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# CAPACITOR BANKS SELECTION IN RADIAL DISTRIBUTION NETWORKS BY COMBINED ALGORITHM

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*This paper presents the combined algorithm for determination of locations and sizes of capacitor banks in the radial distribution networks for practical minimization of power losses and costs of these losses and banks. The method of concessions and modified PSO-method are applied in the proposed algorithm. Additionally the technicalcoenosis criterion is applied for the specific account of technical and economic factors. The proposed approach allows to obtain the efficient practical solutions and increased stabilization of voltage profile in the radial distribution network.*
References 9, tables 4, figures 2.

***Keywords:*** capacitor bank, combined algorithm, radial distribution network, method of concessions, PSO-method, technicalcoenosis criterion.

**I. Introduction.** A significant part of active power losses in radial distribution networks (RDN) occurs due to the flow of reactive currents and therefore the application of capacitor banks (CB) for reactive power compensation is very important.

In this paper, the combined algorithm of CB selection (sizing and placement in distribution network) is developed as a combination of traditional optimization criteria, i.e. power losses and investment cost minimization, and additional technical-coenosis criterion. Also the method of concessions and modified particle swarm optimization (PSO) method are applied.

The comprehensive survey of the literature from the last decade that has focused on the various heuristic optimization techniques applied to CB selection in RDN was presented in [6]. This analysis of published papers shows that PSO is the most popular technique applied because of its advantages, which include simple implementation, small computational load and fast convergence.

**II. Problem description and solution method.** Two main criteria for CB placement and sizing are usually considered as the following two objective functions used to minimize yearly active losses and costs of these losses and shunt CB installation

$$\Delta P_{\Sigma} = \sum_{i=1}^{q} \Delta P_i \to \min, \qquad (1)$$

where $\Delta P_{\Sigma}$ – total active power losses and $\sum_{i=1}^{n} \Delta P_i$ – losses in the $i$ section/node distribution system, $q$ – number of nodes.

$$B_{\min} = K_p \Delta P_{\Sigma} + \sum_{i=1}^{nc} K_{ci} Q_{ci} \to \min, \qquad (2)$$

where $K_p$ – annual cost per unit of active losses $\Delta P_{\Sigma}$ ($/(kW*year)), $K_{ci}$ – annual cost of reactive power $Q_{ci}$ corresponding $i$-th CB ($/(kVAr*year)), $nc$ – total number of CB to be installed.

The function (1) may have many constraints, but in this paper we are considering only one of them, which is the voltage profile constraint: voltage magnitude at each node must be maintained within the limits:

$$U_{\min} \le | \ U_i \ | \le U_{\max}, \qquad (3)$$

where $U_{\min}$, $U_{\max}$ – minimum and maximum permissible voltage.

The analysis of problem solutions by using only traditional criteria (1) and (2) shows that they do not always give a positive result in the presence of such circumstances as inadequate pricing in conditions of economic crises, the technical information imprecision, specific features of the networks requirements and sometimes in terms of engineering logic In these conditions the additional criteria, approaches and analysis methods should be applied.

**III. Solution algorithm.** In order to overcome the above mentioned problems in this paper the application of well-known method of concessions is proposed. It allows to find theoretically only the quasi-

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optimal solution, which can be interpreted as rational and efficient for practical purposes. For the objective function minimum value can be assigned the concession $\delta$,%, as its permissible decline for practical purposes, accuracy requirements etc.

The proposed algorithm for solving the CB selection problem consists of the following procedures.

1. According to the results of PSO procedures application, for each swarm particle $m$ the vector of $n$ CB (in the nodes of RDN) conductivities $\mathbf{y}_1$ is formed. The basic set of solutions $\mathbf{Y}_1$ is formed by these vectors $\mathbf{y}_1$ by criterion (1) with constraint (3). The set of solutions $\mathbf{Y}_1$ is formed of $\mathbf{y}_1$ vectors, which are in the range of allowable concession $\delta_1$ of minimum losses in RDN

$$\mathbf{y}_1(y_1, y_2, ..., y_n) : \Delta P_\Sigma(y_1, y_2, ...y_n) \le (1+\delta_1)\Delta P_{\Sigma\min}.$$

