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SPECTRUM ANALYSIS OF CURRENT IN SINGLE-PHASE HALF-BRIDGE INVERTER IN DOMAIN OF TWO VARIABLES

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The object of this paper is a spectrum analysis of a load current in a single-phase half-bridge inverter with PWM. The analysis is based on an extension of differential equations with one variable of time to partial differential equations with two time variables. The partial differential equation is solved with the use of the two dimensional Laplace transform. The obtained solution is described in the form of a double Fourier series. This result is transformed into such form that permits description as a spectrum in the domain of two discrete variables. References 6, figures 5.
Keywords: spectrum, inverter, extension, two dimensional Laplace transform.

Introduction. Let us consider processes in the load of the single-phase half-bridge inverter shown in Fig. 1. The inverter output voltage is often described with the help of a double Fourier series [2]. The output voltage $u(t)$ for trailing edge naturally sampled modulation [2, 3] in the domain of two variables can be presented by the use of time variables t and τ as shown in Fig. 2. This voltage can be formed by comparison of a sawtooth ramp voltage with a sinusoid voltage. It should be noted that further process analysis in [2] is realized in the domain of one variable.

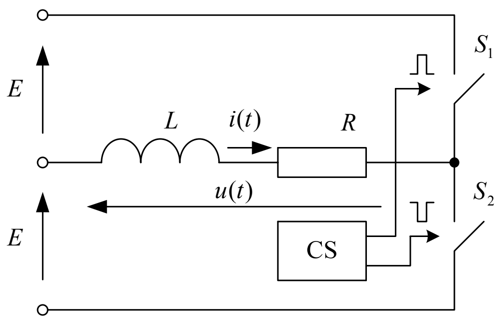

Fig. 1

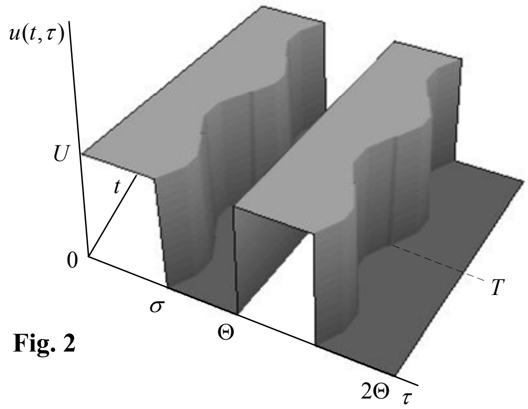

Fig. 2

In order to find a periodical steady-state current in the load of the inverter we expand a differential equation which describes processes in the domain of one variable of time to a partial differential equation which describes process in the domain of two independent variables of time [4,5]. After solving the partial differential equation with the help of a two dimensional Laplace transform one obtains a solution in the form of a double Fourier series. This result is then transformed in order to represent harmonics in the domain of two variables.

Mathematical model. Let us assume that switches S_1 and S_2 are ideal and that the load is linear. Processes in the load of the inverter are described by the differential equation

$$L\, di(t)/dt = -Ri(t) + u(t)\,, \tag{1}$$

where $i(t)$ is a current, $u(t)$ is the output voltage of the inverter.

The output voltage is the periodical in the domain of two variables of time $0 \le t \le T$, $0 \le \tau \le \Theta$. We expand the domain of the differential equation (1) from one independent variable of time t to two independent variables of time t and τ [4,5] in following way

$$L\, \partial i(t,\tau)/\partial t + L\, \partial i(t,\tau)/\partial \tau = -Ri(t,\tau) + u(t,\tau)\,. \tag{2}$$

In order to solve the differential equation (2) we use the two-dimensional Laplace transform [1]

$$F(s,q)=\int_0^\infty\int_0^\infty f(t,\tau)e^{-st-q\tau}d\tau dt .$$

Applying the two-dimensional Laplace transform to (2) one obtains the following

$$L\left(s+q\right)I(s,q)=-RI(s,q)+U(s,q)\,, \tag{3}$$

where $U(s,q)$ and $I(s,q)$ are images of the voltage $u(t,\tau)$ and current $i(t,\tau)$.

We assume that boundary conditions are equal to zero, i.e. $i(t,0)=i(0,\tau)=0$.

Solving (3) for $I(s,q)$ gives

$$I(s,q)=U(s,q)[L(s+q)+R]^{-1}, \tag{4}$$

where $U(s,q)=[(1-e^{-sT})(1-e^{-q\Theta})]^{-1}\int_0^T\int_0^\Theta u(t,\tau)e^{-st-q\tau}d\tau dt$.

