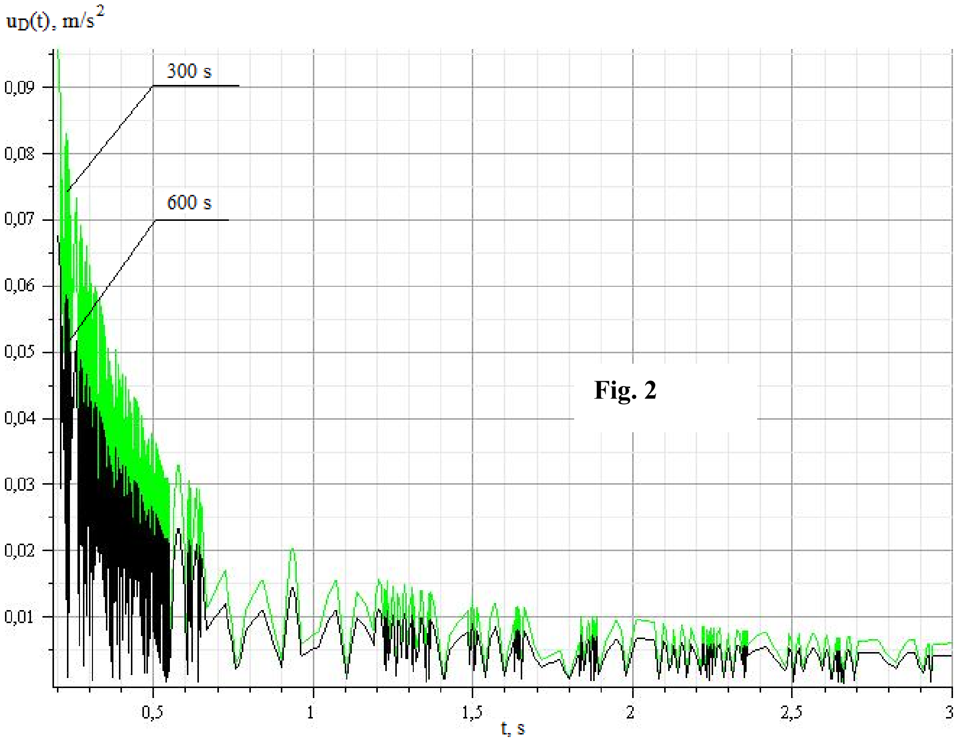

Fig. 2

Thus, based on the proposed spectral method of evaluation of uncertainty of dynamic measurements, the evaluation of the uncertainty of dynamic measurements of vibration acceleration of roller bearings of the electric motor was achieved. This was achieved based on mathematical models of the spectral function of the input signal of the vibration acceleration and frequency characteristics of the measurement transducer for the vibration acceleration. This resulted in obtaining the opportunity to take into account the values of dynamic uncertainties when assessing combined total uncertainty of the measurement of vibration acceleration.

Our research resulted in establishing, (Fig. 1, Fig. 2), the shorter the period of time of the observation of the signal of the vibration acceleration of a moving object, roller bearing in this case, the greater the value of the dynamic uncertainty of the measurement. Therefore, to reduce the impact of dynamic uncertainty of measurements in the assessment of the results of the measurement of vibration acceleration, we must ensure an observation time of at least 600 s.

Conclusions. The proposed spectral method of evaluating the uncertainty of dynamic measurements allows the calculation of the amplitude values of dynamic uncertainties, taking into account the international requirements for the evaluation of the quality of measurements - the concept of uncertainty. It helps to ensure the uniformity of measurements and enables comparison of the results of dynamic measurements made by different measuring devices and testing by different laboratories of leading countries. This method was tested when evaluating the dynamic uncertainty of the measurement of vibration acceleration of the roller bearing of the electric motor, which proved its validity and effectiveness.

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СПЕКТРАЛЬНЫЙ МЕТОД ОЦЕНКИ НЕОПРЕДЕЛЕННОСТИ ДИНАМИЧЕСКИХ ИЗМЕРЕНИЙ

А.Н. Василевский[1], докт.техн.наук, **М.Ю. Яковлев**[2], докт.техн.наук, **П. И. Кулаков**[1], докт.техн.наук
[1-]Винницкий национальный технический университет,
Хмельницкое шоссе, 95, Винница, 21021, Украина,
[2-]Национальная академия сухопутных войск имени гетьмана Петра Сагайдачного,
ул. Героев Майдана, 32, Львов, 79012, Украина,
e-mail: o.vasilevskyi@gmail.com

В статье предложен спектральный метод оценки динамической неопределенности измерительных приборов, который позволяет исследовать точность измерений в динамических режимах эксплуатации в частотной области, оценивать амплитудные значения динамической неопределенности на основе частотной характеристики и спектральной функции входного сигнала. Результаты были апробированы при оценивании неопределенности динамических измерений виброускорения электродвигателей. Установлено, что максимальное значе-

ние динамической неопределенности измерения виброускорения составляет 0,137 м/с^2 при времени наблюдения 600 с и номинальном значении сигнала виброускорения 0,35 м/с^2 на частоте 6 кГц. Библ. 10, табл. 1, рис. 2.

Ключевые слова: динамическая неопределенность средства измерения, оценка качества динамических измерений, спектральная функция, частотная характеристика, виброускорение.

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СПЕКТРАЛЬНИЙ МЕТОД ОЦІНЮВАННЯ НЕВИЗНАЧЕНОСТІ ДИНАМІЧНИХ ВИМІРЮВАНЬ

О.М. Васілевський[1], докт.техн.наук, **М.Ю. Яковлев**[2], докт.техн.наук, **П.І. Кулаков**[1], докт.техн.наук
[1-]Вінницький національний технічний університет,
Хмельницьке шосе, 95, Вінниця, 21021, Україна,
[2-]Національна академія сухопутних військ імені гетьмана Петра Сагайдачного,
вул. Героїв Майдана, 32, Львів, 79012, Україна,
e-mail: o.vasilevskyi@gmail.com

У статті запропоновано спектральний метод оцінювання динамічної невизначеності засобів вимірювань, який дозволяє досліджувати точність вимірювань у динамічному режимі роботи в частотній області, оцінювати амплітудні значення динамічної невизначеності на основі частотної характеристики та спектральної функції вхідного сигналу. Результати були апробовані під час оцінювання невизначеності динамічних вимірювань віброприскорення електродвигунів. Встановлено, що максимальне значення динамічної невизначеності вимірювання віброприскорення складає 0,137 м/с^{2}при часі спостереження 600 с та номінальному значенні сигналу віброприскорення 0,35 м/с^2 на частоті 6 кГц. Бібл. 10, табл. 1., рис. 2.

Ключові слова: динамічна невизначеність засобу вимірювання, оцінювання якості динамічних вимірювань, спектральна функція, частотна характеристика, віброприскорення.

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