2. From the solutions set $\mathbf{Y}_1$ we form another set of solutions $\mathbf{Y}_2 \subset \mathbf{Y}_1$, which is composed of vectors $\mathbf{y}_2$ by criterion (2) and its allowable concession $\delta_2$

$$\mathbf{y}_2(y_1, y_2, ..., y_n) : B(y_1, y_2, ...y_n) \le (1+\delta_2)B_{\min}.$$

The set $\mathbf{Y}_2$ also can be defined by PSO-method separately from the set $\mathbf{Y}_1$. The resulting set can be defined as the intersections of corresponding solution sets

$$\mathbf{Y}_2' = \mathbf{Y}_1 \cap \mathbf{Y}_2 \ne \varnothing.$$

In the case of empty set $\varnothing$ the allowable concessions $\delta_1$, $\delta_2$ then sequentially have to be changed and calculations of $\mathbf{Y}_2$ or $\mathbf{Y}_2'$ have to be repeated.

3. From the set $\mathbf{Y}_2$ (or $\mathbf{Y}_2'$, in case when intersection method is applied) we choose the best solution by additionally applied the Technical-coenosis (TC) criterion. This criterion establish the conformity of ranged values of CB conductivities to hyperbolic (power law) *H*-distribution [4] from the set of solutions $\mathbf{Y}_{TC} \subset \mathbf{Y}_2$. According to TC-approach the electrotechnical systems can be considered as TC [4], where a certain “family” of electrical products (for example CB in our case) can be arranged by particular species-forming parameters [2,4]

$$C(r) = C_1 / r^\beta, r = 1,2,...,d, \tag{4}$$

where $r$ – parametric rank; $C_1$ – parameter value of individual with a rank $r = 1$; $\beta$ – rank factor characterizing the degree of curve distribution steepness, $d$ – total number of ranks.

The values of the first rank and rank factor are very important, since together they define the range of CB in RDN and their quantity: $C_1 \to \min$ with the condition of the best approximation to hyperbola with $0,5 \le \beta \le 1,5$, which corresponds to highly effective state of TC [2,4]. At the same time, to determine the most acceptable H-distribution of the first rank and rank factor we can apply the concession $\delta_3$ for $C_1$ parameter.

The additional TC-criterion for vector of CB conductivities $\mathbf{y}(y_1, y_2, ..., y_n)$ have to be defined by (5) for the following conditions:

– similarity of normalized target vector y to normalized vector in the form of a hyperbolic dependence;

– minimization of the magnitude of CB conductivity of the first rank $\mathbf{y}_{(r=1)}$.

$$\begin{cases} y_{TC}^{nor}(r) = y_{TC(r=1)}^{nor} / r^\beta \\ d_E(\mathbf{y}_{TC}^{nor}, \mathbf{y}_{rank}^{nor}) < \delta_3 \\ y_{TC(r=1)}^{nor} \to \min, \end{cases} \tag{5}$$

where $d_E(\mathbf{y}_{TC}^{nor}, \mathbf{y}_{rank}^{nor})$ – the Euclidean distance between normalized vectors, $\delta_3$ – permissible concessions of the distance between normalized vectors, vector $\mathbf{y}_{rank}^{nor}$ – vector $\mathbf{y}^{nor}$ after ranking, $\mathbf{y}_{rank}^{nor} = \text{sort}(\mathbf{y}^{nor}, descend)$, operator ‘sort’ sorting vector $\mathbf{y}^{nor}$ in the order specified by mode, mode 'descend' indicates descending order [5]. The unit length of normalized vectors is $|\mathbf{y}_{TC}^{nor}|=1$, $|\mathbf{y}_{rang}^{nor}|=1$. The concession $\delta_3$ allows to estimate the similarity of searching vector y to vector in the form of a hyperbolic dependence in accordance with TC-criterion.