The image $U(s,q)$ is calculated as follows

$$U(s,q)=\tilde{U}(s,q)[(1-e^{-sT})(1-e^{-q\Theta})]^{-1},$$

where $\tilde{U}(s,q)=Uq^{-1}\int_0^T\left(1-e^{-q(\sigma+K\cos\omega t)}\right)e^{-st}dt$, $\omega=2\pi/T$; U − a constant voltage that is equal to $2E$.

In order to find the steady-state current we calculate residues at singularity points of the image $I(s,q)$ $s_{\pm n}=\pm jn\omega$, $n=0,1,2,..$, $q_0^2=0$, $q_{\pm m}=\pm jm\Omega$, $m=0,1,2,...$, where $\Omega=2\pi/\Theta$.

We shall not take into consideration a constant value since the inverter output voltage must be zero. At points $q^2=0$, $s=\pm j\omega$ we obtain

$$i_{1,0}(t,\tau)=2\,\mathrm{Re}\lim_{s\to j\omega}\left\{(s-j\omega)\lim_{q\to 0}\left[\frac{d}{dq}\left(q^2I(s,q)e^{q\tau}\right)\right]e^{st}\right\}=\frac{2KU\cos(\omega t-\varphi_1)}{\Theta\sqrt{(R^2+\omega^2L^2)}}, \tag{5}$$

where $\varphi_1=\mathrm{arctg}(\omega L/R)$.

This result corresponds to a first harmonic of the output current. The calculation of residues at points $q=0$, $s=\pm jn\omega$ for $n=2,3,..$, yields the result that they are equal to zero.

Calculating residues at points $s=0$, $q=\pm jm\Omega$ for $m=1,2,..$ one obtains the following

$$i_{0,m}(t,\tau)=2\,\mathrm{Re}\lim_{s\to 0}\left\{s\lim_{q\to jm\Omega}\left[\left((q-jm\Omega)I(s,q)e^{q\tau}\right)\right]e^{st}\right\}=$$

$$=\frac{U\left[J_0(mK\Omega)\sin\left(m(\sigma-\tau)\Omega+\phi_m\right)\right]}{m\pi\sqrt{(R^2+m^2\Omega^2L^2)}}+\frac{U\sin\left(m\Omega\tau-\phi_m\right)}{m\pi\sqrt{(R^2+m^2\Omega^2L^2)}}, \tag{6}$$

where $J_0(mK\Omega)$ is the Bessel function of the first kind; $\phi_{0,m}=\mathrm{arctg}(m\Omega L/R)$.

Calculating residues at points $s=\pm jn\omega$, $n=1,2,..$, $q=\pm jm\Omega$ for $m=1,2,..$ we get

$$i_{n,m}(t,\tau)=2\,\mathrm{Re}\lim_{s\to jn\omega}\left\{(s-jn\omega)2\,\mathrm{Re}\lim_{q\to jm\Omega}\left[\left((q-jm\Omega)I(s,q)e^{q\tau}\right)\right]e^{st}\right\}=2UJ_n(mK\Omega)\times \tag{7}$$

$$\times\frac{\sqrt{R^2+(Ln\omega)^2}\cos(n\omega t-\sigma_{n,m})\sin\left(m(\sigma-\tau)\Omega+n\frac{\pi}{2}\right)+Lm\Omega\sin(n\omega t+\varphi_{n,m})\cos\left(m(\sigma-\tau)\Omega+n\frac{\pi}{2}\right)}{\pi mZ_{n,m}},$$

where $$Z_{n,m}=\sqrt{R^4+L^4\left[(n\omega)^2-(m\Omega)^2\right]^2+2(LR)^2\left[(n\omega)^2+(m\Omega)^2\right]},$$

$$\varphi_{n,m}=\mathrm{arctg}\frac{R^2+L^2(m^2\Omega^2-n^2\omega^2)}{2RLn\omega},\quad \sigma_{n,m}=\mathrm{arctg}\frac{Ln\omega\left[R^2+L^2(n^2\omega^2-m^2\Omega^2)\right]}{R\left[R^2+L^2(n^2\omega^2+m^2\Omega^2)\right]}.$$

The steady-state current corresponding to the solution (2) has the form

$$i(t,\tau)=\sum_{\substack{n=0\\m\neq 0}}^{\infty}\sum_{\substack{m=0\\n\neq 0}}^{\infty}i_{n,m}(t,\tau).$$

In this expression the constant value of the current is not taken into account since the output voltage of the inverter does not have a constant component.