It is known that in PSO-method each particle moves in the multi-dimensional set of solutions [1, 8]. Moreover, the particle position $x_k$ is determined by the interaction with all other particles of swarm. All particles move to the global extremum; the process occurs at the end of the relevant performance of breakpoint criteria. One of the classical optimization algorithms of continuous nonlinear functions, named in [7] as "A modified particle swarm optimizer" is used in this paper. To calculate the position and velocity for $n$-dimension of $m$ particles, the following expressions are used at each $k$ iteration

$$V_{k+1} = \omega V_k + a_1 R(Pbest_k - x_k) + a_2 R(Gbest_k - x_k), \qquad x_{k+1} = x_k + V_{k+1},$$

where $x_k$ – current position of particle, $V_k$ – particle velocity, $a_1$, $a_2$ – constants of acceleration, $Pbest$ – best position found by particle, $Gbest$ – best position found by all particles, $\omega$ – inertia factor, $R$ – random number between 0 and 1.

The distinctive feature of this PSO-algorithm is the ability to control the speed of convergence process using the number of particles in the swarm and the inertia factor. After PSO-method modification, intermediate results of each iteration computation $x_k$ are not discarded, but are grouped in the appropriate set of solutions. In addition, at the beginning of the process a suitable total number of particles and the ratio of inertia are chosen.

As a result of the modified PSO-method application, a corresponding set of solutions is formed. In this set, each $m$ particle corresponds to the $n$-dimensional vector of CB conductivities $\mathbf{y}(y_1, y_2, ..., y_n)$ consisting of $n$ CB, which has to be installed in the corresponding feeder nodes.

The application of criteria (1), (2), (4) and voltage profile constraint (3) with the method of concessions and modified PSO-method gives the combined algorithm of CB selection for radial distribution networks. The fixed CB capacities are defined by using the proposed algorithm for the regime of maximum load in RDN. For the determination of CB steps of regulation the daily schedules, controller switching codes, and other engineering methods have to be used.

**IV. Test results.** The proposed algorithm was used for CB selection in the radial network, with a rated voltage 23 kV, which consists of 27 nodes ($q$=27) and 21 loads ($n$=21), considered in [9]. Permissible voltage [0,95 – 1,05]. The calculation was carried out in the absence of harmonic current sources for a period of 8760 hours per year. The energy losses cost is taken as 0,06 \$/KW•h. Technical data of conductors are taken from [9]. The base values of power and voltage for calculations are the following: $S_b$= 24 MVA, $U_b$=23 kV. The types of wires were chosen previously and this procedure is not described in the example. The conductor selection problem, as it was done in some studies [3, 9], is not considered. The power values of CB are considered with discreteness 150 kVAr. The CB $j$ sizes (reactive power) $Q$ (kVAr) and accepted costs $B_c$ \$/(kVAr*year) are taken as presented in Table I.

**Table I**

| $j$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| $Q$ | 150 | 300 | 450 | 600 | 750 | 900 | 1050 | 1200 |
| $B_c$ | 0,5 | 0,35 | 0,253 | 0,22 | 0,276 | 0,183 | 0,228 | 0,17 |
| $j$ | 9 | 10 | 11 | 12 | 13 | 14 | 15 | |
| $Q$ | 1350 | 1500 | 1650 | 1800 | 1950 | 2100 | 2250 | |
| $B_c$ | 0,207 | 0,2 | 0,19 | 0,187 | 0,21 | 0,176 | 0,197 | |

As the first iteration of the calculation, the CB with fixed capacities (rated power) were installed in each node with load. Thus, the vector of CB conductivities $\mathbf{y}(y_1, y_2, ..., y_n)$ had $n$=21 variables.

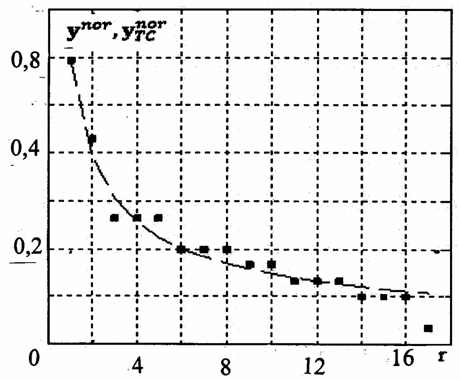

**Fig. 1**

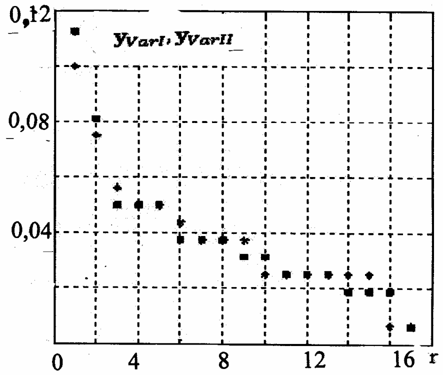

**Fig. 2**

Subsequently, as a result of set of solutions determination, some CB took zero values of the rated power.