Spectrum Analysis. The obtained results are presented in the form of a double Fourier series [6]

$$i(t,\tau)=\sum_{n,m=0}^{\infty}\left[A_{n,m}\cos(n\omega t+m\Omega\tau+\alpha_{n,m})\right]+\sum_{n,m=1}^{\infty}\left[B_{n,m}\cos(n\omega t-m\Omega\tau+\beta_{n,m})\right],$$

where coefficients

$$A_{n,m}=\mu_{n,m}abs(a_{n,m}),\ \alpha_{n,m}=\arg(a_{n,m}),\qquad B_{n,m}=abs(b_{n,m}),\ \beta_{n,m}=\arg(b_{n,m}),$$

$$a_{n,m}=\frac{2}{T\Theta}\int_0^T\int_0^\Theta i(t,\tau)e^{-j(n\omega t+m\Omega\tau)}d\tau dt,\qquad b_{n,m}=\frac{2}{T\Theta}\int_0^T\int_0^\Theta i(t,\tau)e^{-j(n\omega t-m\Omega\tau)}d\tau dt,$$

$$\mu_{n,m}=\begin{cases}0{,}5, & \text{for } n=0, m=0,\\ 1, & \text{for others } n \text{ and } m.\end{cases}$$

After mathematical manipulations of expressions (5–7) one obtains the following:

$$A_{0,m}=\frac{U\left[1-\cos(m\sigma\Omega)J_0(mK\Omega)\right]}{m\pi\sqrt{R^2+(L\omega m)^2}},\quad \alpha_{0,m}=\text{arctg}\frac{-R-J_0(mK\Omega)\left[-R\cos(m\sigma\Omega)+Lm\Omega\sin(m\sigma\Omega)\right]}{-Lm\Omega+J_0(mK\Omega)\left[Lm\Omega\cos(m\sigma\Omega)+R\sin(m\sigma\Omega)\right]},$$

$$\text{or}\qquad \alpha_{0,m}=\text{arctg}(R/Lm\Omega)-\pi\qquad \text{for } \sigma=\Theta/2,$$

$$A_{n,m}=\frac{U\left|J_n(mK\Omega)\right|\sqrt{R^2+L^2(n\omega-m\Omega)^2}}{m\pi Z_{n,m}},\qquad B_{n,m}=\frac{U\left|J_n(mK\Omega)\right|\sqrt{R^2+L^2(n\omega+m\Omega)^2}}{m\pi Z_{n,m}},$$

$$\alpha_{n,m}=\text{arctg}\left(\frac{R}{L(n\omega+m\Omega)}\right)-\frac{n\pi+2m\sigma\Omega}{2}+\arg\left[J_n(mK\Omega)\right],\tag{8}$$

$$\beta_{n,m}=\text{arctg}\left(\frac{L(m\Omega-n\omega)}{R}\right)+\frac{n\pi+2m\sigma\Omega}{2}-\frac{\pi}{2}+\arg\left[J_n(mK\Omega)\right].\tag{9}$$

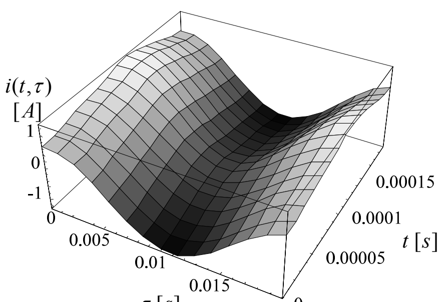

Fig. 3

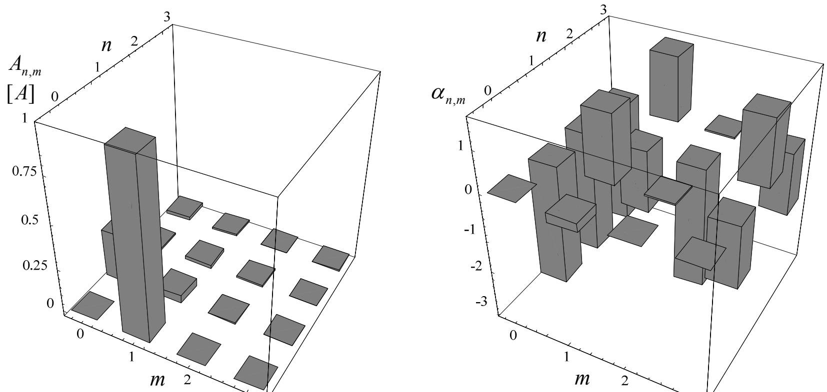

Fig. 4

In further calculations expressions (8–9) are reduced to the range between π and $-\pi$.

Results of calculation. Let us find the steady-state process for element values: $R=10\,\Omega$; $L=0{,}7\ m\text{H}$; $E=100\ \text{V}$; $T=20\ ms$; $\Theta=T/100\ ms$; $K=0{,}1\Theta\ ms$; $\sigma=\Theta/2\,ms$. The time waveform of the steady-state current of the load in the domain of two time variables for $n,m=1,2,3$ are shown in Fig. 3.

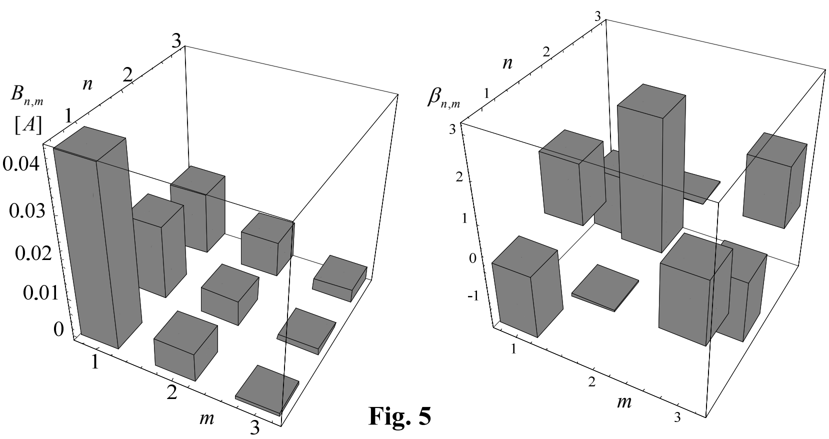

Fig. 5

The spectrum of this current is represented in Fig. 4-5.

As one would expect the first harmonic dominates in the steady-state current.

Conclusions. In this paper a steady-state current in the load of an inverter with a sinusoidal PWM has been presented. In order to find the steady-state process an ordinary differential equation with one variable of time has been extended to a partial differential equation with two time variables. The obtained equation has been solved by the use of the two-dimensional Laplace transform and the result has been presented in the form of a double Fourier series. The expression for the spectrum is obtained by rearranging expressions for the double Fourier series. The spectra in the domain of two variables are represented using four Fourier coefficients, i.e., two amplitudes and two phases. It has been shown that the amplitude of a spectrum decreases with increasing harmonic numbers.

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СПЕКТРАЛЬНЫЙ АНАЛИЗ ТОКА В ОДНОФАЗНОМ ПОЛУМОСТОВОМ ИНВЕРТОРЕ В ПРОСТРАНСТВЕ ДВУХ ПЕРЕМЕННЫХ

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Целью статьи является гармонический анализ тока нагрузки в однофазном полумостовом инверторе с ШИМ. Анализ основывается на расширении дифференциальных уравнений, описываемых одной переменой времени, до уравнений в частных производных, описываемых двумя переменными времени. Дифференциальное уравнение в частных производных решается с помощью двумерного преобразования Лапласа. Полученное решение представляется в форме двойного ряда Фурье. Полученный результат преобразуется в форму, которая позволяет описание в виде спектра в пространстве двух дискретных переменных. Библ. 6, рис. 5.

Ключевые слова: спектр, инвертор, расширение, двумерное преобразование Лапласа.

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СПЕКТРАЛЬНИЙ АНАЛІЗ СТРУМУ В ОДНОФАЗНОМУ НАПІВМОСТОВОМУ ІНВЕРТОРІ В ПРОСТОРІ ДВОХ ЗМІННИХ

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Метою статті є гармонійний аналіз струму навантаження в однофазному напівмостовому інверторі з ШИМ. Аналіз базується на розширенні диференціальних рівнянь, що описуються однією змінною часу до рівнянь у частинних похідних, описуваних двома змінними часу. Диференціальне рівняння у частинних похідних вирішується за допомогою двовимірного перетворення Лапласа. Отримане рішення представляється у формі подвійного ряду Фур'є. Отриманий результат перетворюється на форму, яка дозволяє опис у вигляді спектра в просторі двох дискретних змінних. Бібл. 6, рис. 5.

Ключові слова: спектр, інвертор, розширення, двовимірне перетворення Лапласа.

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