The normalized vector of ranked CB conductivities $\mathbf{y}^{nor}_{rank}$ (marked by squares) is shown in Figure 1. It is similar to the normalized (approximated) vector in the form of a hyperbolic dependence $\mathbf{y}_{TC}^{nor}$ (marked by continuous line), which has a low value of first rank $y_{TC(r=1)}^{nor}$ =0,5919 and rank factor $\beta$ =0,605.

The comparison of CB reactive power QC for two variants respectively is based on the following criteria:

– Variant I: Criteria (1) and (2) are applied,
– Variant II: Criteria (1), (2) and (5) are applied.

The Figure 2 shows the target vector y of ranked CB conductivities (marked by squares), which is similar to normalized vector in the form of a hyperbolic dependence $\mathbf{y}_{\mathrm{var}\,I}$ for Variant I and vector $\mathbf{y}_{\mathrm{var}\,II}$ for Variant II (marked by asterisks), which corresponds to the minimum cost $B_{\min}$ after the corresponding conductivities ranking. These variants of CB placement and sizing are shown in Table II.

**Table II**

| *q* | ***y*, [p.u.]** | | *q* | ***y*, [p.u.]** | | *q* | ***y*, [p.u.]** | | *q* | ***y*, [p.u.]** | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | ***Criteria of Variant*** | | | ***Criteria of Variant*** | | | ***Criteria of Variant*** | | | ***Criteria of Variant*** | |
| | ***I*** | ***II*** | | ***I*** | ***II*** | | ***I*** | ***II*** | | ***I*** | ***II*** |
| 1 | 0 | 0 | 8 | 0,0375 | 0,0375 | 15 | 0 | 0 | 22 | 0,0063 | 0,0063 |
| 2 | 0 | 0 | 9 | 0 | 0 | 16 | 0,050 | 0,050 | 23 | 0,025 | 0,0187 |
| 3 | 0 | 0 | 10 | 0,0563 | 0,0500 | 17 | 0 | 0 | 24 | 0,025 | 0,025 |
| 4 | 0,0750 | 0,0813 | 11 | 0,100 | 0,1125 | 18 | 0,0438 | 0,050 | 25 | 0,025 | 0,0187 |
| 5 | 0 | 0 | 12 | 0 | 0 | 19 | 0,025 | 0,025 | 26 | 0,025 | 0,0313 |
| 6 | 0,0375 | 0,0313 | 13 | 0,050 | 0,0375 | 20 | 0 | 0 | 27 | 0 ,0063 | 0,0187 |
| 7 | 0 | 0 | 14 | 0,0375 | 0,0375 | 21 | 0,025 | 0,025 | - | - | - |

The results of CB selection by variants I and II are shown in Table III. In comparison with the case of Variant I, the active losses $\Delta P$ in the case of Variant II icreased by 0,16% and the cost increased by 0,18%. When using TC-criterion in case of Variant II, the slight increase of cost allows to use this criterion for practical purpose of CB selection.

**Table III**

| *Applicable criteria* | *Criteria Of Var I* | *Criteria of Var II* |
|---|---|---|
| Active losses (p.u.) | 0,03131 | 0,03136 |
| Cost B (usa/year) | 197556 | 197907 |
| Cost CB (usa) | 3172 | 3274 |
| Magnitude of conductivity of CB of the first rank $y_{(r=1)}$ [p.u.] | 0,1 | 0,1125 |

In the Column 3 of Table IV the worst case of CB disconnection is presented. In this case the maximum quantity of nodes, with voltage deviation less than 0,95 p.u., was found. It is obvious that in the case of Variant II the result is better. All investigated negative voltage deviations was greater than value $U$<0,92.

The results show that the proposed combined algorithm (Variant II) allows to obtain the reduction of quantity of nodes with voltage deviation less than 0,95 p.u. in all cases of each one CB disconnection. At the same time active losses and cost slight increase. Combined algorithm is useful for practical purposes.

**V. Conclusion.** The combination of traditional criteria of yearly active losses and costs minimization with additional TC-criterion was proposed in order to obtain the specific solution for CB sizing and placement in RDN. The method of concessions was used to solve the multi-criteria problem in conjunction with PSO-method. The proposed algorithm allows to find the vector of ranked CB conductivities in correspondence with specific hyperbolic (power law) *H*-distribution according to the considered TC-approach.

**Table IV**

| 1 | 2 | | 3 | |
|---|---|---|---|---|
| *U*, [p.u.] | The quantity of nodes with voltage deviation less than 0,95 p.u. in all cases of each one CB disconnection | | The quantity of nodes with voltage deviation less than 0,95 p.u. in the worst case of one CB disconnection | |
| | *Criteria of Var I* | *Criteria of Var II* | *Criteria of Var I* | *Criteria of Var II* |
| *U*<0,95 | 100 | 82 | 19 | 15 |
| *U*<0,94 | 23 | 19 | 12 | 8 |
| *U*<0,93 | 6 | 2 | 6 | 2 |

In comparison with a traditional one, the application of the proposed algorithm allows to improve the voltage profile stabilization by decreasing the quantity of nodes with voltages, which are out of permissible range when single capacitor bank is alternately disconnected. The results show that proposed algorithm of CB placement and sizing in radial distribution networks can be quite effective for practical purposes.

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## ВИБІР МІСЦЬ ВСТАНОВЛЕННЯ КОНДЕНСАТОРНИХ УСТАНОВОК У РАДІАЛЬНИХ РОЗПОДІЛЬНИХ МЕРЕЖАХ

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*Для мінімізації електричних втрат, вартості електричних втрат та конденсаторних установок (КУ) представлено комбінований алгоритм для вибору потужності і місць розташування КУ у розподільних мережах із радіальною конфігурацією. У запропонованому алгоритмі використовуються метод поступок та модифікований PSO-метод. Додатково введено техноценологічний критерій: приналежність ранжованих змінних до класу нелінійних показових послідовностей. Запропонований підхід дозволяє отримувати ефективні практичні рішення та підвищену стабілізацію профілю напруги у радіальній розподільній мережі. Показано позитивні результати використання запропонованого алгоритму, зокрема зменшення кількості вузлів з напругою, нижчою за нижній поріг, у разі відключенні будь-якої КУ.* Бібл. 9, табл. 4, рис. 2.

***Ключові слова***: конденсаторна установка, комбінований алгоритм, радіальна розподільна мережа, метод поступок, PSO-метод, техноценологічний критерій.

## ВЫБОР МЕСТ УСТАНОВКИ КОНДЕНСАТОРНЫХ УСТАНОВОК В РАДИАЛЬНЫХ РАСПРЕДЕЛИТЕЛЬНЫХ СЕТЯХ

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*Для минимизации электрических потерь, стоимости электрических потерь и конденсаторных установок (КУ) представлено комбинированный алгоритм для выбора мощности и мест расположения КУ в распределительных сетях с радиальной конфигурацией. В предложенном алгоритме используются метод уступок и модифицированный PSO-метод. Дополнительно введено техноценологичный критерий: принадлежность ранжированных переменных к классу нелинейных показательных последовательностей. Предложенный подход позволяет получить эффективные практические решения и повышенную стабилизацию профиля напряжения в радиальной распределительной сети. Показаны положительные результаты использования предложенного алгоритма, в частности, уменьшение количества узлов с напряжением ниже нижнего порога, при отключении любой КУ.* Библ. 9, табл. 4, рис. 2.

***Ключевые слова:*** конденсаторная установка, комбинированный алгоритм, радиальная распределительная сеть, метод уступок, PSO-метод, техноценологический критерий.

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